# High-Leverage Actions Ensure All Your Students Are Common Core Ready

**Diane J. Briars**

**President-Elect, NCTM**

**djbmath@comcast.net**

***NCTM 2013 Las Vegas Regional Conference***

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# FYI

Electronic copies of slides are available by request

djbmath@comcast.net

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Today's Goals

- Analyze CCSS content knowledge and mathematical practices for middle grades and high school students.
- Identify high-leverage actions and instructional practices to help students meet these expectations.

# High-Leverage Actions

Research-informed actions that produce the greatest benefits for your efforts.

# Reactions to CCSS

Andy Isaacs, 2011

# How Familiar Are You with CCSS-M?

*Rate your knowledge*

*on a scale of*

*6 (high) to*

*1 (low)*

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Key Features of CCSS-M

- **Focus:** Focus strongly where the standards focus.
- **Coherence**: Think across grades, and link to major topics
- **Rigor:** In major topics, pursue conceptual understanding, procedural skill and fluency, and application

# Curriculum Standards, Not Assessment Standards

**Define, evaluate, and compare functions. (8.F)**

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

# Dimensions of Mathematical Understanding in CCSS

## Skill-algorithm understanding

from the rote application of an algorithm through the selection and comparison of algorithms to the invention of new algorithms (calculators and computers included)

## Property-proof understanding

from the rote justification of a property through the derivation of properties to the proofs of new properties

## Use-application understanding

from the rote application of mathematics in the real world through the use of mathematical models to the invention of new models

## Representation-metaphor understanding

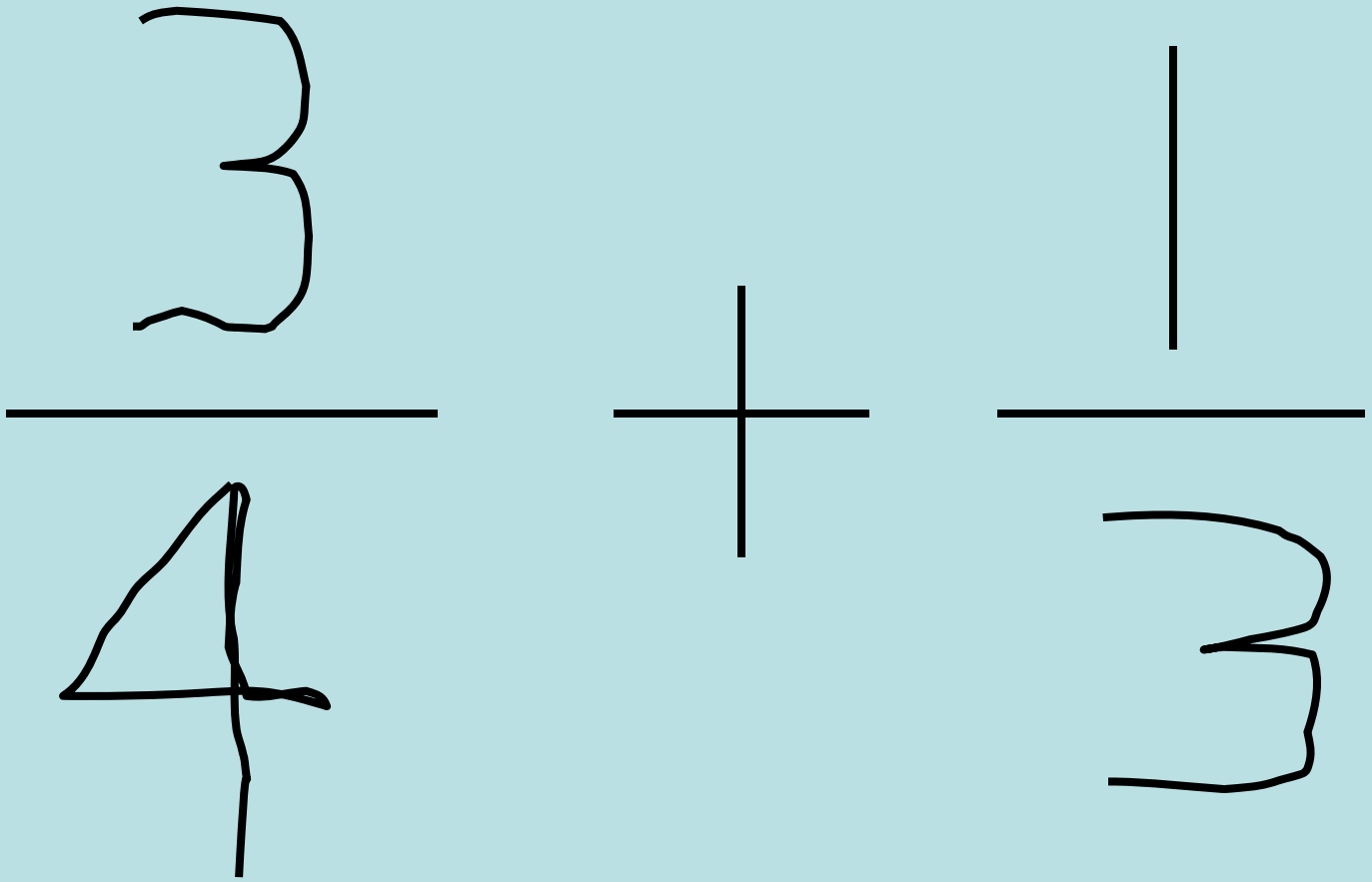
from the rote representations of mathematical ideas through the analysis of such representations to the invention of new representations

