

The Mathematics of Angry Birds

NCTM 2013

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The Main Objectives

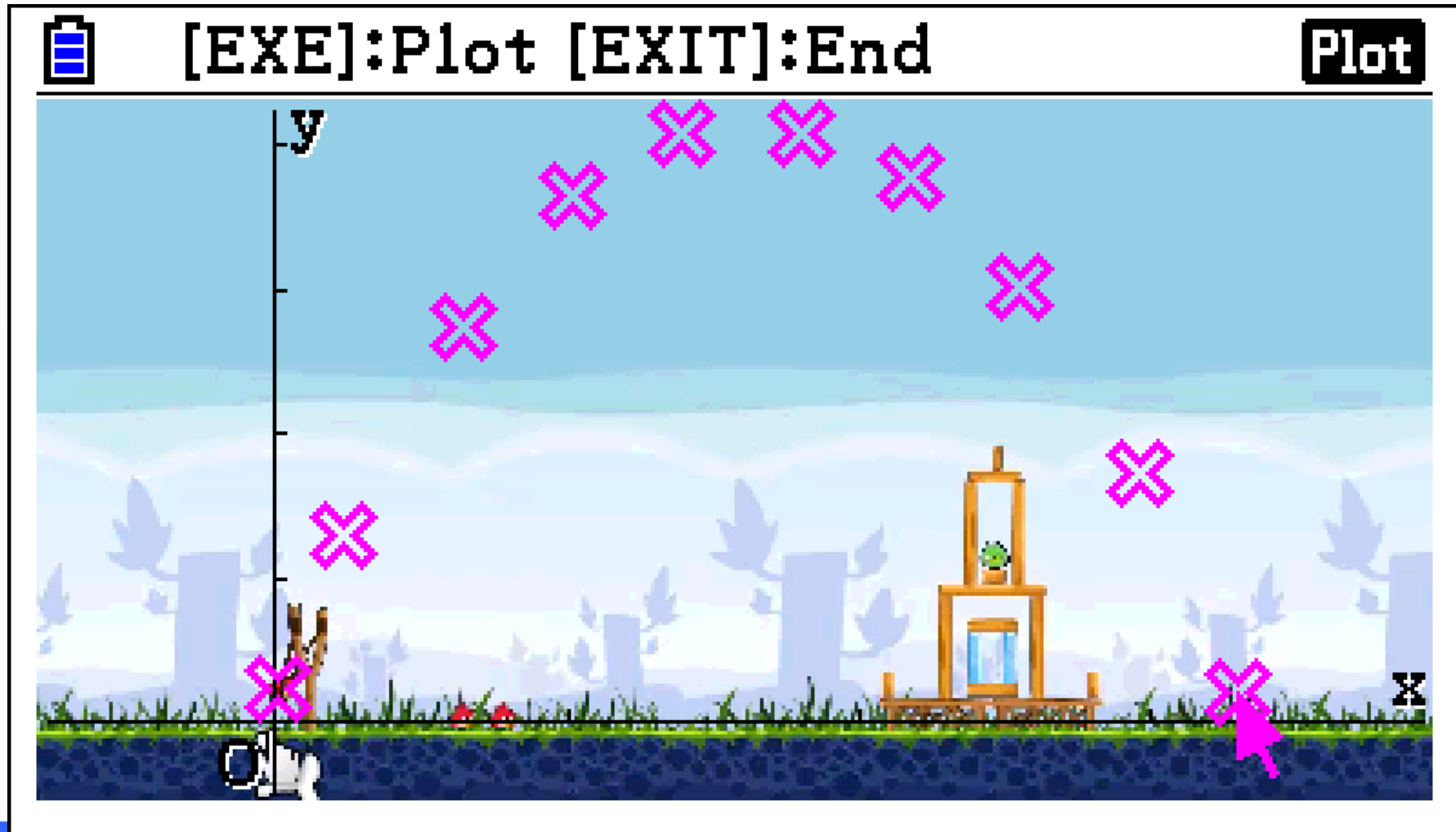
- Use data from the flight of an angry bird to develop models for the motion
- Explore the major variables of angle and initial velocity
- Explore the parametric relation

