The Mathematics of Angry Birds

NCTM 2013

John Diehl
Hinsdale Central HS (Retired)

Ismael Zamora
Hinsdale South HS, IL

The Main Objectives

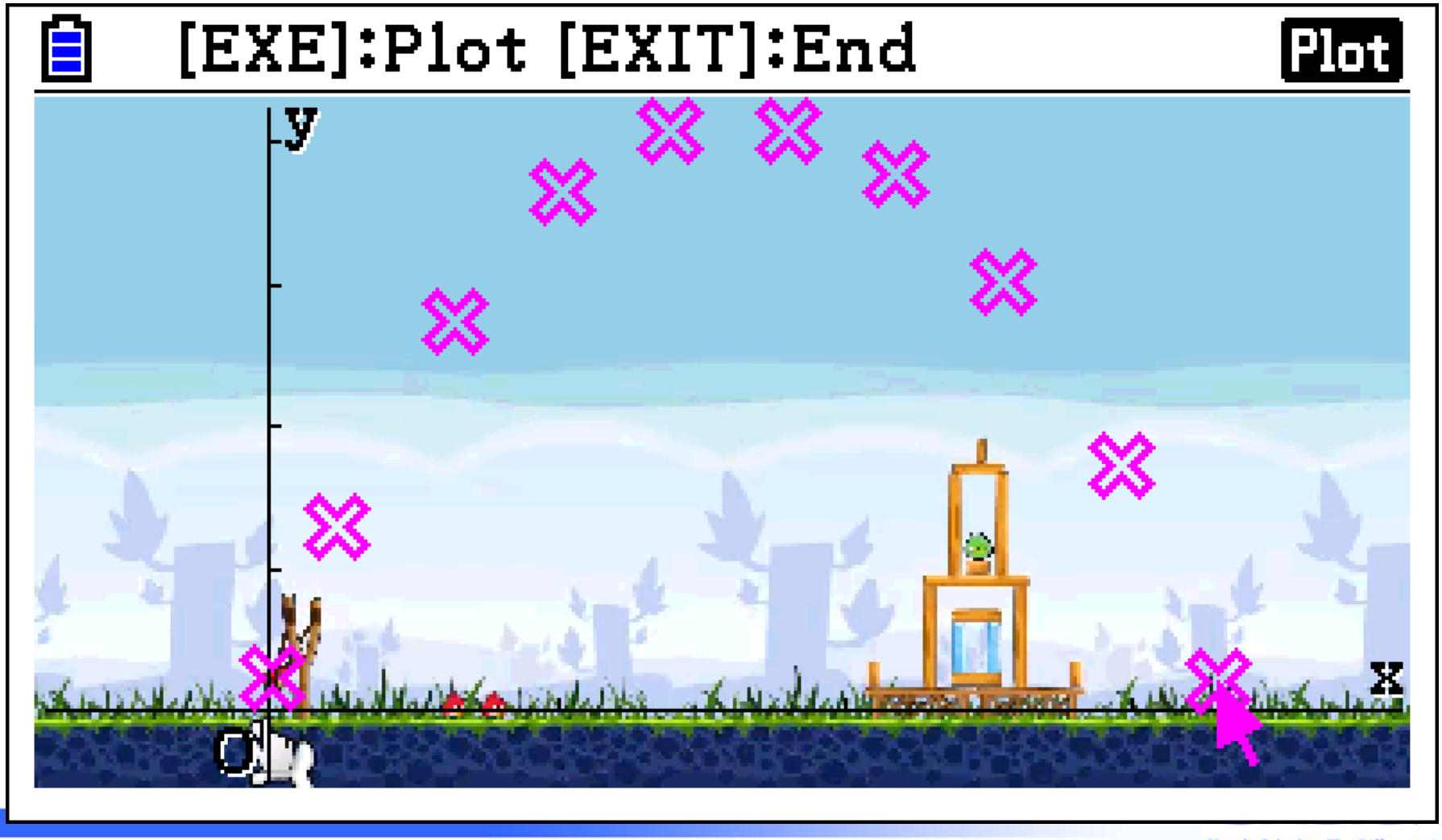
- Use data from the flight of an angry bird to develop models for the motion
- Explore the major variables of angle and initial velocity
- **Explore the parametric relation**