## Vocabulary

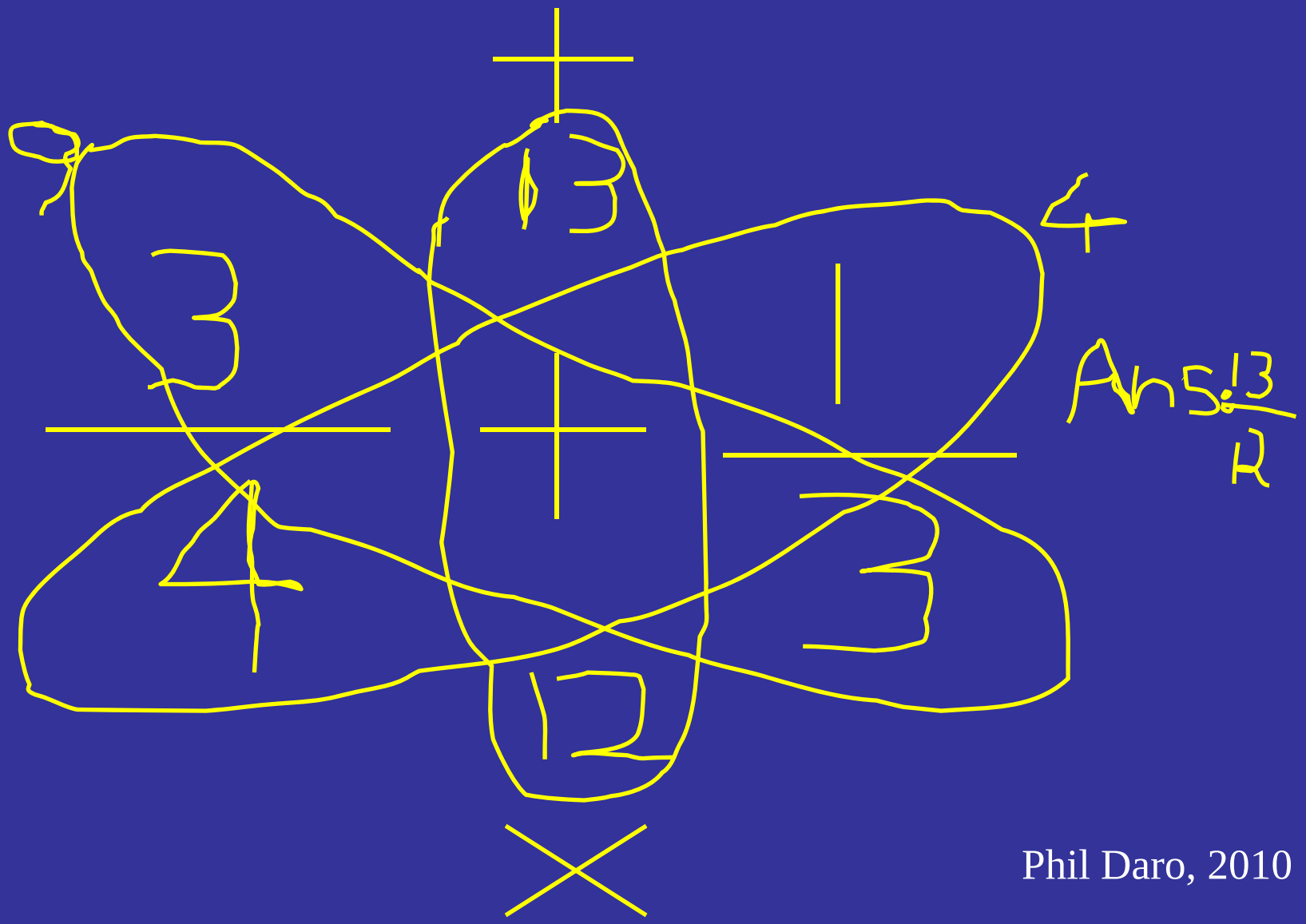
## Problem Solving

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Zalman Usiskin, 2012



Phil Daro, 2010




Phil Daro, 2010

# Other “Butterflies”?

# Key Instructional Shift

From emphasis on:

**How to get answers**

To emphasis on:

**Understanding mathematics**

# High-Leverage Action 1

Teach for ***mathematical understanding***, not ***answer-getting***.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

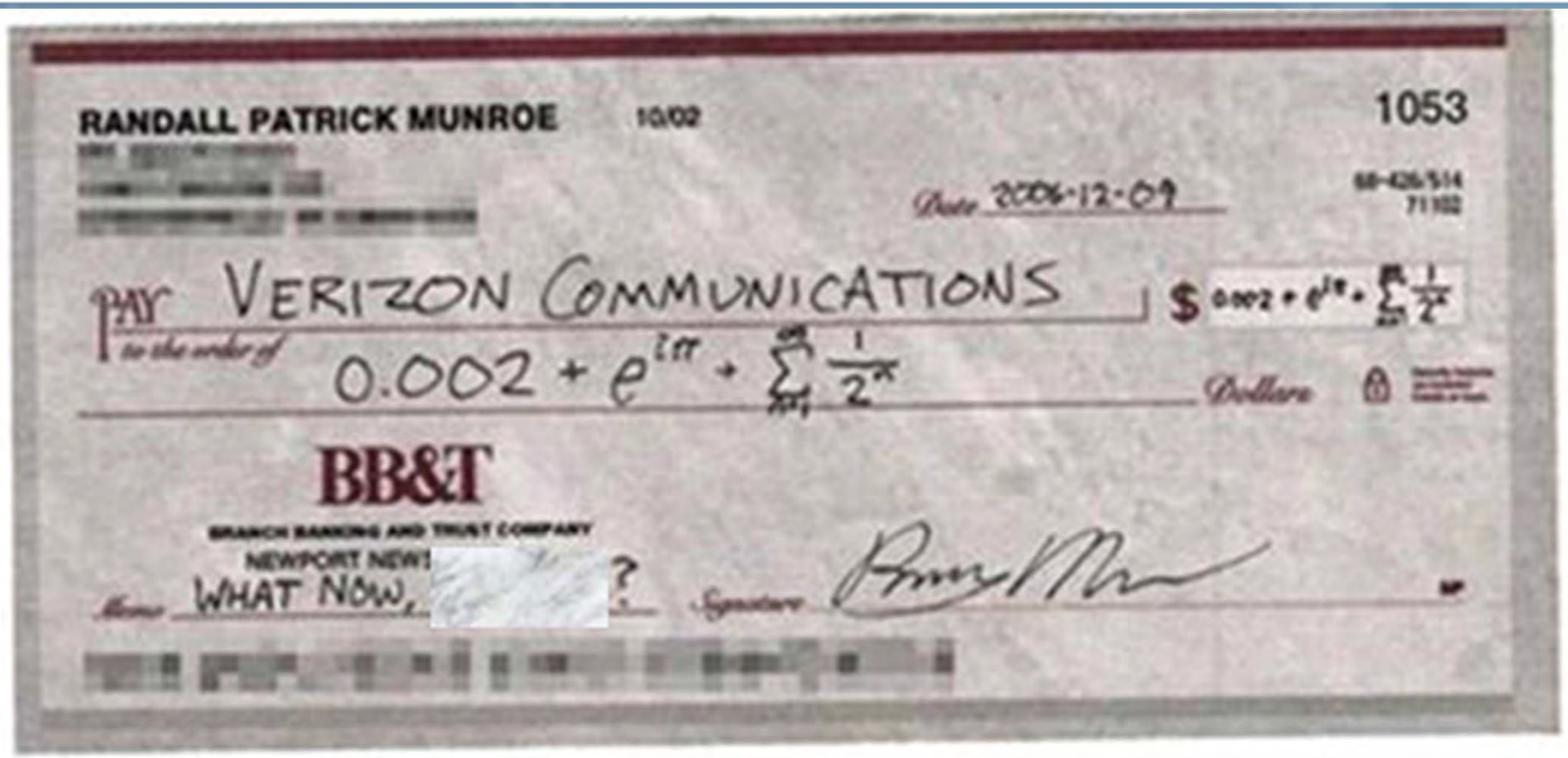
# Consider This Situation:


..\..\Videos\RealPlayer Downloads\verizon operator gets math lesson - YouTube.flv

# Verizon Call

- What is the caller's issue?
- What mathematics are the Verizon employees able to do? What do they not understand?
- Take a moment and then share with a "shoulder partner".

