Previewing Calculus Concepts in Grades 6 through 11.

Ways you can preview Calculus concepts in MS and early HS……

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Problem 1

You are driving on the interstate when you pass a state trooper. Worried that you may get a ticket, you travel at exactly 75 mph for the next three hours.

Sketch a graph of the problem.

Questions to go over:

What are the units of the sides of the rectangle?

Horizontal units are hours.
Vertical Units are miles per hour.

What is the area of the rectangle, including units?

The area is 225 miles.

What does that mean in terms of the problem?

That is the distance you drove in the 3 hours.

What did we just figure out?

Actually, we just calculated $\int_0^3 75dt$, the area under the "curve".

If your MS students can graph the original problem, then they can do calculus!

Story Problem 2

Pat, a student in your 3rd period class, is standing near the green door of the red brick school when her school bus arrives. Pat speeds up at a steady pace until she is moving at 4 ft/sec. This takes 2 seconds. She maintains this pace for 8 seconds before stopping to talk to her friend Chris, who is standing five feet from the bus. Stopping takes 1 second.

Draw a graph of the situation.

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Could your students draw the graph of this problem?

Stop and think of the questions your text would ask about the graph....

The next page has a list of the questions I will ask.... Including a few questions that are not previewing Calculus, but are non-traditional questions.

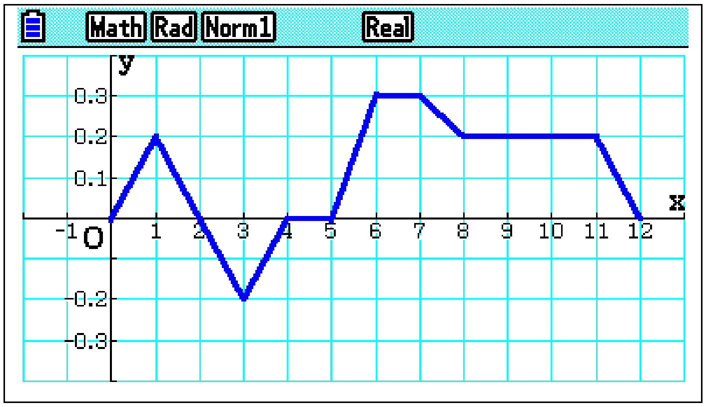
Calculus based questions for MS or early HS:

1. How far did Pat walk in the 11 seconds?
2. What was Pat's average speed?
3. How fast was she moving at 1 second? At 7 seconds? At 10¼ seconds?
4. What are the **units** of the slopes of the three line segments?
5. What does the slope of the line segments tell you about the situation? What are the 3 slopes (include units)?
6. How far was Pat from the bus when the bus first arrived?
7. Was Pat walking, jogging, or running?
8. What information in the problem is not needed?
9. What gender are the two people in the problem. Justify your answer.
10. What Calculus concepts does this problem preview?

Solutions to the problems:

1. Pat walked 38 feet (the area of the trapezoid).
2. Her Average speed was $3.\frac{7}{11}$ft/sec (38 feet in 11 seconds).
3. 2 ft/sec @ 1; 4 ft/sec @ 7; & -3 ft/sec @ 10¼
4. The units of slope are ft/sec. (feet per second).
5. The slope of the line segments tell you Pat's *acceleration*. The slopes are 2ft./sec^2 for the first segment; 0 ft./ sec^2 for the horizontal one; and -4ft./ sec^2 for the last segment.
6. Pat was anywhere from 38 to 43 feet from the bus when the bus first arrived. (Remember, in the story problem, Chris was 5 feet from the bus door. Chris could be anywhere on a semicircle with a 5 foot radius). I asked the question with 43 feet in mind. I never thought of the 38 foot answer until 1 teacher said 38 feet and explained by asking me "What if Chris is 5 feet *past* the bus??" -- Unexpected solutions with good explanations are another advantage of posing these kinds of questions.
7. Pat's speed of 4 ft./sec is about 2.5 mph., she was walking.
8. Several things were not needed: Did you need to know the students' names (maybe), but did it matter that Pat is in *your* class, let alone in your *3rd period class*? The color of the door and the school (as well as it being brick)
9. Pat is a girl ("...until *she* is moving...") and Chris is up to you as long as you <u>justify</u> your choice!
10. This problem previews definite integrals (area under the curve) and derivatives (rate of change of the graph); graph analysis; and justifying your answers, which is important for any AP test.