$$x = (v_0 \cos \theta)t + x_0$$

$$y = -0.5gt^2 + (v_0 \sin \theta)t + y_0$$



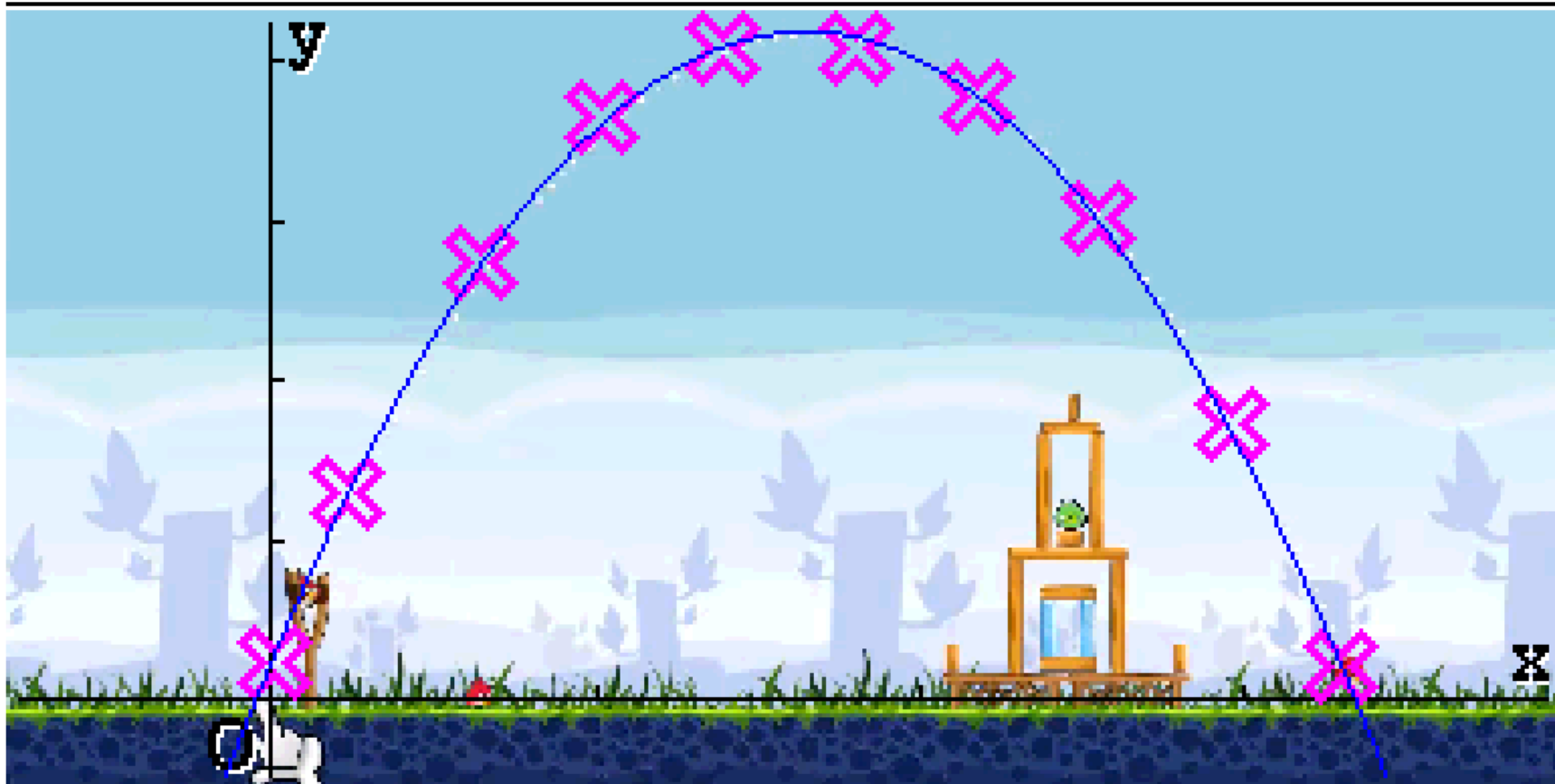
A Demonstration of Data Collection



Create a Model by Regression



Press [OPTN]



More About These Coefficients Later!