# The Outcome




NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Verizon Call

- What would your middle school or high school students do in this situation?
- What does this have to do with CCSS?

# Common Core State Standards for Mathematics

Two type of standards:

- **Standards for Mathematical Practice**
- Standards for Mathematical Content

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Standards for Mathematical Practice

"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education."

(CCSS, 2010)

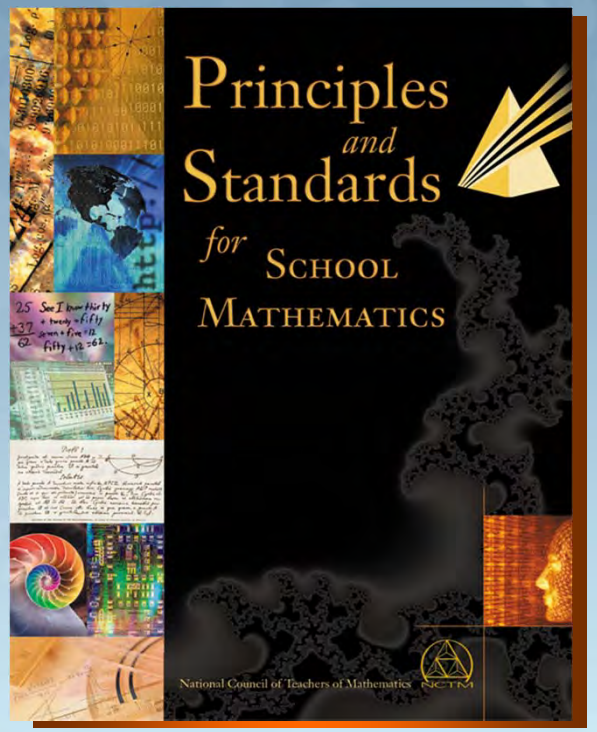
NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Underlying Frameworks

## National Council of Teachers of Mathematics

### 5 **Process** Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations

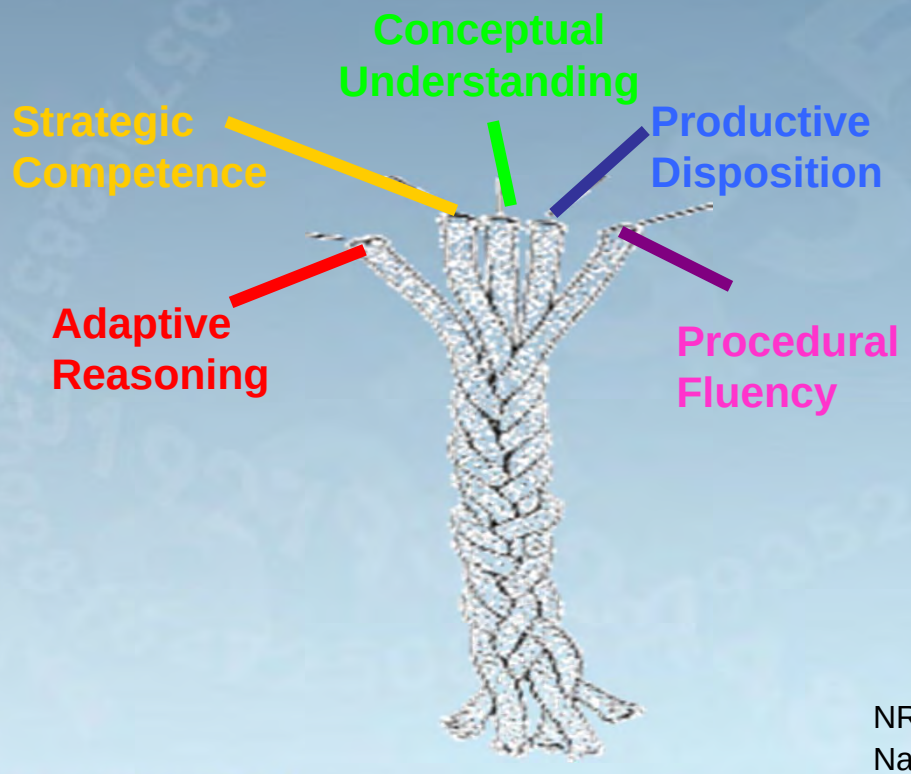



NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Underlying Frameworks

## Strands of Mathematical Proficiency





NRC (2001). *Adding It Up*. Washington, D.C.: National Academies Press.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Strands of Mathematical Proficiency

- ***Conceptual Understanding*** – comprehension of mathematical concepts, operations, and relations
- ***Procedural Fluency*** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic Competence*** – ability to formulate, represent, and solve mathematical problems
- ***Adaptive Reasoning*** – capacity for logical thought, reflection, explanation, and justification
- ***Productive Disposition*** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

# Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

**2. Reason abstractly and quantitatively**

**3. Construct viable arguments and critique the reasoning of others**

Reasoning and explaining

**4. Model with mathematics**

**5. Use appropriate tools strategically**

Modeling and using tools

**7. Look for and make use of structure.**

**8. Look for and express regularity in repeated reasoning.**

Seeing structure and generalizing

# Verizon Call

- Which mathematical ***practices*** are needed to complete the task?
- In which mathematical ***practices*** were the Verizon operators lacking proficiency?

# SMP 2:
# Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations.

They bring two complementary abilities to bear on problems involving quantitative relationships:

- the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
- the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# High-Leveral Action 2.
# Develop Students' Proficiency in the Standards for Mathematical Practice

- Analyze the Standards for Mathematical Practice
- Use the Mathematical Practice as the primary vehicle for learning the Content Standards
- Provide opportunities for students to develop the Standards for Mathematical Practice as "habits of mind" (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards

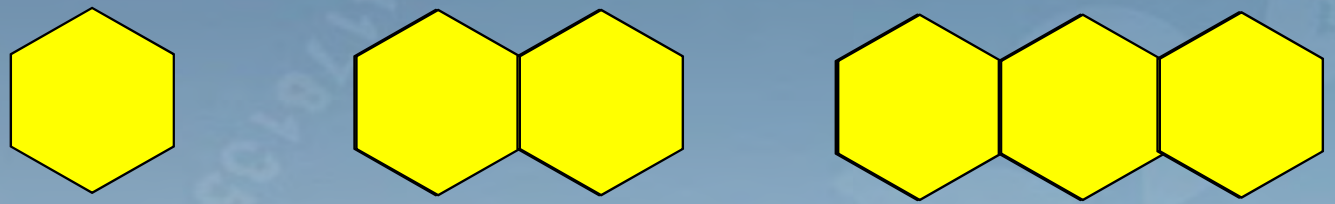
# Grade 6:
# Expressions and Equations

**Apply and extend previous understandings of arithmetic to algebraic expressions.**

**6.EE.3.** Apply the properties of operations to generate equivalent expressions.

**6.EE.4.** Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).

# Hexagon Trains



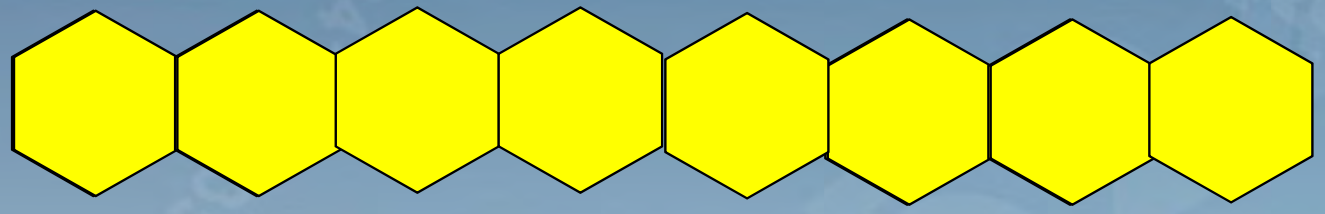
Train 1 Train 2 Train 3

- Compute the perimeter for the first four trains.
- Determine the perimeter for the tenth train without constructing it.
- Write a description /expression that could be used to compute the perimeter of any train in the pattern.
- Find as many different ways as you can to represent the perimeter of any train.

# What Mathematical Practices Did You Use?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# Hexagon Trains



- Explain what each student was thinking to find the perimeter of the $n^{th}$ train.
- Connect your explanation to the picture of the tables.

  Terri: $1 + 4n + 1$

  Tim: $1 + 2(2n) + 1$

  Jerry: $5 + 4(n - 2) + 5$

  Linda: Multiply n times 6, then subtract n-1 times 2.

# What Mathematical Practices Did You Use?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# Grade 7: The Number System

**7.NS.1.** Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

***What instructional and/or assessment tasks would you use?***

# Properties of Integer Addition and Subtraction

Decide if the statements below are true or false. Then, justify your answer mathematically; that is, explain your reasoning in a way that will convince someone else that you are correct.

1. The sum of a negative integer and a positive integer is always positive.
2. The sum of two negative integers is always negative.
3. The difference between two negative integers is always positive.

# Properties of Integer Addition and Subtraction

Ms. Lora is discussing properties of arithmetic with integers with students, asking them to say whether a statement is true or false and provide some reasoning to justify their conclusion.

1. For the statement "The sum of a negative integer and a positive integer is always positive." Keisha says "This is false. The sum *can* be positive, like 10 + -3 = 7. But, it can also be negative. For example, -9 + 3 is -6."

   Is Keisha's reasoning correct? Explain why you think so.

# Properties of Integer Addition and Subtraction

2. For the statement "The sum of two negative integers is always negative." Mike says, "This is true. I tried lots of examples, like -3 + -2, -10 + -27, and even ones with big numbers, like -2,000 + -5,000. All the sums were negative. So this must be true."

   Did Mike prove that the sum of two negative integers is always negative? Explain why you think so.

# Properties of Integer Addition and Subtraction

3. For the same statement "The sum of two negative integers is always negative." Dev says, "I agree with Mike that the statement is true, but I don't think giving examples is good enough to prove that it is always true. I wonder if I could use the number line to show that when you add two negative numbers together, the sum is always negative?"

   Is Dev's reasoning correct? Explain why you think so.

   How could Dev use a number line to prove that the sum of two negative integers is always negative?

# Properties of Integer Addition and Subtraction

Which mathematical ***practices*** are needed to complete this task?

# Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Engaging Students in the Standards for Mathematical Practice

**SMP 3. Construct viable arguments and critique the reasoning of others**

- Make conjectures
- Justify their conclusions and communicate them to others
- **Recognize and use counterexamples**
- Compare the effectiveness of two plausible arguments
- Listen and respond to the arguments of others for sense making and clarity

**SMP 8: Look for and express regularity in repeated reasoning.**

- Notice if calculations are repeated
- Look both for general methods and for shortcuts.
- Maintain oversight of the process of solving a problem, while attending to the details.
- Continually evaluate the reasonableness of intermediate results.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Promoting Proficiency in the Standards for Mathematical Practice and Conceptual Understanding

"Not all tasks are created equal, and ***different tasks will provoke different levels and kinds of student thinking***."

Stein, Smith, Henningsen, & Silver, 2000

"***The level and kind of thinking in which students engage determines what they will learn***."

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Higher-Level Tasks

- Doing mathematics
  - Hexagon Train Task
- Procedures with connections
  - Using a 10 x 10 grid, identify the decimal and percent equivalents of 3/5.
  - Explain how the graph of $y = -3x^2 + 7$ compares to the graph of $y = x^2$. Sketch the graph of $y = -3x^2 + 7$.

# Lower-Level Tasks

- Memorization
  - What are the decimal equivalents for the fractions ½ and ¼?
  - What is sin 30°, cos 30°and tan 30°?
- Procedures without connections
  - Convert the fraction 3/8 to a decimal.
  - A rectangular carpet is 12 feet long and 9 feet wide. What is the area of the carpet in square feet?
  - Factor $x^2 - 3x - 10$

# Research Tells Us

Learners should:

- Acquire conceptual knowledge as well as skills to enable them to organize their knowledge, transfer knowledge to new situations, and acquire new knowledge.
- Engage with challenging tasks that involve active meaning-making

# Core Implementation Issue

Do *all* students have the opportunity to engage in mathematical tasks that promote students' attainment of the mathematical practices on a regular basis?

# Opportunities for *all* students to engage in challenging tasks?

- Examine tasks in your instructional materials:
  - Higher cognitive demand?
  - Lower cognitive demand?
- Where are the challenging tasks?
- Do *all* students have the opportunity to grapple with challenging tasks?
- Examine the tasks in your assessments:
  - Higher cognitive demand?
  - Lower cognitive demand?

# High-Leverage Action 3

Regularly incorporate high cognitive demand tasks into your instruction and assessment.

# Common Core State Standards for Mathematics

Two type of standards:

- Standards for Mathematical Practice
- **Standards for Mathematical Content**

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Key Advances in Content Grades 6 – 12

- Ratios and Proportional Relationships
- The Number System
- Expressions and Equations
- Functions
- Modeling

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Strategies for Solving Proportions

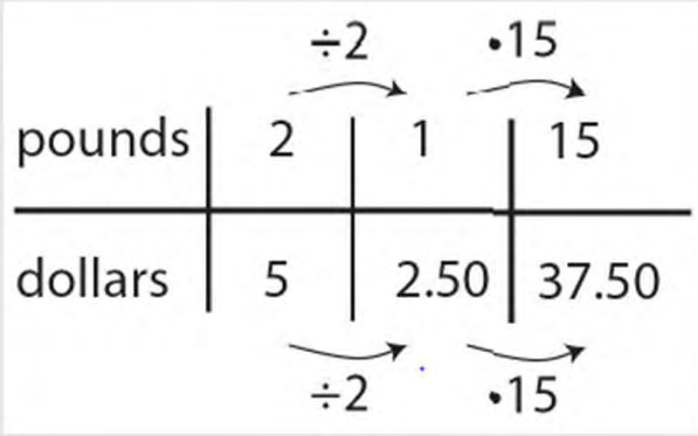
If 2 pounds of beans cost $5, how much will 15 pounds of beans cost?