AP Calculus AB/Calculus BC 2009 Free Response Question 1



Caren rides her bicycle along a straight road from home to school, starting at home at time t = 0 minutes and arriving at school at time T = 12 minutes. During the time interval 0 ≤ t ≤ 12 minutes, her velocity v(t), in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

a. Find the acceleration of Caren's bicycle at time t = 7.5 minutes. Indicate units of measure.

b. Using the correct units, explain the meaning of $\int_0^{12} |v(t)|dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|dt$.

c. Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

d. Larry also rides his bicycle along a straight road home to school in 12 minutes. His velocity is modeled by the function $\boldsymbol{w}$ given by $\boldsymbol{w}(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $\boldsymbol{w}(t)$ is in miles per minute for 0 ≤ t ≤ 12 minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

Note: All of the questions in the AP test question can be explored by students starting in 6th grade as long as you avoid the calculus terms and symbols and ask some additional questions.

Adaptations for Part a:

1. You could ask this problem as written, if your students have *mastered* the rate of change ideas that from the differential calculus problems. Otherwise, it might help to get to the Part a question by asking the following:
    a. Note: Make sure that students realize that the vertical values are in tenths of a mile per minute, not in full miles per minute. They will need to know that in order to correctly answer all of the questions.

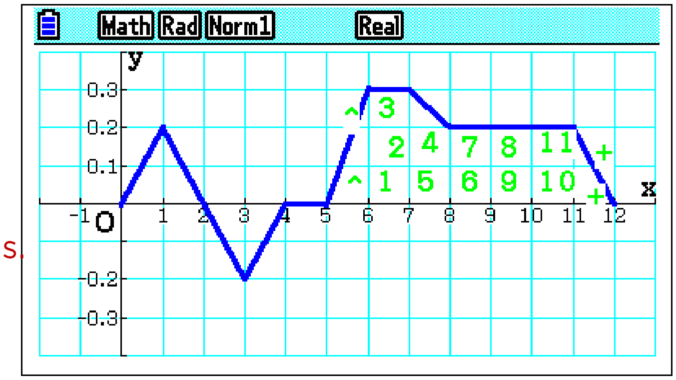
    b. What are the units of the rate of change of the graph? (Ans. The units of the rate of change are in miles per minute *squared* (or tenths of a mile per minute *squared*). If your students have investigated rate of change in prior velocity versus time graphs, this will not be difficult for most of them. If they have not spent time looking at other velocity versus time graphs, you should discuss that the rate of change would be calculated by taking mile (or tenths of a mile) per minute by minutes, which is why they get minutes squared as the denominator of the rate of change.)

    c. What is the rate of change of the graph at 7.5 seconds, including units? What does this tell you about Caren's bicycle ride? (Ans. The rate of change at 7.5 seconds is –0.1 miles/second^2. This tells us that Caren is slowing down (or decelerating) at that rate.

    d. You could ask about her acceleration at other points along her trip to reinforce this concept. As we pointed out earlier, students can not state what the acceleration would be at 1, 3, 4, 5, 6, 7, 8, or 11 seconds since these are all vertices of an angle in the graph. That means the slope of the segment before and after these times are two different numbers, so there is no definitive answer to the question of acceleration.

Adaptations for Part b:

1. What is the total area of the two triangles and the shape at the left of the graph? (Ans. The total area of the three shapes is 1.8 miles. Each triangle has an area of 0.2 miles, so the two triangles equal 0.4 miles. The shape at the left has an area of 1.4 miles.
We got this by counting unit squares. There are 11 full squares. The partial square at the right with the plus sign in it and the triangle above it add to another square. The same is true of the trapezoid between 5 & 6 and the small triangle above it. The trapezoid next to the square numbered "2" is half a square. Adding the other half square above square "4" brings the total to 14 squares. Since each square represents 0.1 miles, the area of the right hand shape is 1.4 miles. That plus the area of the two triangles means the total area of all three shapes is 1.8 miles)



2. What does this area tell you about Caren's bicycle ride? (Ans. The total area of the three shapes tells us the total distance Caren rode her bicycle.)

3. How can you use the three areas from Question 1 to arrive at the distance Caren lives from school? (Ans. The area of each triangle is how far Caren rode before realizing how she had forgotten her homework. Since she rode out and then back home, these two areas do would cancel each other out.)

4. How far does Caren live from school? (Ans. Caren lives 1.4 miles from school.)

Adaptations for Part c:

1. No adaptations are necessary, but we suggest adding a few questions. Besides asking "At what time does she turn around to go back home? Give a reason for your answer." You can also ask the additional questions below. (Ans. She turns around after two minutes. This is when her velocity becomes negative, indicating she is backtracking to get her homework.)

2. How far has Caren pedaled when she turns around? (Ans. Caren has pedaled 0.2 miles, the area of the triangle above the X-axis.)