$$x = (v_0 \cos\theta)t + x_0$$

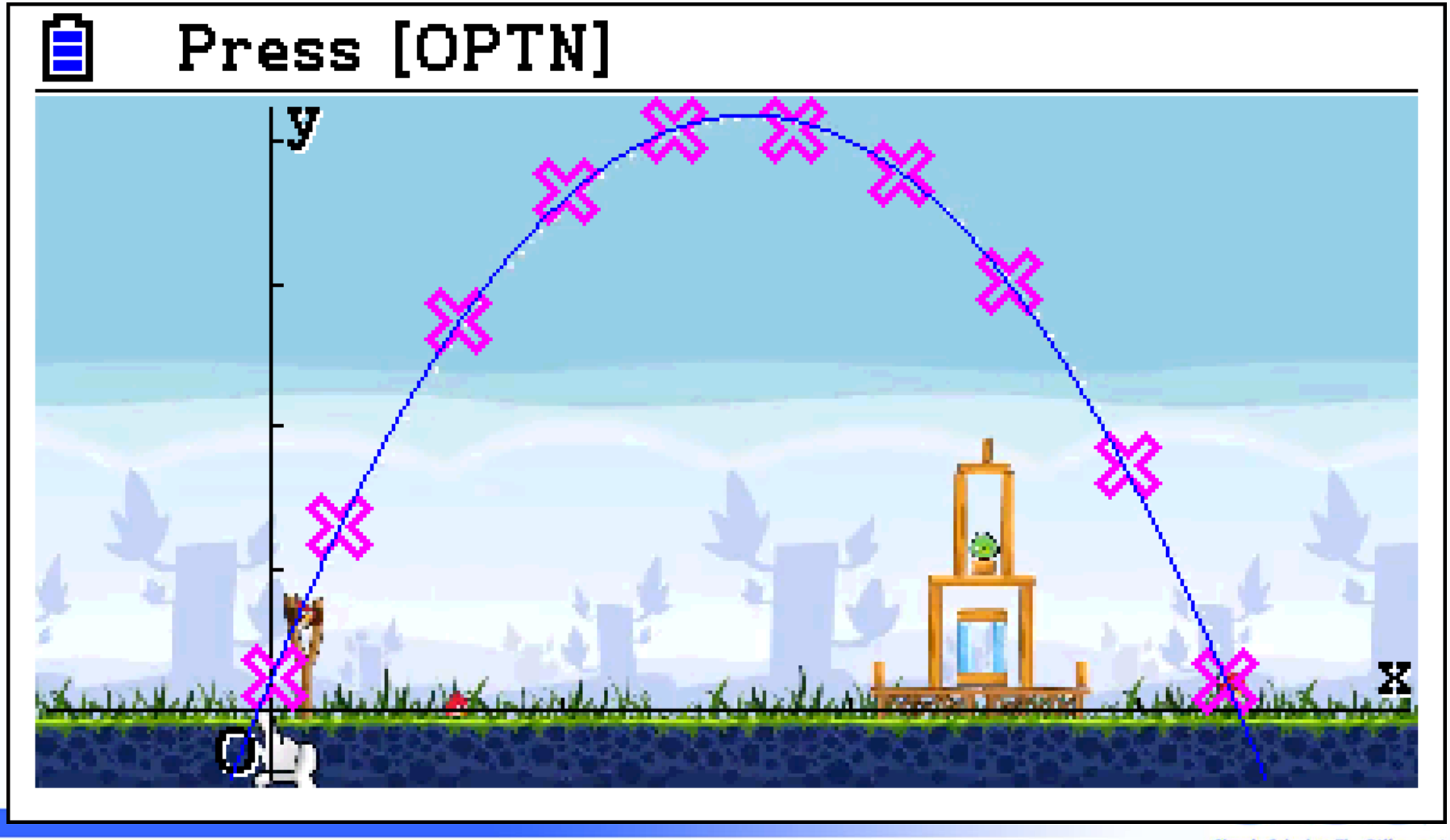
$$y = -0.5gt^2 + (v_0 \sin\theta) + y_0$$

CASIO
Simply Calculate The Difference!