QuadReg

$$a = -0.0145123$$

$$b = 2.39544602$$

$$c = 5.98181537$$

$$r^2 = 0.99938941$$

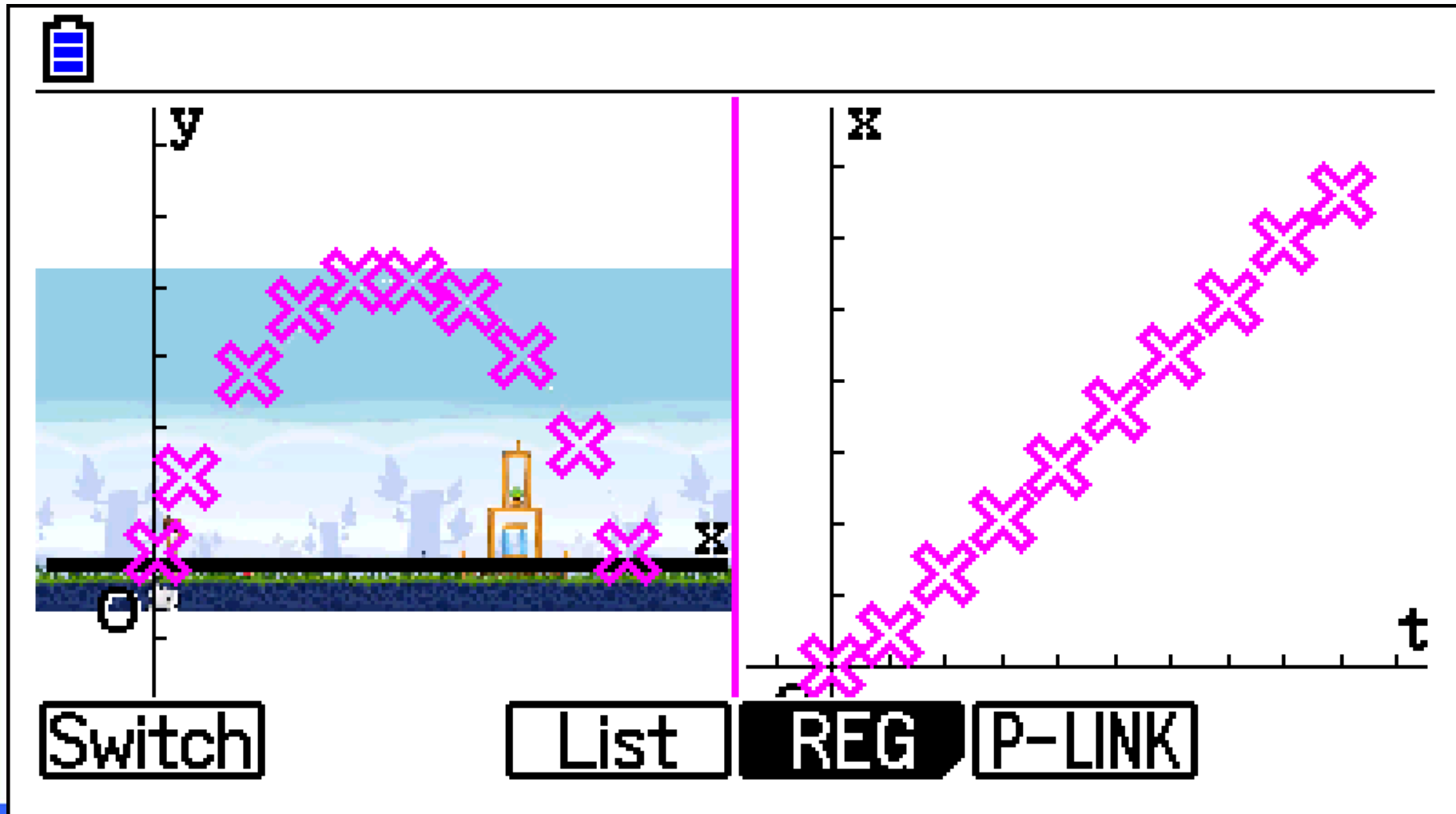
$$MSe = 1.1348274$$

$$y = ax^2 + bx + c$$

COPY

DRAW

Let's Explore Horizontal Position vs. Time



Interpret Both Coefficients of the Model



LinearReg (a+bx)