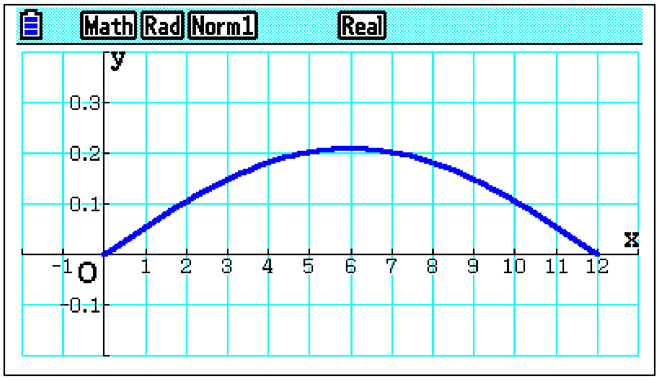
3. What is happening at 1 minute? At 3 minutes? And, from 3 to 4 seconds? (Ans. At one minute, Caren is slowing down. Her velocity is still positive, so she is still pedaling away from her house. At 3 minutes, her velocity changes from going faster towards her house to still pedaling towards her house, but less quickly. From 3 to 4 minutes, she is not pedaling at all. This must be when she goes into her house to get her homework.)

4. What is happening between 8 and 11 seconds? (Ans. From 8 to 11 seconds, Caren is pedaling towards school at a steady velocity of 0.2 miles/minute.)

Adaptations for Part d:

1. Once students have had enough trigonometry to know how to enter the equation and set the View Window for the given function, you could have them do that, then find the area under the curve for Larry using one of the methods discussed in adaptation 2.

2. Give the students a graph of the function w(t) as given in the problem. Ask, "How far does Larry ride to school?" This is asking what is the area under the curve. Students can find the area in several Ways that we already have shown, but we suggest letting them count unit squares.



How many do you get when you count the unit squares under this curve? Give your answer to the nearest full square.
(Ans. You should get 16 squares. This means that Larry lives 1.6 miles from his school.)

Besides counting unit squares, you could do Riemann Sums. To do this you would need to find the sum of the Y values of the curve at 0.5, 1.5, 2.5, …, 10.5, and 11.5 minutes. If your students have had practice with Riemann sums, you could Trace the curve to these 12 values during class and have a student write the Y-values down. The class could then add these values together to get the Riemann Sum estimate for the area under the curve. The sum of the 12 Riemann Sum rectangles equals 1.601 miles.

3. Who lives closer to school: Caren or Larry? Show the work that leads to your answer. (Ans. Larry lives 0.2 miles closer to school than Caren does. The work from questions 1 and 2 above has already been shown.)

AP Question: 2000 AP Calc Free Response question AB-2

Two runners, A and B run on a straight track for 10 seconds (0≤t≤10). The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. {webinar note: the given graph is the blue graph on the PRIZM screen}   The velocity, in meters per second of Runner B is given by the function v, defined by:

$$v(t) = \frac{24t}{2t+3}$$

(I would give middle school, Algebra 1, or Geometry students the graph of Runner B's velocity. Algebra 2 or above can create their own graph)

a) Find the velocity of Runner A and runner B at time t=2 seconds. Indicate units of measure.
b) Find the acceleration of Runner A and Runner B at time t=2 seconds. Indicate units of measure.
c) Find the total distance run by Runner A and the total distance run by Runner B over the interval 0≤t≤10 seconds. Indicate units of measure.

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Could your students solve these based on what we have done if they were given the graph of the Runner B?

Additional questions (some written by MS and HS teachers in Hawaii in the Spring of 2012 plus some TW questions):

1. The graphs cross at two places. What is happening when the graphs intersect? They are both running at the same speed.
2. How fast is each runner running at 9 seconds? A: 2.7m/sec B: 4.5 m/sec
3. Who is ahead in the race at the two times when the graphs intersect? Justify your answer. At 2.1 secs, B is ahead. At 7.5 secs. A is ahead (based on area between the curves).
4. Who wins a 100 meter race? By what distance do they win? A wins by 46.1cm
5. Will the person who won the 100 meter race continue to lead or will he be caught? Since A keeps same speed, and B increases, B will pass A

6. If the person is caught, when will that occur? What happens after that time? B passes A between 12 & 13 seconds (closer to 12 seconds)
7. Is the model is valid for a full minute? Assuming the runners are sprinters, they would most likely not be able to keep up the same pace for a full minute.

Let’s look at a Distance vs. Time story problem:

John is standing at the class room door, when he decides to move to the 4 feet to the water fountain. It takes him 2 seconds to do that. For the next 8 seconds, he drinks and stands at the fountain.   He returns to the door in 1 more second.