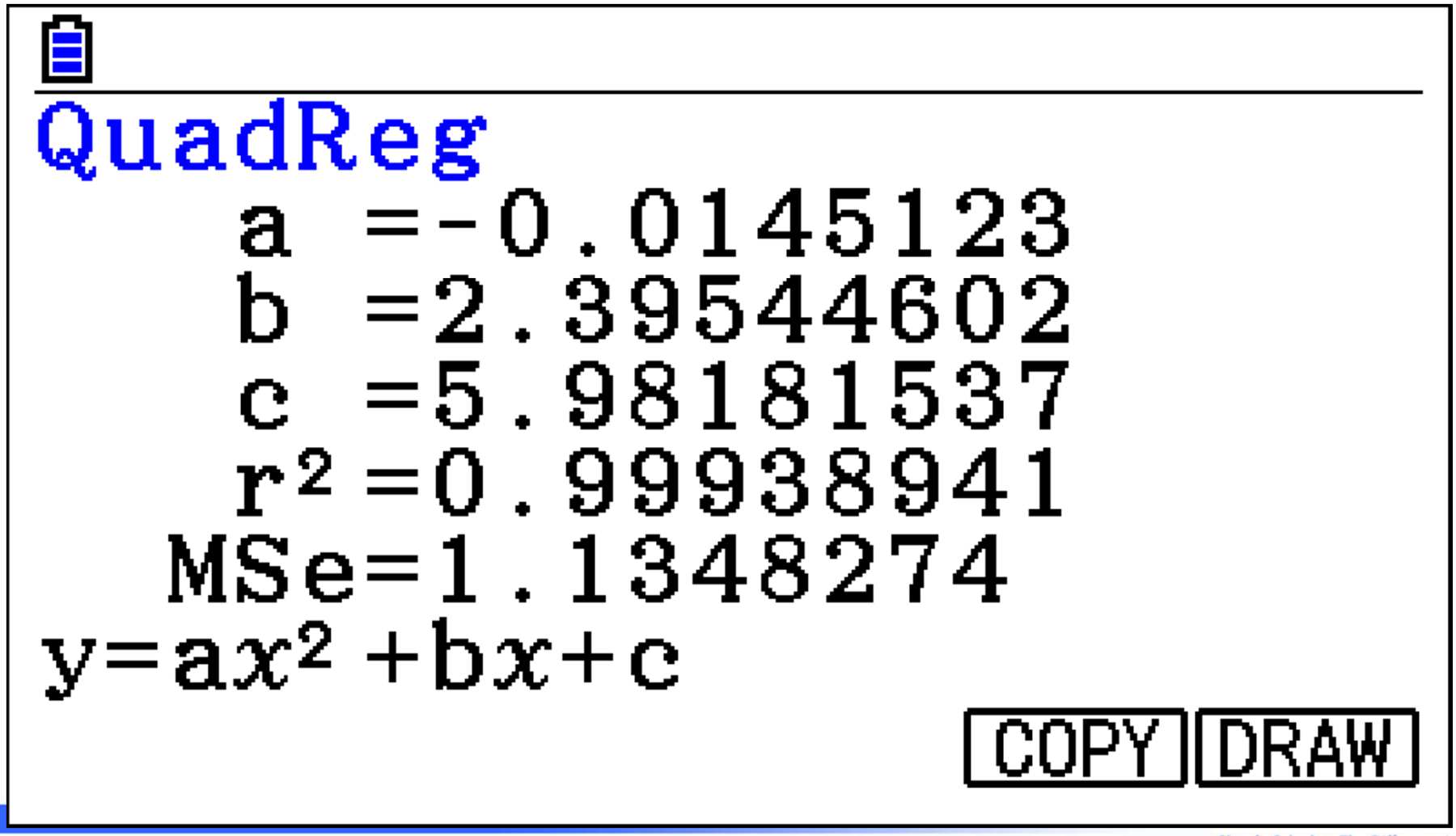
A Demonstration of Data Collection



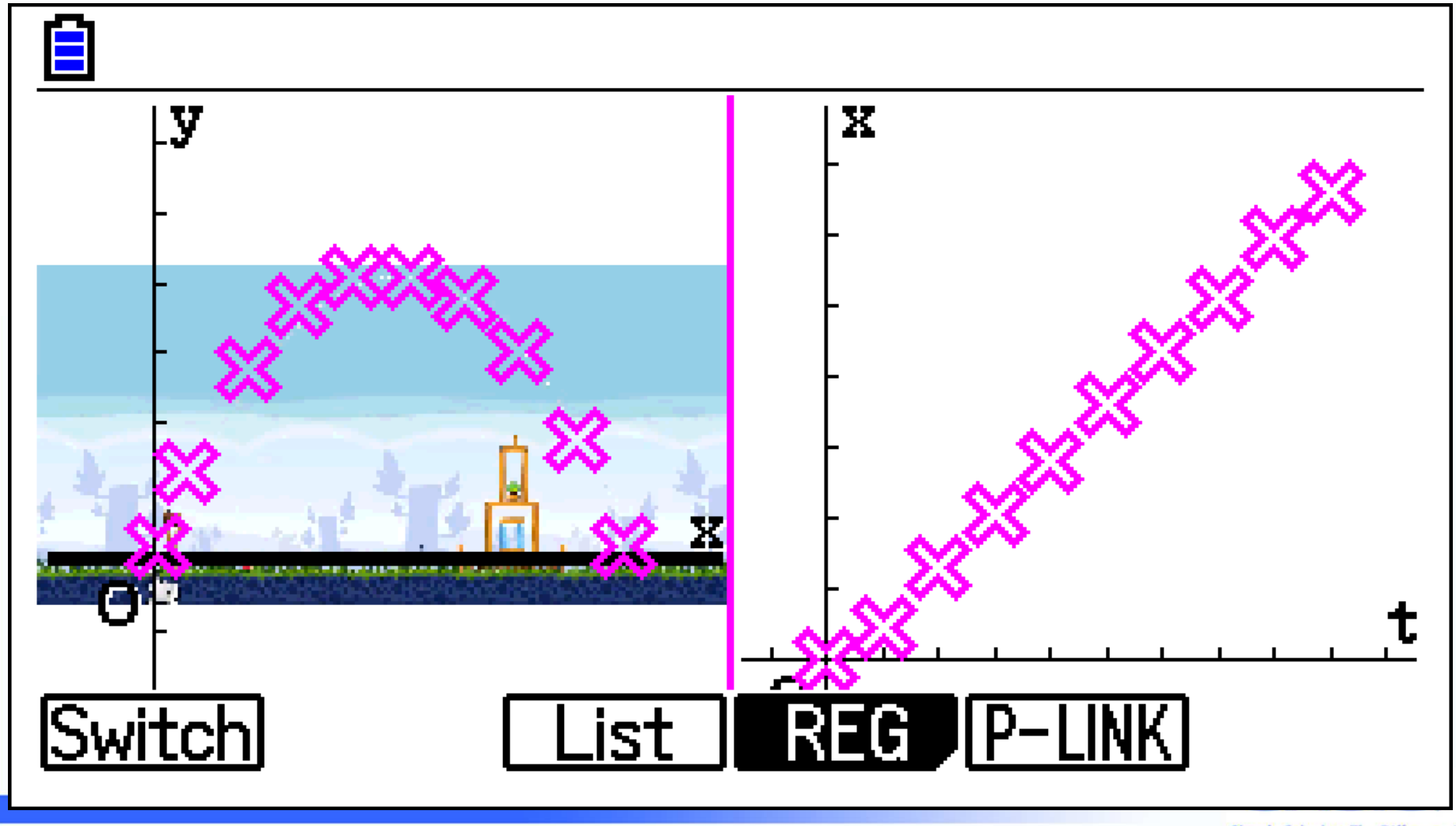
Create a Model by Regression