a = -4.4002886

b = 18.866378

r = 0.99933381

r² = 0.99866807

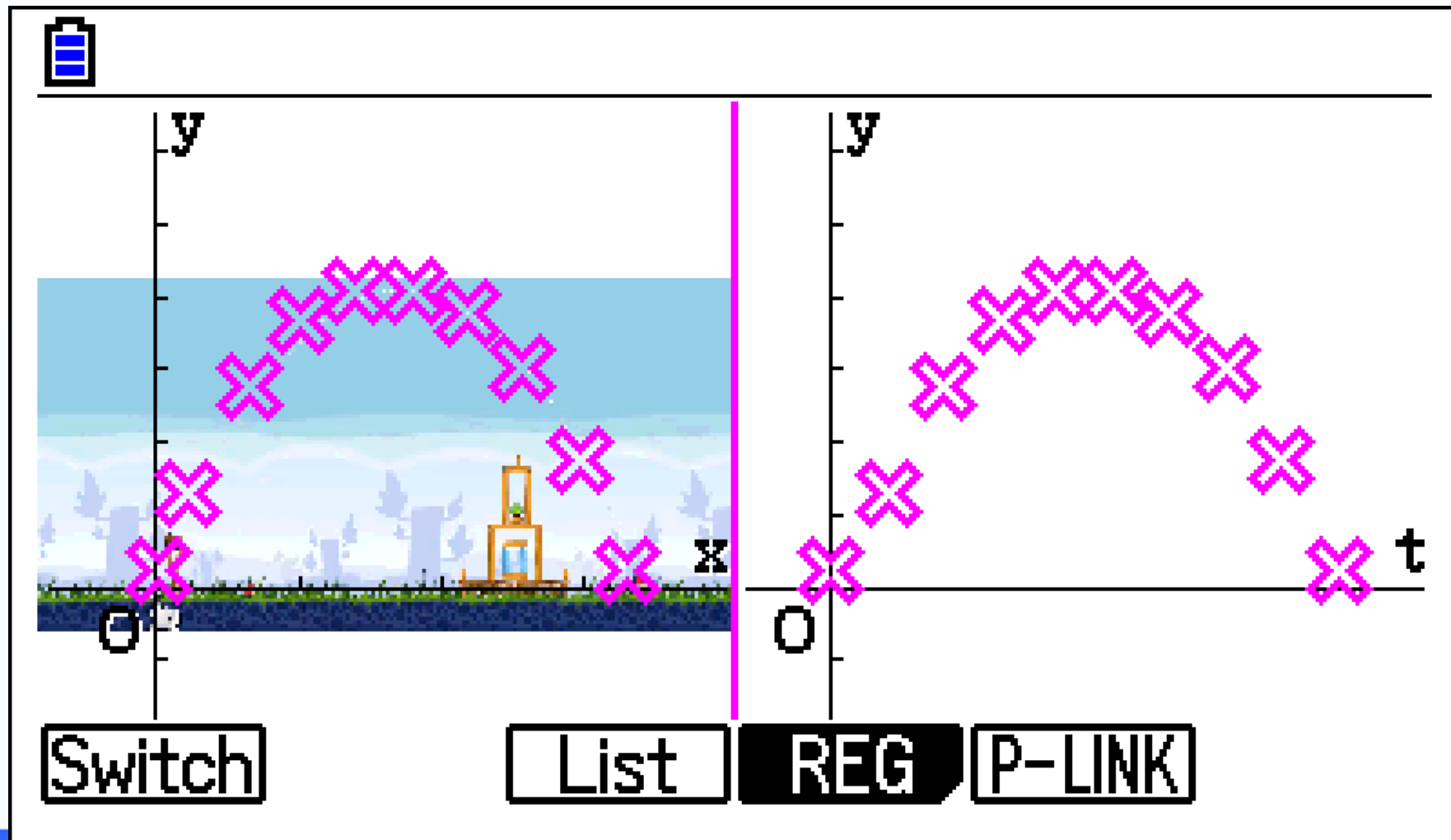
MSe = 4.89551753

$x = a + bt$

COPY

DRAW

Let's Explore Vertical Position vs. Time



Interpret the Coefficients of this Model



QuadReg

$$a = -4.9358911$$

$$b = 45.1151026$$

$$c = -0.2461388$$

$$r^2 = 0.99177329$$

$$MSe = 15.2901893$$

$$y = at^2 + bt + c$$

COPY DRAW

Let's Explore the Angle and Initial Velocity

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$

- There are several ways to compute the values