# If 2 pounds of beans cost $5, how much will 15 pounds of beans cost?

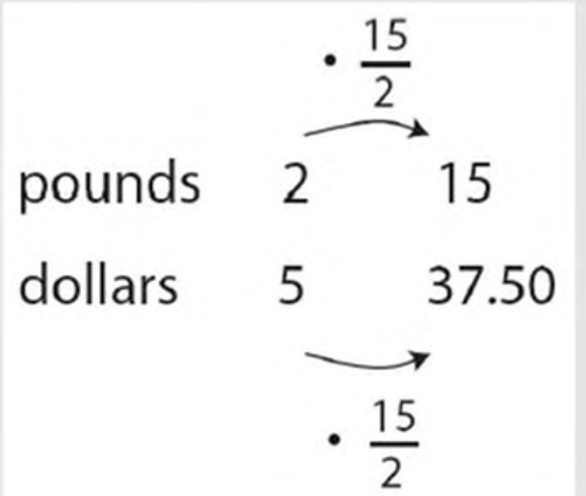
## Method 1

| pounds | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 1 | 15 |
|---|---|---|---|---|---|---|---|---|---|
| dollars | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 2.50 | 37.50 |

## Method 2




## Method 3




NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# If 2 pounds of beans cost $5, how much will 15 pounds of beans cost?

CCSS de-emphasizes means/extremes as solution method

$$\frac{2}{5} = \frac{15}{x}$$

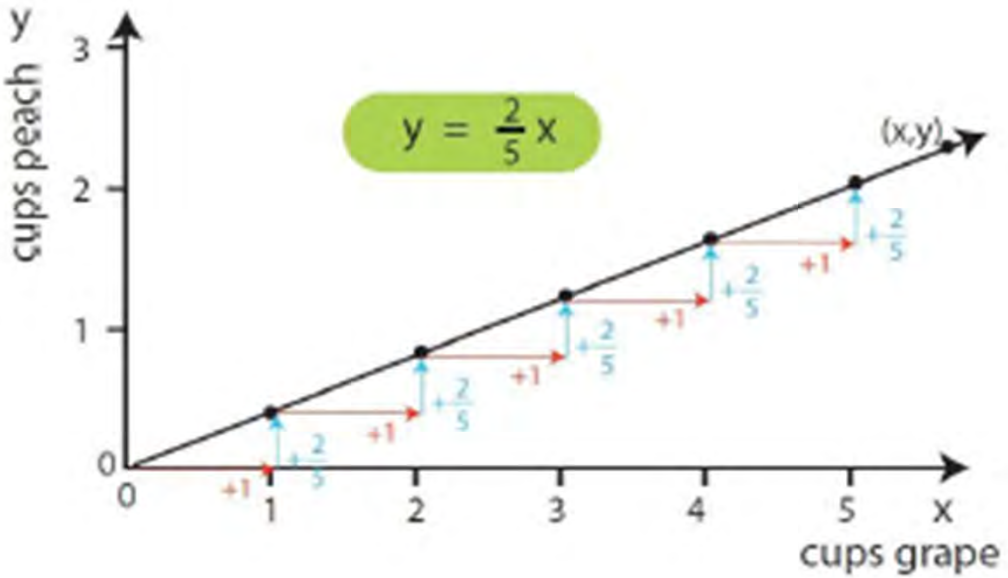
$$2x = 5 \cdot 15$$

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

## Correspondence among a table, graph, and equation of a proportional relationship

For every 5 cups grape juice, mix in 2 cups peach juice.

| x cups grape | y cups peach |
|---|---|
| (0) | (0) |
| 5 | 2 |
| 1 | $\frac{2}{5}$ |
| 2 | $2 \cdot \frac{2}{5}$ |
| 3 | $3 \cdot \frac{2}{5}$ |
| 4 | $4 \cdot \frac{2}{5}$ |
| x | $x \cdot \frac{2}{5}$ |




On the graph: For each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

When you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.

When you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.

When you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.

When you go $x$ units to the right, you go up $x \cdot \frac{2}{5}$ units.

Starting from $(0, 0)$, to get to a point $(x, y)$ on the graph, go $x$ units to the right, so go up $x \cdot \frac{2}{5}$ units.

Therefore $y = x \cdot \frac{2}{5}$ $\quad y = \frac{2}{5}x$

# Ratios & Proportional Relationships

Key Advances:

- Emphasize understanding unit rates associated with ratios.
- Expect students to represent proportional relationships by tables, equations, and graphs, and understand informally that that unit rate indicates the steepness of the graph of the line (informal introduction to slope).
- Expect students to solve problems involving proportional relationships using various methods, such as equivalent ratios and unit rates.

# Number and Operations in Base Ten

Conceptual Understanding Facilitates Skill Acquisition:

1. Develop conceptual understanding building on students' informal knowledge
2. Develop informal strategies to solve problems
3. Refine informal strategies to develop fluency with standard procedures (algorithms)

| | Numbers and Operations in Base Ten |
|---|---|
| 1 | Use place value understanding and properties of operations to add and subtract. |
| 2 | Use place value understanding and properties of operations to add and subtract. |
| 3 | Use place value understanding and properties of operations to perform multi-digit arithmetic.<br>***A range of algorithms may be used.*** |
| 4 | Use place value understanding and properties of operations to perform multi-digit arithmetic.<br>***Fluently add and subtract multi-digit whole numbers using the standard algorithm.*** |
| 5 | Perform operations with multi-digit whole numbers and with decimals to hundredths.<br>***Fluently multiply multi-digit whole numbers using the standard algorithm.*** |
| 6 | Compute fluently with multi-digit numbers and find common factors and multiples.<br>***Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.*** |