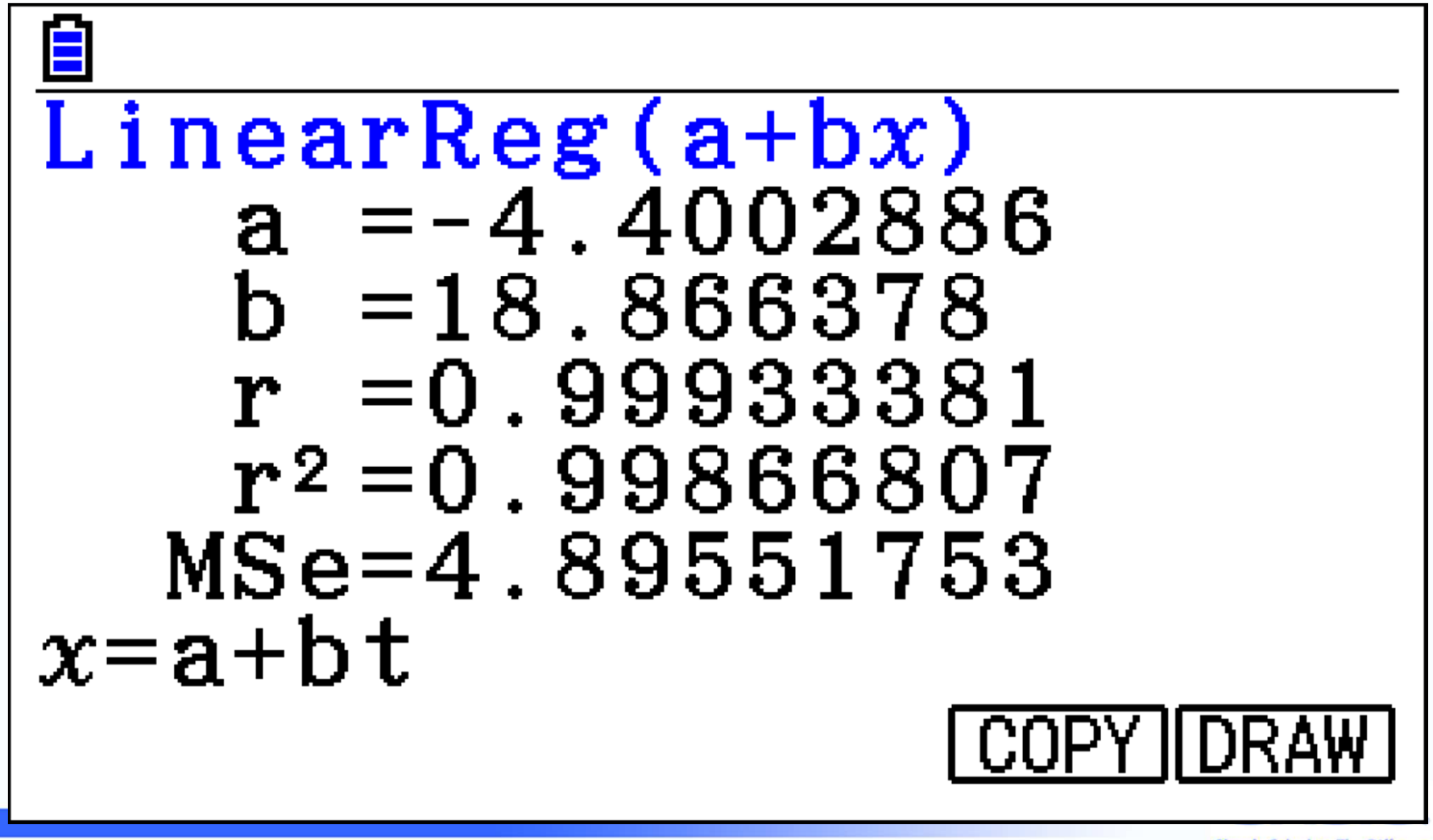
More About These Coefficients Later!



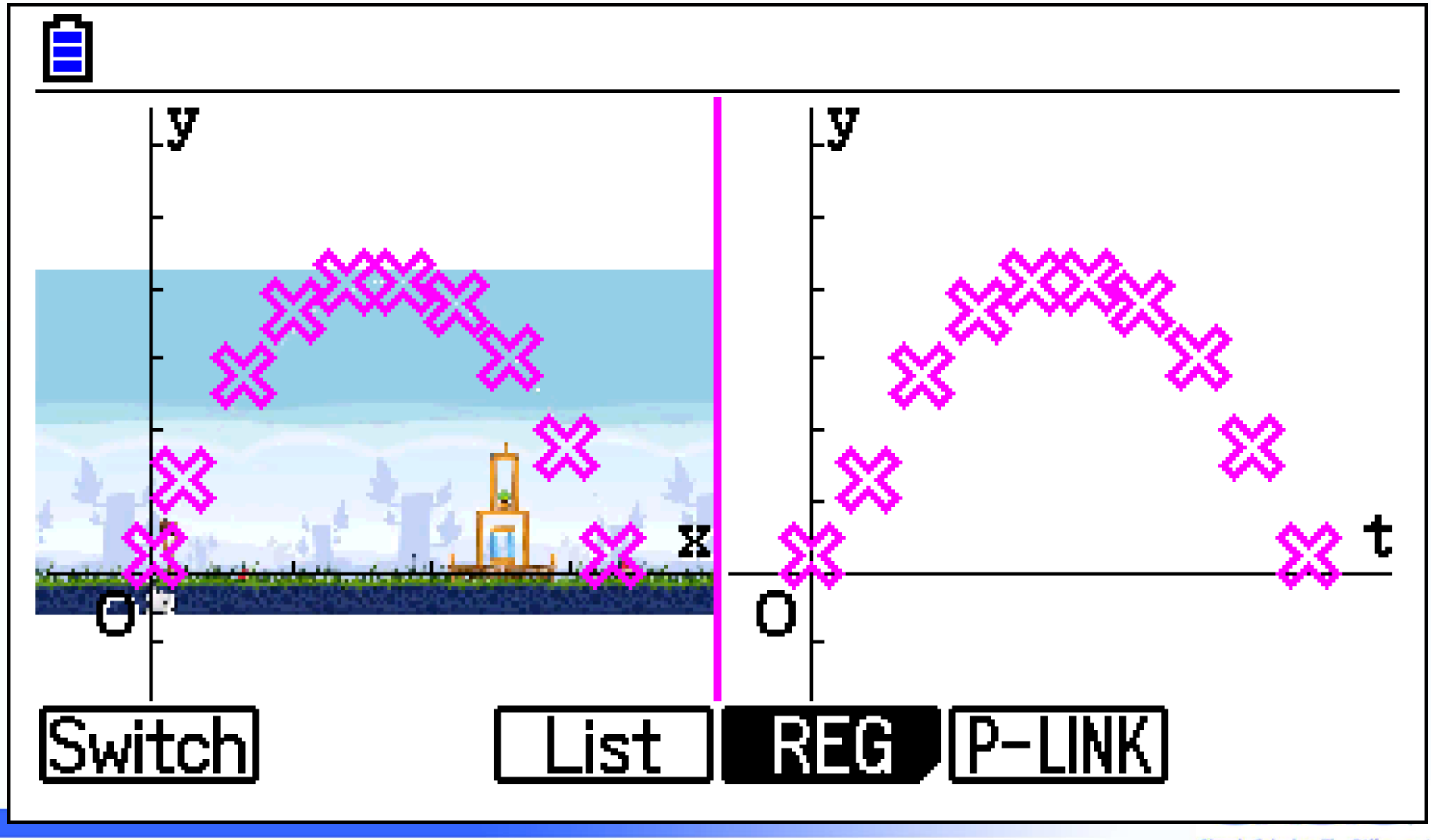
Let's Explore Horizontal Position vs. Time