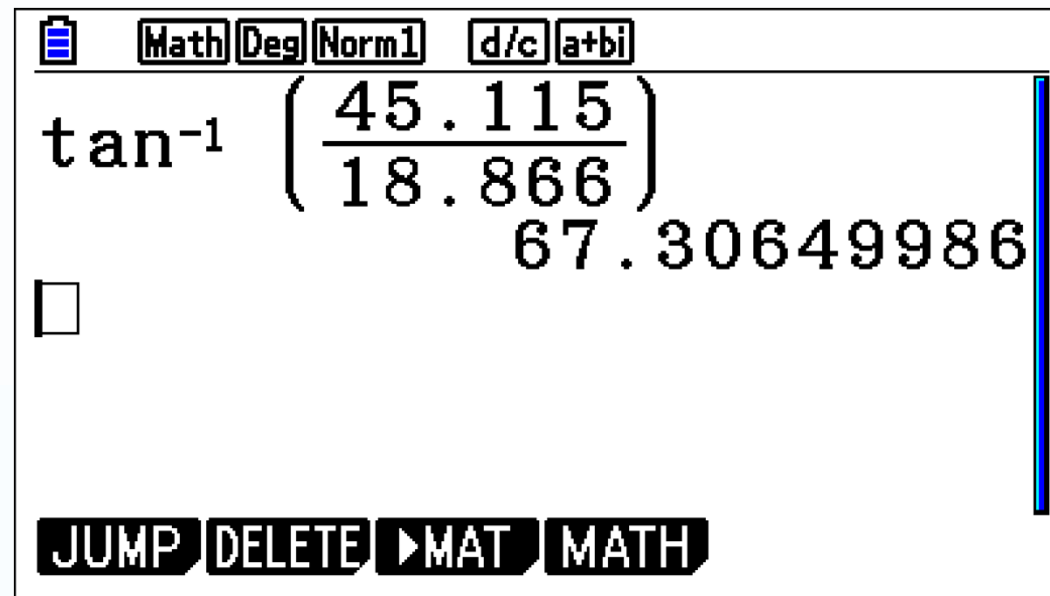


Method 1 - Division

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$

$$\tan(t) = 45.115/18.866$$



Method 1 - Substitution

$$v_0 \cos(t) = 18.866$$

$$v_0 = 18.866 / \cos(t)$$

The image shows a Casio calculator screen with the following display and controls:

- Mode indicators: Math, Deg, Norm1, d/c, a+bi
- Function key: \tan^{-1}
- Input: $\left(\frac{48.90080961}{18.866} \right)$
- Result: 67.30649986
- Input: 18.866
- Function key: \cos
- Label: Ans
- Result: 48.90080961
- Bottom buttons: JUMP, DELETE, MAT, MATH