Draw a graph of the situation.

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Could your students draw the graph of this problem?

Stop and think of the questions your text would ask about the graph….

Calculus based questions for MS or early HS:

1. What are the units of the rate of change of the three segments? What does this tell you about the situation?
2. When is John moving the fastest? Justify your answer without any calculations.
3. What is his speed at 1 second? At 4 seconds? At 10.5 seconds? Is the average speed from 0 to 10 seconds greater or less than the average speed from 0 to 4 seconds? Use the graph (or “logic”) to justify your answer.
4. What is the average speed from the start to 10 seconds? From 0 to 4 seconds?
5. What is his speed at exactly 2 seconds? Justify your answer
6. What Calculus topics did we preview?

Solutions for the Distance vs. Time problems:

1. The units of the rate of change are feet per second.
2. Rate of change tells you the velocity, speed if you don't care about the direction of the movement.
3. He was moving fastest when he returned to the door, sine the line is the steepest.
4. His speed was 2ft/sec @ 1 sec.; o ft/sec @ 4 sec.; and
   -4 ft/sec at 10.5 seconds.
5. Graphically, the average speed from 0 to 10 seconds is less than the average from 0 to 4 seconds, since the slope of the line connecting the points is shallower from 0 to 10 seconds. "Logically", since John is standing still from 4 to 10 seconds, his average speed must be decreasing over that time. Avg speed from the start to 10 seconds is $\frac{2}{5}$ ft/sec ($\frac{4}{10}$).
   From 0 to 4, seconds, avg speed is 1 ft/sec. ($\frac{4}{4}$).
6. The speed from 0 to anything short of 2 seconds is 2ft/sec. The speed any tiny amount after 2 seconds is 0ft/sec. You could justify any value from 0 to 2 as the speed as long as you had a good justification for it. You could say the speed was 1 ft/sec, since it is the average of the 2 values. Or, you could say the model is inaccurate, that you *cannot* go from 2 ft/sec to a stop instantaneously, then say that the person was slowing down before 2 seconds, but did not come to a complete stop until after 2 seconds, so he was going 1¾ feet per second at exactly 2 seconds.
7. This previews: average rate of change; derivatives, differentiable and non-differentiable functions; graph analysis; limits; and delta epsilon proofs.

Human cannon ball problem.

The circus is coming to town, and for advertising, they stage a human cannon ball exhibition from the top a building across the street from a town park. The cannon is aimed up at a 56 degree angle, and the initial velocity of the person being shot out of the cannon is 60 feet per second. When she exits the cannon, the human cannon ball is 40 feet above ground. The circus places a net 10 feet off the ground to catch the cannon baller.

Sketch the graph.

I used a graph on the PRIZM to answer most of these:

Where is the cannon baller traveling the fastest? Hint: The correct answer is *not* when she exits the cannon. This question can be answered without a graphing calculator if you have been paying attention so far, or if your students have worked on several distance or height vs. time problems… Let's pretend the PRIZM graph is a MS student sketch and the tangent line is a ruler… Ask your class". Who can tell me where the max vertical velocity is on my sketch?

With a PRIZM, find when her height is 10 feet by using G-Solve where you put in the Y value, and the PRIZM finds the X value. The Trace the graph to that time (

Traditional questions:
What is her maximum height? When does it occur? Her maximum height occurs at 1.5544+ seconds and it is 78.6608+ feet.
When is the cannon baller at a height of 50 feet? At 0.216+ and at 2.89+ secs.
How high is the person after 2 seconds? 75.4845 feet
How long until she gets to the net? at X=3.6259909229 seconds, she is exactly 10 feet above ground

TW questions:
Is the initial velocity a reasonable velocity? (a research questions for students)
What is not finished with the sketch (or my graph)? Needs to get down
What are the units of the rate of change? (feet –vertical feet – per second)
What does the rate of change tell you about the situation? (The cannon baller's vertical speed.)
When is the person moving the fastest? (immediately before hitting the net.)

What is the biggest velocity? at X=3.6259909229 seconds, she is exactly 10 feet above ground, and her vertical speed is 66.28 ft/sec downward
Is that a reasonable velocity? (a research questions for students)
When is the person going up at a speed of 20 feet per second? 0.92943 sec.
Down at 20 feet per second? 2.1796 sec.
Can you find a place where the downward vertical velocity is equal to the upward vertical velocity when the person exited the cannon? When does that happen? It occurs at 3.10889 seconds into the flight.
What is the average speed from start to the net? - 8.2736 ft/sec
Does the person ever go exactly that vertical velocity? Yes, at 1.813007 secs.
Find some values for the rate of change and create a scatter diagram of them.