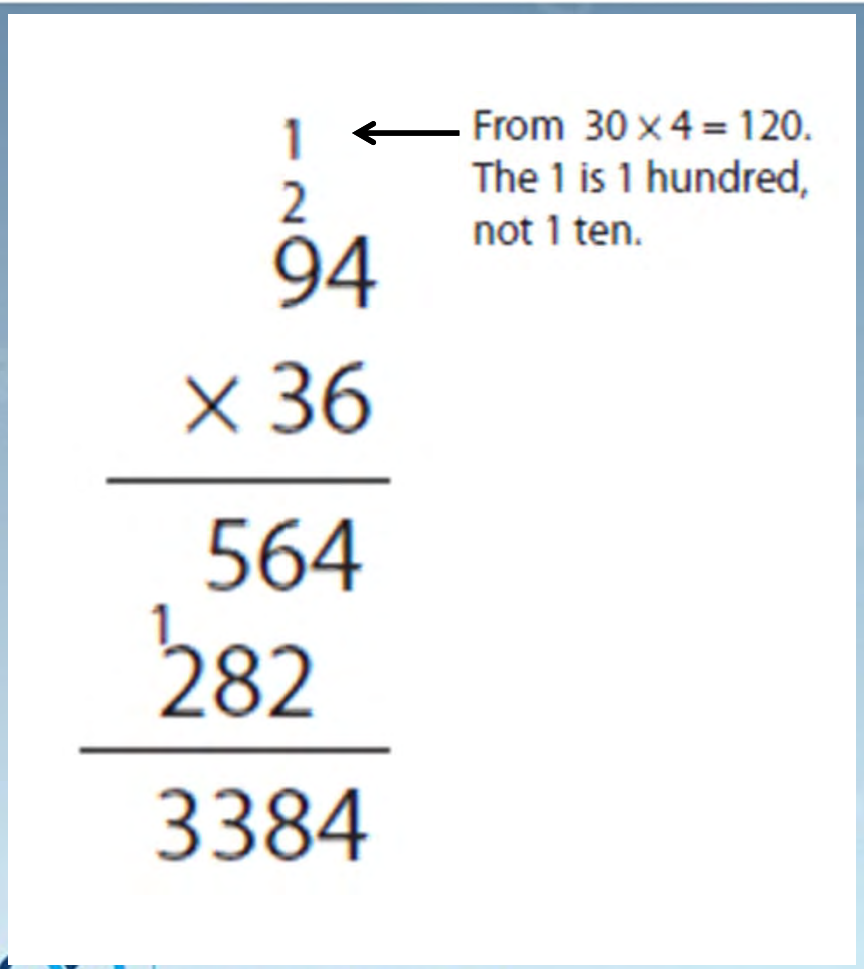
# What is Meant by "Standard Algorithm?"

"In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, **minor variations in methods of recording standard algorithms are acceptable**."

(Fuson & Beckmann, 2013; NBT, p13)

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Multiplication Algorithms



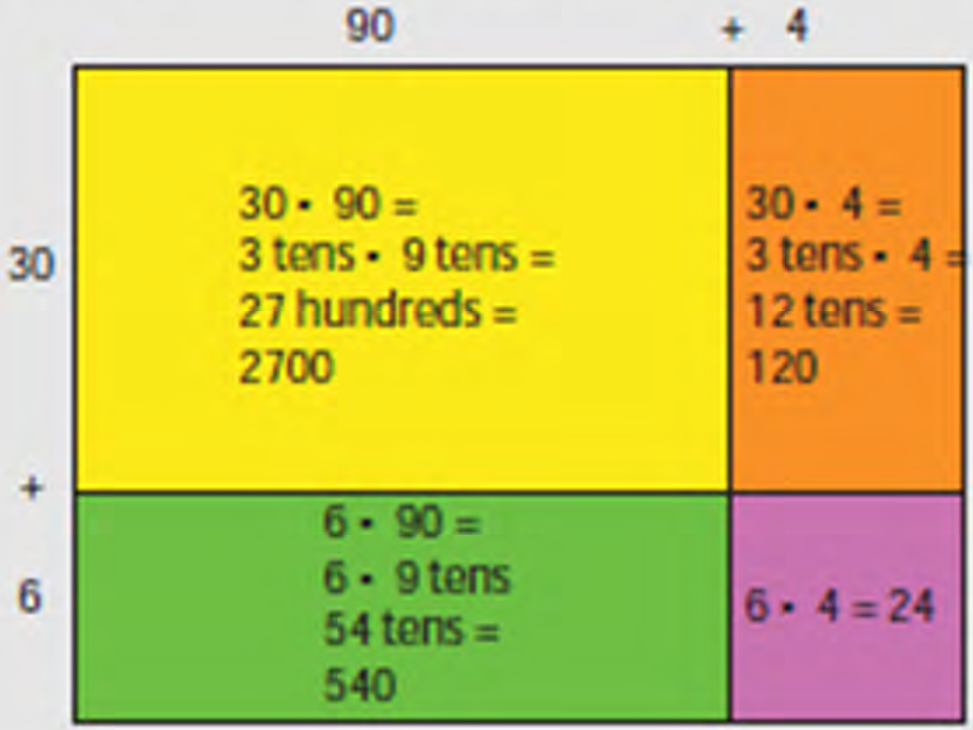

$$
\begin{array}{rcl}
94 & = & 90 + 4 \\
\times\, 36 & = & 30 + 6 \\
\hline
6 \times 4 & = & 24 \\
6 \times 90 & = & 540 \\
30 \times 4 & = & 120 \\
30 \times 90 & = & 2700 \\
 & & \overset{1}{\phantom{0}} \\
 & & 3384
\end{array}
$$

NCTM NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Fuson & Beckmann, 2013, p. 25

# Multiplication Algorithms

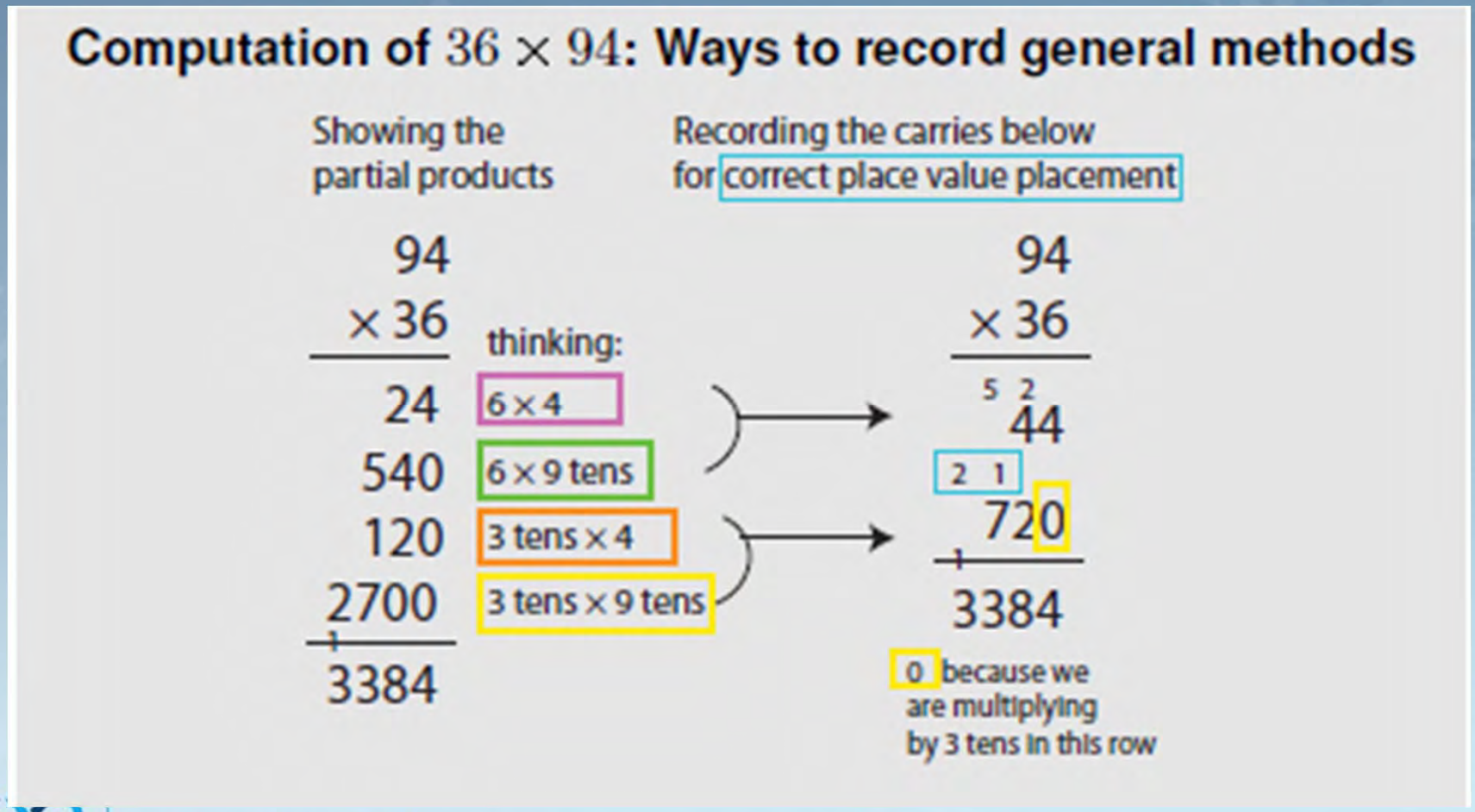
**Computation of $36 \times 94$ connected with an area model**