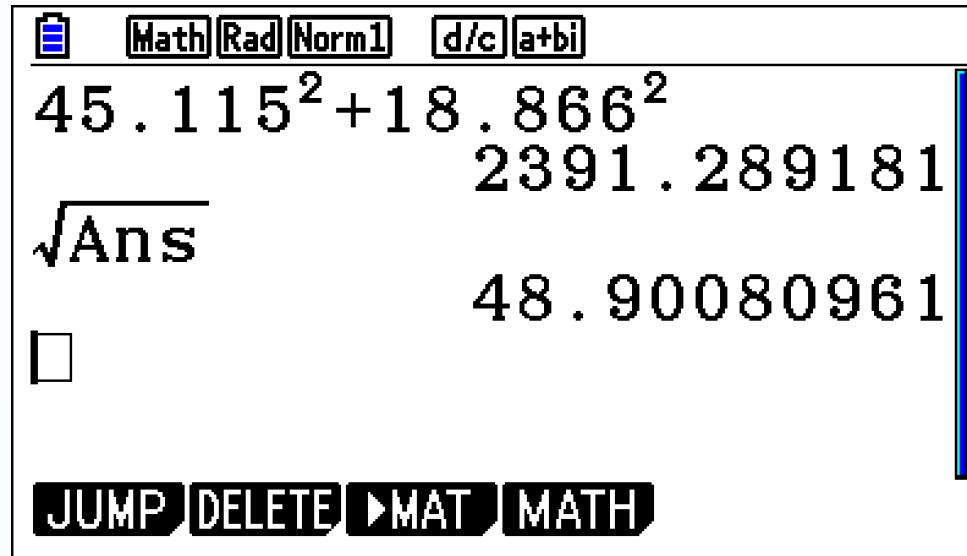


Method 2 – Trig Identity

$$[v_0 \sin(t)]^2 + [v_0 \cos(t)]^2 = v_0^2$$

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$



The image shows a Casio calculator screen with the following display:

- Mode: Math, Rad, Norm1, d/c, a+bi
- Input: $45.115^2 + 18.866^2$
- Result: 2391.289181
- Function: $\sqrt{\text{Ans}}$
- Result: 48.90080961
- Buttons: JUMP, DELETE, MAT, MATH