> What kind of relationship seems to be true?
> Find the line of best fit using Modify on PRIZM.
> What do the slope and Y-intercept of the line of best fit tell you about the situation?

Can your Algebra 1 students find the graph of the derivative of a parabolic graph? **We just did!!**

We also looked at the parametric equation form of this problem. That let us find that she hits the net 121 feet from the building. We also saw that the dx/dt = 0, since we ignored wind resistance and gravity only impacts vertical velocity.

Ferris wheel questions for MS and HS:

1. What does the Y-intercept tell you about the ride? Start ht.
2. When do you get to the top of the Ferris wheel? 15 & 45 sec
3. When are you going up and when are you going down?
    a. Up 0 to 15 & 30 to 45 secs
    b. Down 15 to 30 secs and 45 to 60 secs
4. What are the units of the rate of change of the graph? Ft/sec
5. Where are you on the ride when the vertical rate of change the largest? (at what times during the ride does this occur?)
    a. You would be even with the center hub of the ride (9 and 3 o'clock) at 7.5, 22.5, 37.5, & 52.5 seconds
6. Where are you on the ride when the vertical rate of change is equal to zero? (at what time during the ride does this occur?)
    a. Top or bottom of the ride, at 0; 15; 30; 45; & 60 seconds
7. When is the vertical rate of change increasing?
    a. Same as 3a
8. Are there times when the vertical rate of change is positive, but the changes are getting smaller? Yes, 7.5 to 15, 37.5 to 45 secs (larger?) Yes (positive and getting larger) , from 0 to 7.5 and 30 to 37.5 secs
9. What is the average vertical speed from when you are at the "bottom" of the ride until you are at the "top" of the ride?
    a. 40/15=8/3 ft/sec… the vertical distance is 40 and it takes 15 seconds.
10. What was your height above ground 5 seconds before the ride?
    a. Probably 0 feet, you were most likely on the ground 5 seconds before you got on the ride… students need to think about this one, and be willing to make a conjecture. I accept other heights if they explain their answer well. :-}
11. How many people got on the ride after you did? (I am assuming one thing about the graph most of you have drawn.)
    a. None, if someone had gotten on the ride after you, it would have stopped, assuming it is not one of the wheels that keeps going around and never stopping like at Navy Pier in Chicago.
12. Does your sketch show that you got off the ride or that you are still on it? (What is the end behavior of the graph of your ride?)
    a. No you have not gotten off, you would need to go back to 0 for height above ground… If you were the first one off, it would just show your height going to 0… if two people got off b4 you, the sinusoidal would continue, but would have two flat spots for when the ride stopped to let people off ;-}

These next questions require a graphing calculator.

13. When are you at the same height as the center of the Ferris wheel?
    a. 7.5, 22.5, 37.5, & 52.5 secs
14. How high are you off the ground 3 seconds after your ride began?
    a. 7.81966+ feet (trace to 3)
15. What is your vertical speed 3 seconds after you got on the ride?
    a. 2.4621 ft/sec… its dy/dx of the graph!
16. When is your vertical velocity –3 ft/sec?
    a. By trial and error and symmetry, find dy/dx = -3 at 18.813; but also at 26.187; 48.813; and 56.187 seconds
17. What is the value of the greatest vertical velocity? Where does this occur?
    a. ±4.1887 ft/sec, whenever you are even with the center (if you include negative speeds). Times are same as in 13a
18. Find the time where your increasing vertical velocity starts to decrease. (reword the question if you use it)
    a. @ 7.5 and 37.5 seconds… when you are above the center headed to the top of the wheel, velocity decreases, but is still positive
19. What is your average vertical velocity between:
    a. 5 and 20 seconds?
        i. 1.333.. ft/sec – the slope of the segment connecting those two points { (34 – 14)/(20 – 5) = 20/15 = 4/3=1.333…}
    b. Find where the instantaneous velocity (dy/dx) on the graph equals the average velocity from 5 to 20 seconds.
        i. At 13.4533 seconds, dy/dx = 1.3333. This also occurs at 1.5467 seconds (Can you find the other two times when dy/dx = 1.3333? What about all four places where you were on the ride when dy/dx = -1.3333 ? Two hints: use the symmetry of the graph….and the fact that the answer to question 13b occurs 1.5467 seconds before the first maximum height on the height vs. time graph! I am not giving you the answer here… if you need help finding it, ask your HS 's Trig or Pre-Calc teacher)
        ii. This question and answer perview the Mean Value theorem in calculus. In the problem we did earlier of Joe going to the water fountain, you **cannot** find a place on the distance vs. time graph where the instantaneous velocity equals the average velocity between 1 and 7 seconds, since Joe's distance graph is not a smooth curve like the Ferris Wheel graph is. The problem you had finding a velocity at exactly 2 seconds (since it was a "corner" of the trapezoid) precludes using the Mean

value Theorem. Joe's graph is non-differentiable, while the Ferris wheel graph is differentiable.

c. What is your average vertical velocity between 10 and 20 seconds?
    i. 0 ft/sec, your height at 10 and 20 seconds are the same, so the average vertical change is 0/2 = 0 ft/sec

These next questions would be for Algebra 2/ Trigonometry classes or Pre-Calculus classes, after students had learned about trigonometry function graphs and transformations of the graphs.