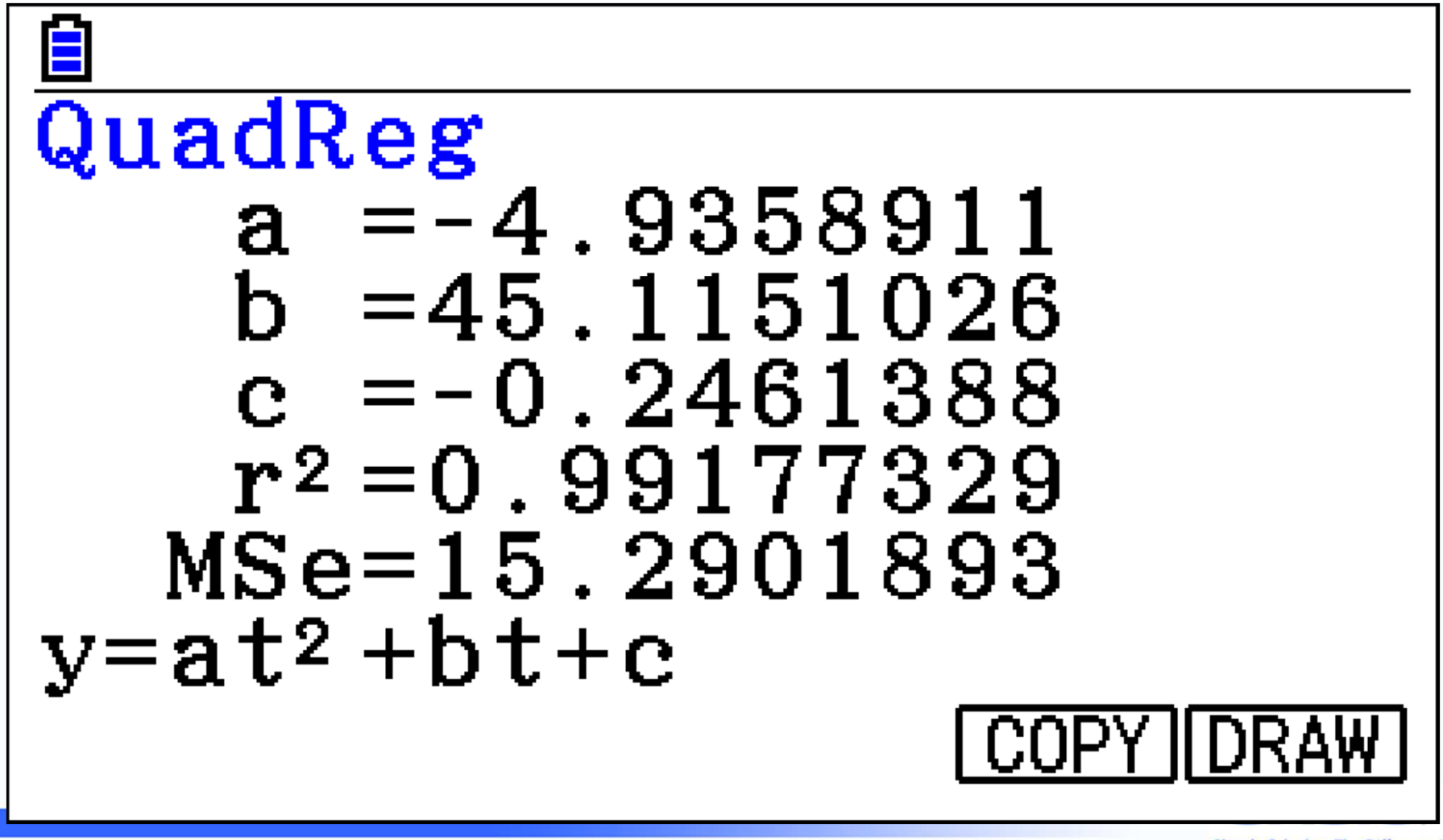
Interpret Both Coefficients of the Model