Checking the Parametric Model

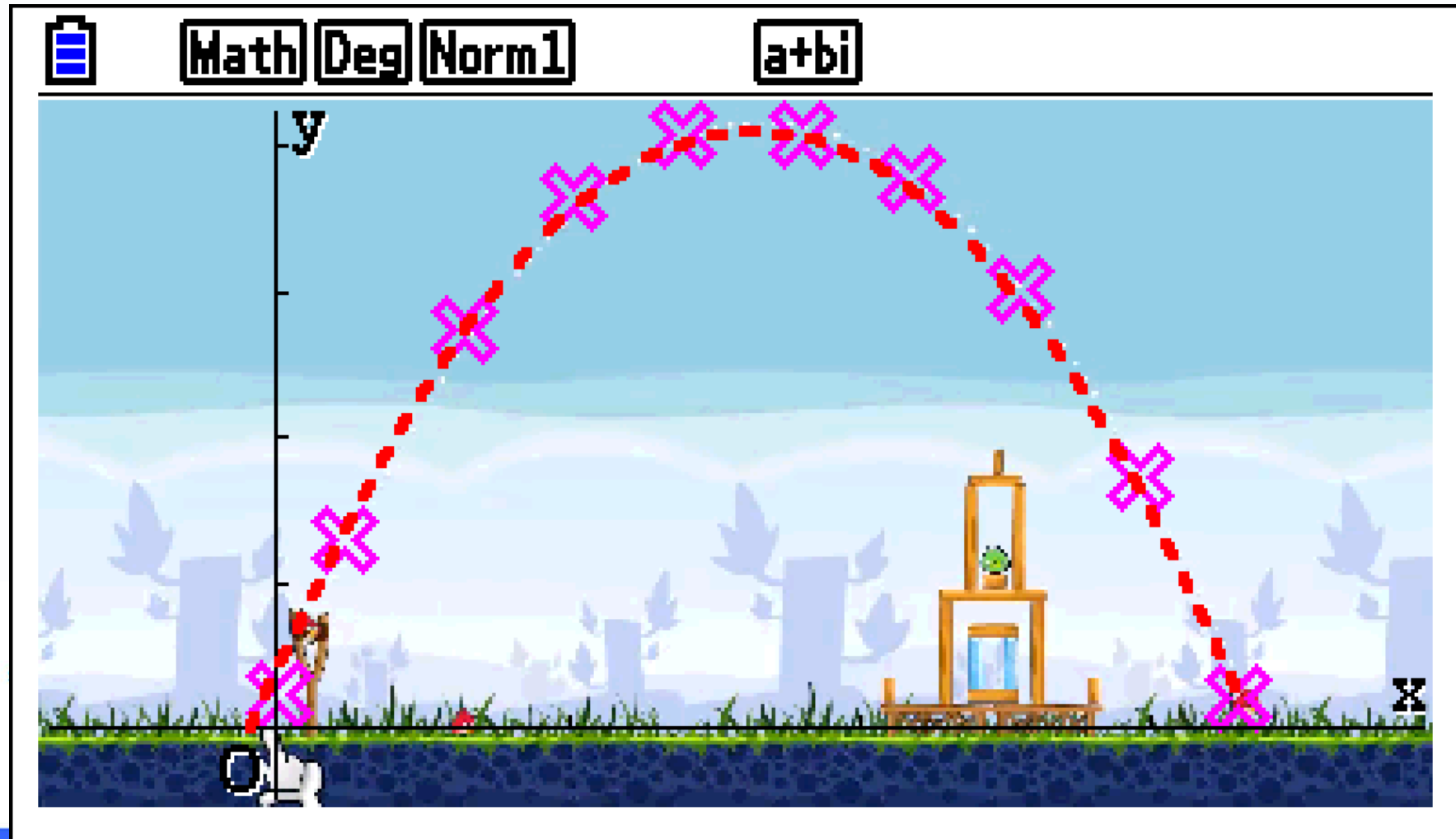
$$X = 48.90 \cos(67.31)T - 4.40$$

$$Y = -4.94T^2 + 48.90 \sin(67.31)T - 0.25$$

```
Math Deg Norm1 a+bi
Graph Func : Param
Xt1 (48.9cos 67. [----]
Yt1 -4.94T^2 + (48.
Xt2: [---]
Yt2:
Xt3: [---]
Yt3:
[SELECT] [DELETE] [TYPE] [TOOL] [MODIFY] [DRAW]
```



Checking the Parametric Model



Revisiting the (x, y) Model

$$x = (v_0 \cos \theta)t$$

$$\frac{x}{v_0 \cos \theta} = t$$

$$y = \frac{-1}{2}gt^2 + (v_0 \sin \theta)t + y_0$$

$$y = \frac{-1}{2}g\left(\frac{x}{v_0 \cos \theta}\right)^2 + (v_0 \sin \theta)\left(\frac{x}{v_0 \cos \theta}\right) + y_0$$


$$y = \frac{-g}{2(v_0 \cos \theta)^2}x^2 + (\tan \theta)x + y_0$$



CASIO

Simply Calculate The Difference!

Revisiting the (x, y) Model


 **Math** **Deg** **Norm1** **d/c** **a+bi**

-4.9

$(48.9 \cos 67.3)^2$
-0.01375989768

tan 67.3 2.39057695

DEL-LINE **DEL-ALL**

 **QuadReg**

a = -0.0145123
b = 2.39544602
c = 5.98181537
r² = 0.99938941
MSe = 1.1348274

$y = ax^2 + bx + c$

COPY **DRAW**



Explorations

Use the graphs of (t,x) , (t,y) , and (x,y) to compute the results:

Q1 What is the bird's position at time $t=2.5$ seconds?

Q2 How long is the bird in flight?

Q3 What is the time when the bird is at maximum height?

Q4 What is the maximum height?

Q5 At what time(s) is the bird at height 60 meters?

Q6 How far did the bird fly horizontally?

Q7 What is the height when the horizontal position is 150 meters?

Q8 What is the horizontal position when the height is 60 meters?