20. Find the equation of your ride on the Ferris wheel from the time you got on the last open car for two complete revolutions. Graph this equation.
    a. The equation of your height vs. time for this Ferris Wheel is: $y = 20\sin\left(\frac{\pi}{15}(x - 7.5)\right) + 24$
21. Sketch the tangent line using your PRIZM and find where the slope of the tangent line is positive, negative, and zero.
22. Are there places on the graph where your vertical speed is positive (or negative) and the change in your speed is also positive?
23. What about places where the vertical speed is positive (or negative) and the change in your speed is negative?
24. Find enough values of your vertical velocity in order to find a regression equation for your velocity. Graph your vertical velocity on the same axes as the graph.
    a. The equation for the graph of the derivative of the original graph is: $y = 4.1887\sin(0.20943951x)$
25. Move the graph up so that the center line of the velocity graph is on the same Y= value as the center line of the height graph.
    a. Add 24 to the velocity graph from question 24 above.
26. Compare the two graphs:
    a. Periods
    b. Amplitudes
    c. Trig function
        i. Periods are the same and the amplitude of the velocity graph is smaller.
        ii. When you look at the original graph and the velocity graph as a pair, one is a cosine graph while the other is a sine graph. There is a phase shift to get from one to the other and there is a sign change to consider as well. This fits the Calculus facts that dy/dx of sine equals cosine, while dy/dx of cosine equals the opposite of the sine.

Both of these can be previewed using the two graphs in Ferris wheel problem.

Here is a way to alter the question in order to have students analyze the situation in a different way, especially if your students have worked on the Ferris wheel idea before, or if you have a strong Honors or gifted class or once they get to a class where they have seen the Ferris wheel several times already:

27. Sketch a graph of your **horizontal distance** from the start of the ride.
    a. How does this change your graph and the questions?
        i. The graph will have both positive and negative values, since you are going back and forth past your starting point. The shape of the graph will be the same as the vertical distance graph, but it will be shifted vertically and possibly horizontally, depending on your answer to question 27b. (Whether you start out going to positive values or negative values depends on your answer to question 27b). The questions now ask about the horizontal distance and all the answers are shifted in time.
    b. You have a choice of how your graph is drawn, which did you choose?
        i. When the ride starts, did you decide to make the change in X positive since you are going forward (or because a friend not on the ride sees you moving to the right (+)). Or, did you make your graph start out going into negative numbers because the friend not on the ride sees you moving to the left?

Note, if you have students work this problem in Geometry, there are many other questions you could have them explore:

28. How far do you go on your ride?
    a. In two revolutions, you travel 251.3274123 feet .... (2* (40π)ft.). NOTE: if your students use 22/7 not they will get a slightly different answer for questions 28 and 29.
29. What is your **circular velocity** on the Ferris wheel?
    a. 4.18879 ft/sec (the circumference divided by 30 seconds)

30. If there is a bug halfway between you and the center of the Ferris wheel, is the bug's circular velocity half of yours? How far would the bug travel? What is the bug's circular velocity? Justify your answers.

   a. Your circular velocities would be the same, since you each complete one circle in the same 30 seconds.
   b. The bug's distance travelled and circular velocity would both be half of yours, half the answers for 28 & 29.

In Calculus or Pre-Calc, ask students to sketch the vertical velocity vs. time graph from the sketch of height vs. time, before they do the analysis or write the equation for height vs. time.

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Hot tea / Cold tea Problem

You make some hot tea, but decide that it is too warm outside, so you fill your empty cup with ice and pour in the hot tea.

Graph the temperature of the tea in the cup for the next three hours. (The horizontal axis will use minutes as the time.)

I have actual data (the temperature was taken every 10 seconds). If you want my data, email me and I will send it to you. However, it would be better to get your own data. Check with science teachers in your school in order to get the equipment you do not have the equipment to measure the temperature over three hours.

