Comparing and Ordering Fractions

Which is the larger number?

$\frac{3}{4}$ $\frac{2}{3}$

Which symbol >, < or = makes this true?

$\frac{1}{3}$ $\square$ $\frac{2}{5}$

Place the following in order from smallest to largest.

$\frac{3}{5}$ $\frac{8}{10}$ $\frac{3}{4}$ $\frac{7}{12}$ $\frac{1}{2}$

Some Key Questions to Ask When Teaching Fractions

Introduction of Fractions Bars

Objective: To name fractions from fraction bars. To identify similarities and differences among fraction bars.

Materials: Fraction Bars®, overhead Fraction Bars® (optional)

Vocabulary: fraction, similarities, differences, pattern

Introductory Activities

Discover the Concept of a Fraction

Distribute a set of fraction bars to each group of 2 to 5 students.

Each fraction bar in this set represents one whole or one unit such as one whole cracker or one whole brownie. Look through your set of fraction bars with your group. Discuss in what ways your bars are all alike and in what ways they are different. Record your findings in a table with two columns headed "Similarities" and "Differences."

After 5–10 minutes, ask volunteers from each group to suggest the similarities and differences they have found.

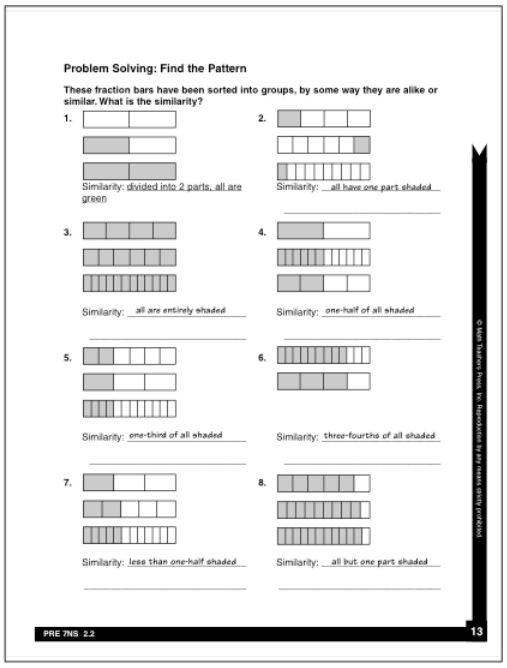
Similarities	Differences
same shape	colors
same size	divided into different
congruent	number of parts
same width, length	number of shaded parts
same area\perimeter	is different
same thickness	number of bars of any
same weight	one color
same material	
* all divided into	
parts of equal size	

*It is very important that the last similarity, i.e., each bar is divided into parts of equal size, be verbalized. This is the essential concept of a fraction. To guide this discovery, ask **What do the lines do to the fraction bar?** (divide the bar into parts of equal size)

Naming and Identifying Fraction Bars

The following 3 activities will assess students' abilities to translate from a concrete model to a written or spoken name.

1. Display or draw a fraction bar. Ask a volunteer to name the shaded part of each bar. To encourage students to visualize the number of parts the whole is divided into, ask 3 questions: **How many parts has the whole been divided into? How many parts are shaded? What fractional part is shaded?**



Problem Solving: Find the Pattern

These fraction bars have been sorted into groups, by some way they are alike or similar. What is the similarity?

1. Similarity: divided into 2 parts, all are green
2. Similarity: all have one part shaded
3. Similarity: all are entirely shaded
4. Similarity: one-half of all shaded
5. Similarity: one-third of all shaded
6. Similarity: three-fourths of all shaded
7. Similarity: less than one-half shaded
8. Similarity: all but one part shaded

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2. Write a fraction on the chalkboard, e.g., ¾. Ask students to find a bar to match the fraction or draw a picture. Repeat with other fractions.
3. Say a fraction name aloud, e.g., 7/12. Ask students to find a bar to match the fraction or draw a picture. Repeat with other fractions.

What's My Secret?

With a partner or small group, students take turns selecting a subset of fraction bars which are alike in one way. Others in the group try to guess the secret. Demonstrate an example by showing all the bars of one color and have students guess the secret of the sorting. Other ways the students will sort by: everything shaded, nothing shaded, one part shaded, equivalent parts (such as ½, 2/4, 3/6, 5/10, 6/12) shaded, more than ½ shaded, and so on.

About This Page

This page provides examples of ways students may have sorted their fraction bars. Illustrate the first problem with overhead fraction bars or by drawing a picture.

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Adding and Subtracting Fractions with Like Denominators

Problem	Notes and drawing with fraction bars
$\frac{2}{6}+\frac{3}{6}$	
$\frac{4}{10}+\frac{3}{10}$	
$\frac{4}{10}-\frac{3}{10}$	
$\frac{11}{12}+\frac{7}{12}$	
$\frac{11}{12}-\frac{7}{12}$	

Some More Key Questions to Ask When Teaching Fractions

Adding and Subtracting Fractions with Unlike Denominators

Problem	Notes and drawing with fraction bars
$\frac{1}{3}+\frac{2}{5}$	
$\frac{2}{3}-\frac{3}{6}$	
$\frac{3}{4}+\frac{2}{3}$	
$\frac{3}{4}-\frac{2}{3}$	

Very important for students to create their own strips so they will be able to re-create it when needed.

Multiples Strips

1															
2															
3															
4															
5															
6															
7															
8															
9															
10															
11															
12															
13															
14															
15															

Sample of completed multiples strips

Multiples Strips

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

Division

What does 12 ÷ 4 mean?

What does $\frac{2}{3} \div \frac{1}{6}$ mean?

Problem	In words and then drawing with fraction bars	Solution
$\frac{2}{3} \div \frac{1}{12}$		
$\frac{5}{10} \div \frac{1}{5}$		
$1 \div \frac{1}{2}$		
$1 \div \frac{2}{3}$		

Problem	In words and then drawing with fraction bars	Solution
$1 \div \frac{3}{4}$		
$\frac{8}{10} \div \frac{1}{10}$		
$2 \div \frac{1}{6}$		
$\frac{1}{4} \div \frac{3}{4}$		

Mathematics | Grade 3

Number and Operations—Fractions[5]	3.NF
Develop understanding of fractions as numbers.	
1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.	
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.	
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

Mathematics | Grade 4

Number and Operations—Fractions[3]	4.NF
Extend understanding of fraction equivalence and ordering.	
1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record results of comparisons with symbols >, =, or <, justify the conclusions, e.g., by using a visual fraction model.	
Build fractions from unit fractions by applying & extending previous understandings of operations on whole numbers.	
3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$. a. Understand addition/subtraction of fractions as joining/separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition &subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	

[3] Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Mathematics | Grade 5

Number and Operations—Fractions	5.NF
Use equivalent fractions as a strategy to add and subtract fractions.	
1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*	
2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*	
Apply and extend previous understandings of multiplication and division to multiply and divide fractions.	
3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-lb sack of rice equally by weight, how many lbs of rice should each person get? Between what two whole numbers does your answer lie?*	
4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)* b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	

5. Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n\ b)$ to the effect of multiplying a/b by 1.	
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1] a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 =1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally?	

[1] Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.