Let's Explore Vertical Position vs. Time



Interpret the Coefficients of this Model



Let's Explore the Angle and Initial Velocity

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$

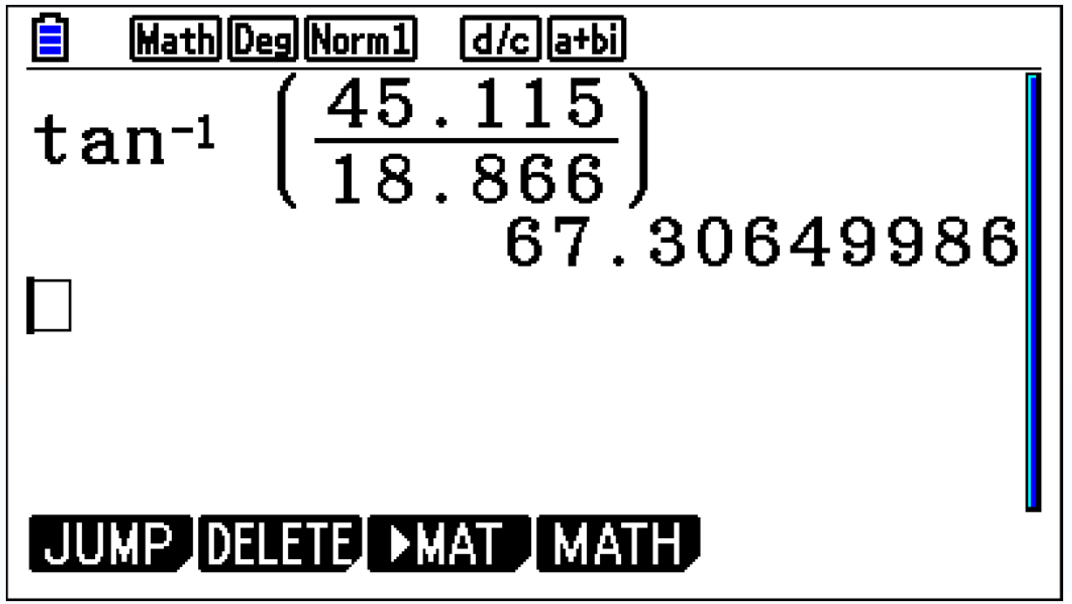
- There are several ways to compute the values