Hot Tea/Cold Tea Questions:

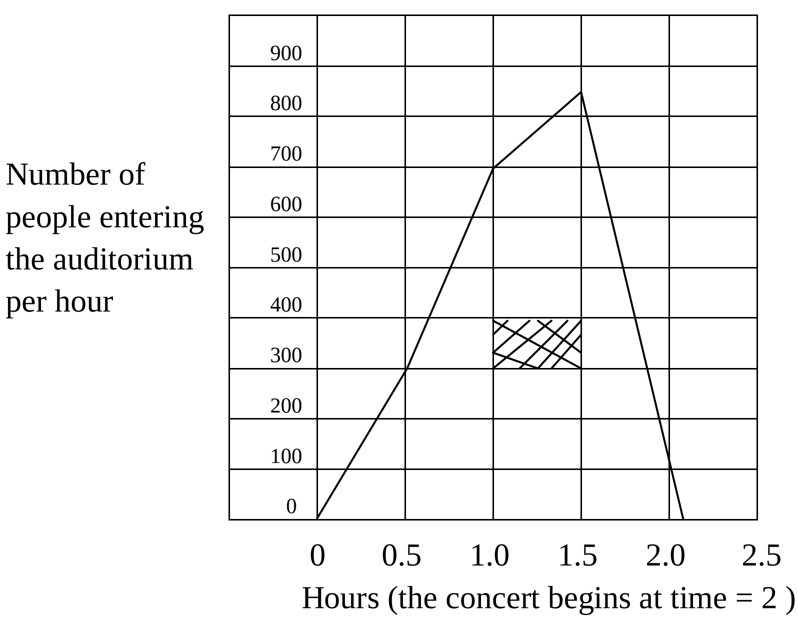
1. What are the units of the rate of change?
2. What does the rate of change tell you about the situation.
3. Where is the rate of change the most?
4. Where is the minimum temperature?
5. What is a reasonable value for the Y-intercept? The minimum value? The value after 4 hours?
6. Does the temperature stay the same for any significant period of time?
7. What kind of curve is it from the high temperature to the low temperature?
8. What kind of curve is it from the low temperature to the "final" temperature?
9. (Find the logistic curve using at least 10 data points.) ← Algebra 2 or Pre-Calc question
10. What is the "final" temperature called?
11. If you chose four or five points to create an exponential growth equation for the rising temperatures, Investigate how the predicted temperatures of the exponential equation compare to the logistic curve values and the data. ← Algebra 2 or Pre-Calc question
12. When is the rate of change positive and increasing? Positive but decreasing? What is that point called?
13. Would it help you in Pre-Calculus and/or Calculus if your students had experience noticing where this occurred in problems appropriate for MS or Algebra 1?
14. Show what the average rate of change from the low temperature to the "final" temperature is.
15. Is there a point in time where the instantaneous rate of change equals this average rate of change? (NOTE: This previews the Mean Value Theorem)
16. What others can your department come up with.....

AP Calculus AB/Calculus AB 2009 Free Response (Integral) Question #2

The rate at which people enter an auditorium for a rock concert is modeled by the function $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; R(t) is measured in people per hour. No one is in the auditorium at time t – 0, when the doors open. The doors close and the concert begins at time t = 2.

a. How many people are in the auditorium when the concert begins?
b. Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
c. The total wait time for all people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function *w* models the total wait time for all the people who enter the auditorium before time *t*. The derivative of *w* is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time t = 1.
d. On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

Middle School adaptations for the Auditorium Entry AP Calculus Problem



MS Algebra 1 Questions

1. How many people do you think attended the concert?
    a. Whatever you write is fine with me, it's probably a guess
2. What is happening at 45 minutes?
    a. People are entering the auditorium at approximately 500 people per hour.
3. What geometric concept does the shaded part of the graph represent?
    a. The area of the rectangle that is shaded.
4. In the context of the problem and using the correct units, how would you interpret the meaning of the shaded region of the graph?
    a. The value of the area is (½ hour)*(100 people per hour) = 50 people. This is the number of people who entered the auditorium for that portion of the graph.
5. What does the area between the graph and the horizontal axis represent?
    a. The total area between the graph and the axis equals the total number of people who entered the concert, if you include the people who entered after 2.0.
6. Calculate the "areas" for the horizontal intervals

   People will get different answers if they estimate some values differently than I did:
    a. $0 \le t \le 0.5$; ½ (0.5)(280) = 70 people
    b. $0.5 \le t \le 1.0$; ½ (0.5)(280 + 700) = 245 people
    c. $1.0 \le t \le 1.5$; ½ (0.5)(700 + 840) = 385 people
    d. $1.5 \le t \le 2.0$. ½ (0.5)(840 + 110) = 238 people (round up 237.5)
7. What does the sum of all of these values equal? What does this value represent?

   The sum equals 938 people, which is how many people entered the auditorium before the concert started.
8. What does the area after 2.0 tell you? What is its value?
    a. The area after 2.0 tells you how many people arrived late. 6 people arrived late. I rounded 5.5 up to 6. I estimated the time after 2.0 to be $\frac{1}{5}$th of the half hour from 2.0 to 2.5, which makes it $\frac{1}{10}$th of an hour. Number of people = $\frac{1}{2}\left(\frac{1}{10}\right)(110) = 5.5$.
9. Where is the maximum value of the graph? A student says this is when the auditorium is the most full. Is the student correct? Explain your reasoning.
    a. The maximum value of the graph occurs at 1.5 hours, when the Y value is approximately 840 people per hour. This is not when the auditorium is most full, since 629 people entered after 1.5 on the graph.
10. When is the auditorium the most full? Justify your answer.
    a. The auditorium was most full when the graph gets back to the X-axis after the 2.0 mark, approximately 2 hours and 6 minutes. That is when no one else enters the auditorium.

For both of the first two web URL's scroll down to where you find the tests listed in order to look at the questions:

http://apcentral.collegeboard.com/apc/members/exam/exam_information/1997.html

Gets you to the page in AP Central with the different AB Calculus test questions.

http://apcentral.collegeboard.com/apc/members/exam/exam_information/8357.html

Gets you to the page in AP Central with the different Statistics test questions.

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FYI: A ***really great*** page to check out:

Here is a URL for a web page written by Dixie Ross. Scroll down and you will find links to 8 different adaptations of AP Calculus and AP Statistics questions for Middle School, Algebra 1 and 2, Geometry, and Pre-Calculus students:

http://apcentral.collegeboard.com/apc/members/courses/teachers_corner/29924.html

If you scroll to the bottom of this web page, you will see a link to the answers . Below the answer link are other links back to interesting parts of the AP Central web pages.

In my opinion: If you are using these ideas, make sure to tell the students that the questions they worked on were ones that you adapted from an AP question. Also, tell the other teachers in your building and your district know you are doing this, so they may use them too.

Casio Education Online Resources

1) **PRIZM "Getting Started" Webinar:** Whether you will be using the handheld unit or the software, this webinar is a great introduction to the calculator basics. Feel free to view this as often as you would like, and at your own pace!
http://www.casioeducation.com/resource/HTML/previous_webinars.html

2) **PRIZM Online Training:** The CASIO Prizm Online Training is a great way to introduce teachers to the full spectrum of functionality that CASIO graphing technology has to offer. This course is suitable for first-time users, as well as those with more familiarity, who have a few questions about a particular application or functionality. While there are three assessments, they are not mandatory and users can take as much or as little of this course as they like. It is very flexible, completely self-paced and free to take!
http://www.casioeducation.com/prizmtraining

3) **Casio Educators Resources Webpage:** This page houses different items that many educators can find resourceful. Among them are archived webinars, upcoming webinars, links to Casio Online Courses, Activities and Sample Questions, and our Correlation Engine.
http://www.casioeducation.com/educators/

4) **CASIO Activities and Sample Questions:** Here you will find excerpts from our workbooks and our new "Fostering Mathematical Thinking" Series, as well as additional resources for your classroom. All downloads are free, so please share these with others!
http://www.casioeducation.com/educators/activities

5) **PRIZM 90-day trial software:** For those of you who already have a calculator, we also have a trial version of the Software that is valid for 90-days. It runs exactly as the full software does, but just expires after 90-days. *Coming soon: MAC 90-day Trial Software!
http://www.casioeducation.com/prizm_trial

6) **CASIO Picture Conversion Engine:** To take your own images and load them on the Manager Software, you need to use the Picture Conversion Engine. The files located on the software and handheld units are all proprietary and protected calculator files, to ensure test security. Once a file is converted, it can be used in class with the Manager Software. Check out how easy it is to create your own lessons with the Conversion Engine!
http://www.casioeducation.com/prizm_convert

7) **Casio YouTube Channel:** This is the official YouTube channel for Casio Education. Check out all the different videos on this site- student & teacher testimonials, guided tours, webinars, PR videos and more.
http://www.youtube.com/user/CasioPrizm

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8) **SeeMathRun YouTube Channel:** This YouTube channel is for teachers and students and offers small videos on everything from Fractions to Calculus, TI side-by-side comparisons, Casio side-by-side comparisons, and more; there are over 30 videos and counting every month. Check out these videos and feel free to make your own requests for specific topics!
http://www.youtube.com/user/Seemathrun?ob=0&feature=results_main

Check This Out!

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