Method 1 - Division

$v_0 \sin(t) = 45.115$

$v_0 \cos(t) = 18.866$

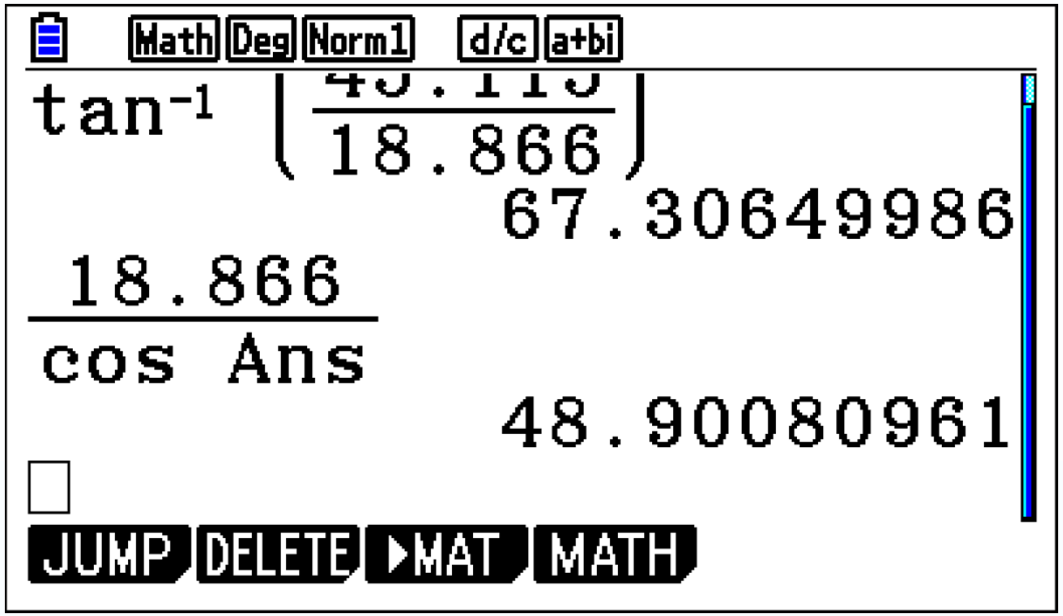
$\tan(t) = 45.115/18.866$



Method 1 - Substitution

$v_0 \cos(t) = 18.866$

$v_0 = 18.866 / \cos(t)$

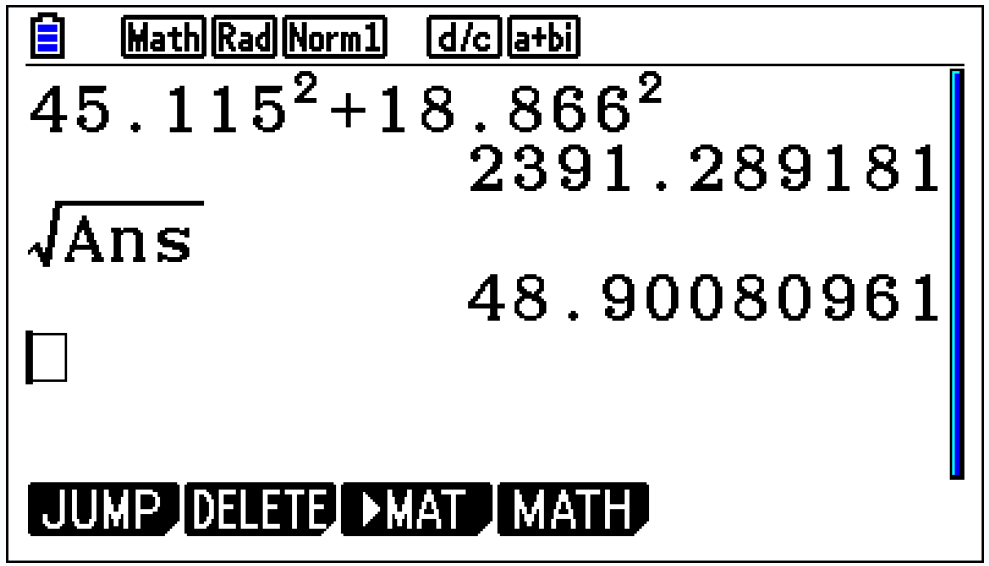


Method 2 – Trig Identity

$$[v_0 \sin(t)]^2 + [v_0 \cos(t)]^2 = v_0^2$$

$$v_0 \sin(t) = 45.115$$

$$v_0 \cos(t) = 18.866$$

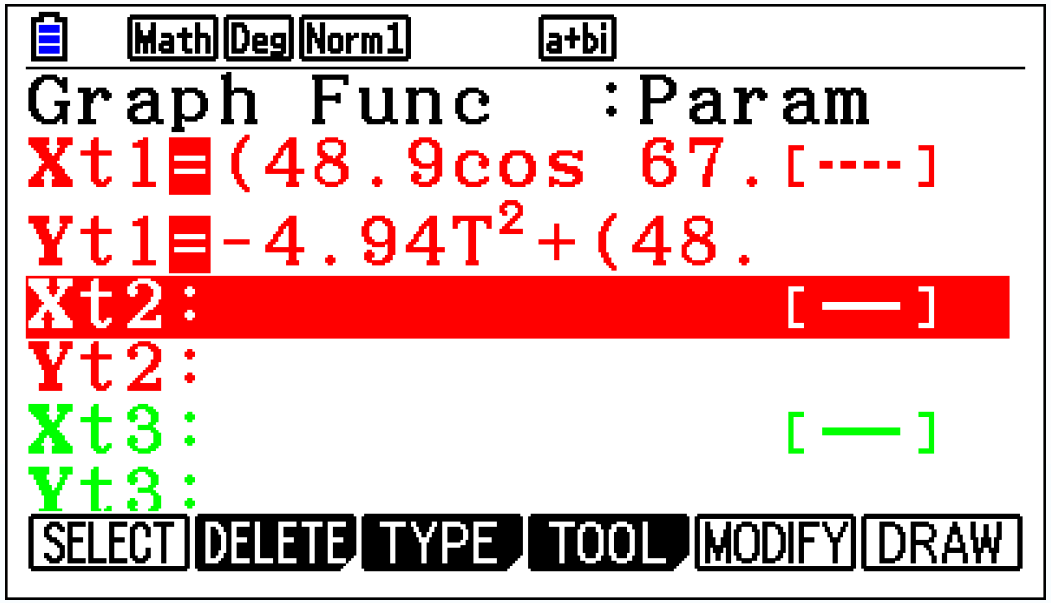


CASIO
Simply Calculate The Difference!

Checking the Parametric Model

$$X = 48.90 \cos(67.31)T - 4.40$$

$$Y = -4.94T^2 + 48.90 \sin(67.31)T - 0.25$$



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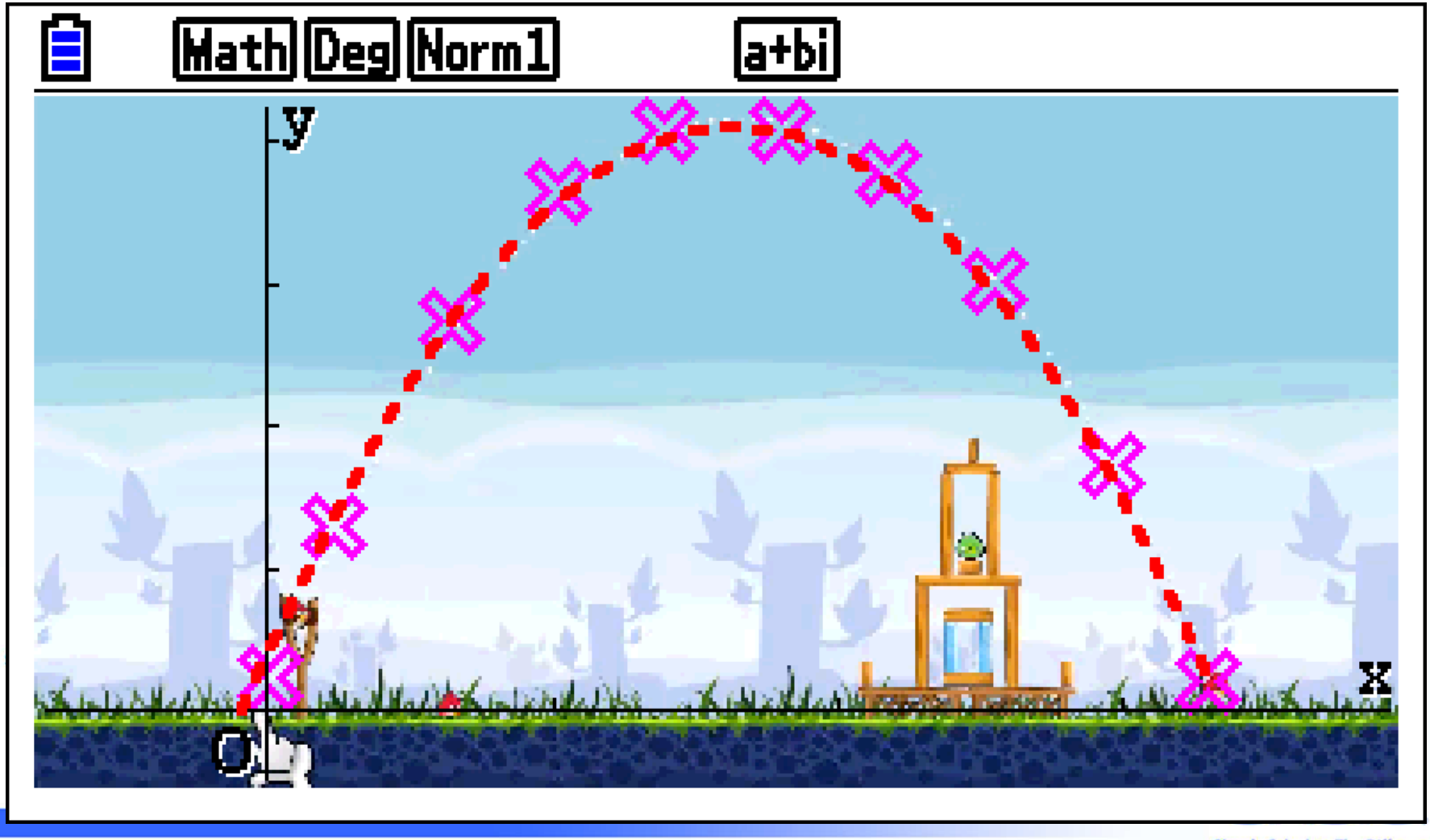
Checking the Parametric Model



Revisiting the (x, y) Model

$$x = (v_0 \cos\theta)t$$

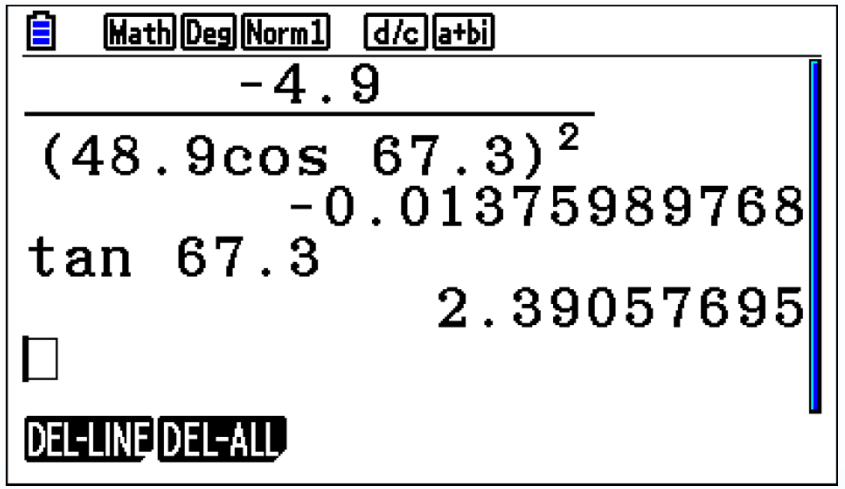
$$\frac{x}{v_0 \cos\theta} = t$$

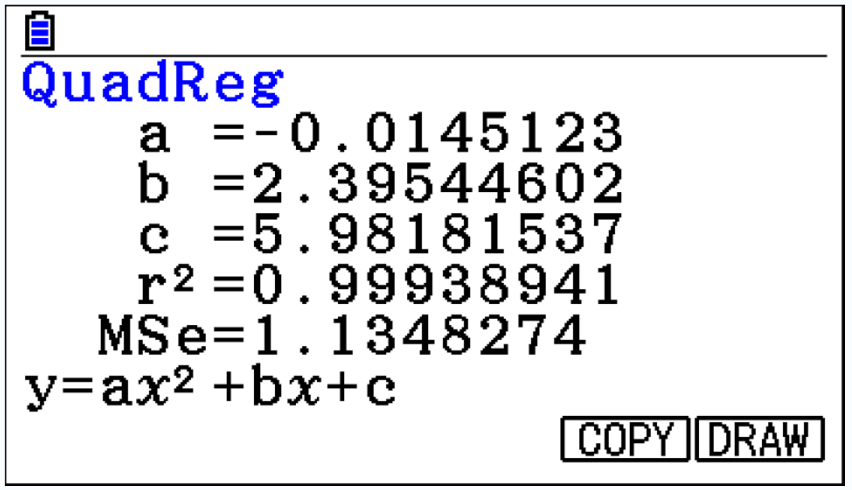
$$y = \frac{-1}{2} g t^2 + (v_0 \sin\theta)t + y_0$$

$$y = \frac{-1}{2} g \left(\frac{x}{v_0 \cos\theta}\right)^2 + (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) + y_0$$

$$y = \frac{-g}{2(v_0 \cos\theta)^2} x^2 + (\tan\theta)x + y_0$$

Revisiting the (x, y) Model





Explorations

Use the graphs of (t,x), (t,y), and (x,y) to compute the results:

Q1 What is the bird's position at time t=2.5 seconds?

Q2 How long is the bird in flight?

Q3 What is the time when the bird is at maximum height?

Q4 What is the maximum height?

Q5 At what time(s) is the bird at height 60 meters?

Q6 How far did the bird fly horizontally?

Q7 What is the height when the horizontal position is 150 meters?

Q8 What is the horizontal position when the height is 60 meters?