*The products of like base-ten units are shown as parts of a rectangular region.*

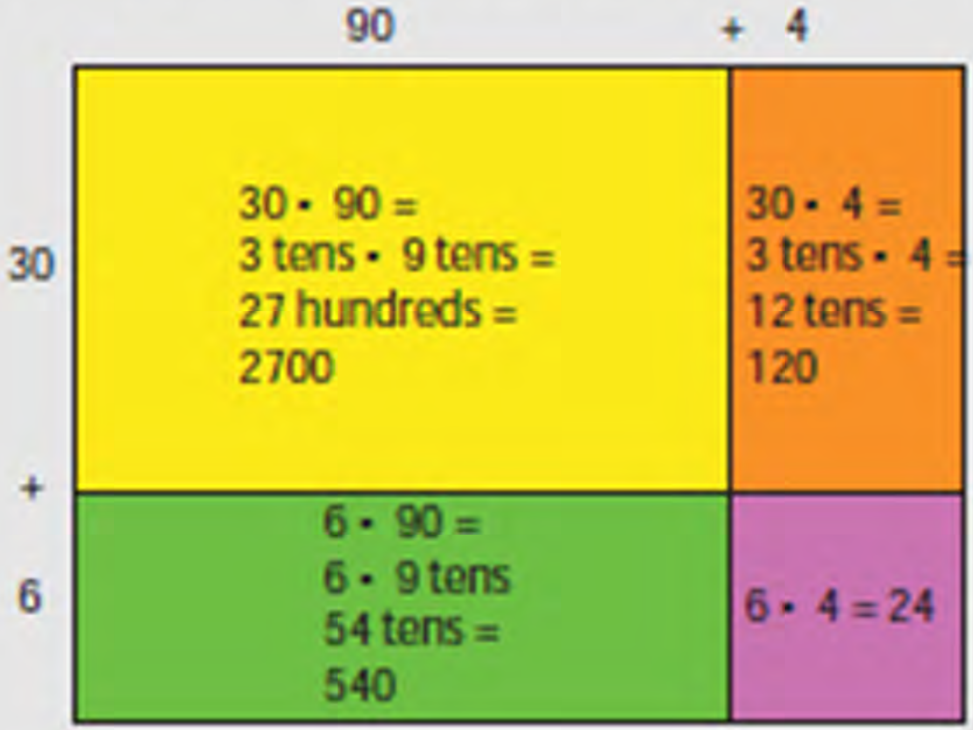
CCSS Numbers and Operations in Base-Ten Progression, April 2011

# Multiplication Algorithms



# Multiplication Algorithms

**Computation of $36 \times 94$ connected with an area model**




*The products of like base-ten units are shown as parts of a rectangular region.*

CCSS Numbers and Operations in Base-Ten Progression, April 2011

# “Rewrite” vs. “Simplify”

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$p^{-3/5} = \frac{1}{p^{3/5}} = \frac{\sqrt[5]{p^2}}{p}$$

# The Number System

Key Advances:

- Algorithms: Standard multiplication algorithm; proficiency with standard division algorithm delayed until grade 6.
- Emphasize on number line models, e.g., absolute value introduced in Grade 6 with negative numbers.
- Bi-directionality of expectations: use rational numbers to solve problems and create problems that can be modeled by particular operations.
- Emphasis on properties, especially distributive property.
- “Rewriting” instead of “simplifying”; writing fractions in lowest terms not a major emphasis.

# Expressions and Equations

Key Advances:

- Using variables and algebraic expressions and equations to describe situations and solve problems in Grade 6.
- Solve equations of the form $px + q = r$ and use the distributive property in Grade 7.
- In Grade 8:
  - All students work with radicals and integer exponents
  - Understand connections between proportional relationships. lines, and linear equations.
  - Analyze and solve linear equations and pairs of linear equations.

# Functions

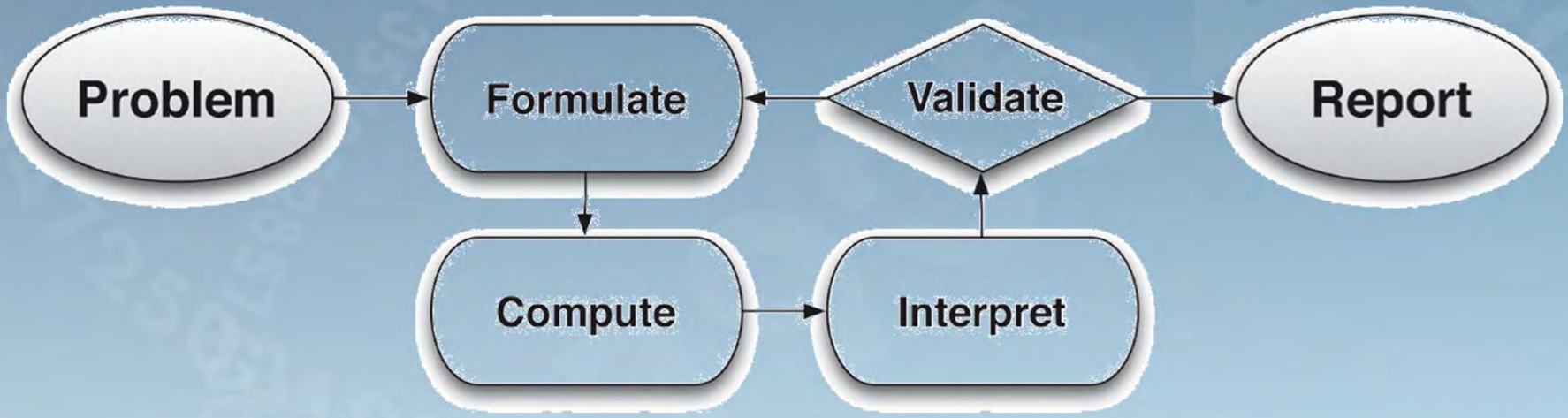
Key Advances:

- In Grade 8:
  - Concept of *function* is introduced
  - Explore and compare functional relationships represented in different ways, with emphasis on linear functions.
  - Use functions to model relationships between quantities.
- Build as well as interpret functions
  - Build functions that model relationships between two quantities
  - Build new functions from existing functions

# Modeling

- Practice K-12
- Content Conceptual Category HS Standards embedded in other categories (★)
- Examples of models:
  - Equations: Writing total cost as a product of unit price and number bought
  - Geometric shape to represent physical object

# CCSS Modeling Cycle




NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Examples of Situations to be Modeled

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.

# What tasks would you use to assess students' proficiency with this standard?

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

# TV Sales-Part A (PARCC Grade 7)

A store is advertising a sale with 10% off all items in the store. Sales tax is 5%.

A 32-inch television is regularly priced at $295.00. What is the total price of the television, including sales tax, if it was purchased on sale? Fill in the blank to complete the sentence. Round your answer to the nearest cent.


The total cost of the television is $ _____ .

Submit Answer

# TV Sales-Part B (PARCC Grade 7)

Write your answers to the following problem in your answer booklet.

A store is advertising a sale with 10% off all items in the store. Sales tax is 5%.

Adam and Brandi are customers discussing how the discount and tax will be calculated.

Here is Adam's process for finding the total cost for any item in the store.

- Take 10% off the original price.
- Then, add the sales tax to the discounted price.

Adam represents his process as:

$$T = \underbrace{0.9p}_{\text{sale price}} + \underbrace{0.05(0.9p)}_{\text{sales tax}}$$

Here is Brandi's process for finding the total cost for any item in the store.

- Determine the original price of the item, including sales tax.
- Then, take 10% off.

Brandi represents her process as:

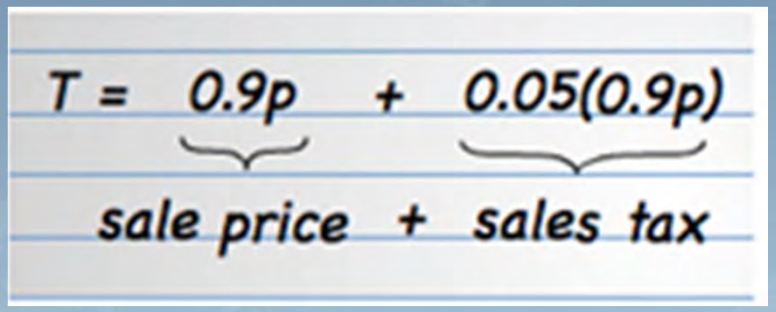
$$T = \underbrace{1.05p}_{\text{T.V. price plus tax}} - \underbrace{0.10(1.05p)}_{\text{10\% off discount}}$$

In both equations, $T$ represents the total cost of the television and $p$ represents the regular price.

Are they both correct? Use the properties of operations to justify your answer.

# TV Sales-Part B (PARCC Grade 7)

## Adam's Process



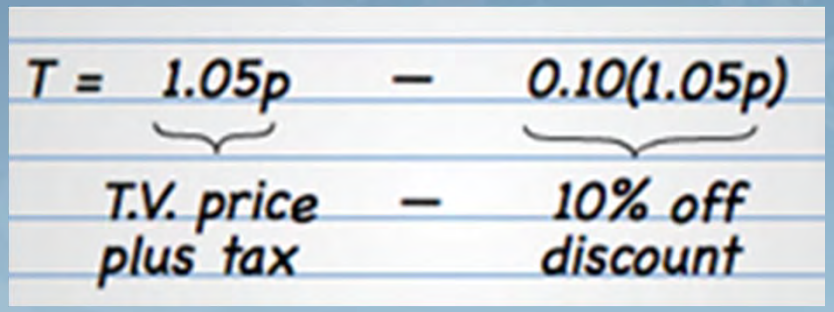

T = (1)0.9$p$ + 0.05(0.9$p$)

= (1 + 0.05)(0.9$p$)

= (1.05)(0.9$p$)

= (1.05)(0.9)$p$

## Brandi's Process




T = (1)(1.05p) – (0.10)(1.05p)

= (1 – 0.10)(1.05$p$)

= (0.9)(1.05$p$)

= (1.05)(0.9)$p$

# Tasks Clarify Expectations

- Range of content
- Depth of knowledge
- Type of reasoning and evidence of it
- Types of applications

# Tasks Clarify Expectations

## PARCC

"[The prototypes] are designed to shine a light on important elements of the CCSS . . . "

## SBAC

"The sample items and tasks illustrate the knowledge and skills students will be expected to demonstrate on the Smarter Balanced assessments, giving educators clear benchmarks to inform their instruction."

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Analyze PARCC & SBAC Prototypes

- Compare and contrast them to typical assessment items.
- To what extent do they:
  - Assess conceptual understanding as well as procedural skills?
  - Include higher cognitive demand tasks? Evidence?
  - Assess the Standards for Mathematical Practice? Evidence?
- What are the characteristics of tasks that assess conceptual understanding and/or the Standards for Mathematical Practice?

# Analyze PARCC & SBAC Prototypes

View actual prototypes at:

PARCC:
http://www.parcconline.org

SBAC:
http://smarterbalanced.org

# High-Leverage Action 4

Analyze the CCSS-M expectations—practices and content--in the original CCSS-M document, in the CCSS-M progression documents, and prototype assessment tasks.

[Common Core State Standards for Mathematics] represent a significant departure from *what* mathematics is currently taught in most classrooms and *how* it is taught. Developing teachers' capacity to enact these new standards in ways that support the intended student learning outcomes will require considerable changes in mathematics instruction in our nation's classrooms. Such changes are likely to occur only through sustained and focused professional development opportunities for those who teach mathematics.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Sztajn, Marrongelle, & Smith, 2011

# High-Leverage Action 5
# Work in Collaborative Teams

- Work in grade level/course and cross-grade level/course teams to develop a common understanding of CCSS content standards
- Analyze progressions of CCSS standards across grades
- Understand and agree upon common unit goals
- Develop common unit assessments and analyze the results
- Develop a plan to transition from current course scope and sequence to CCSS scope and sequence

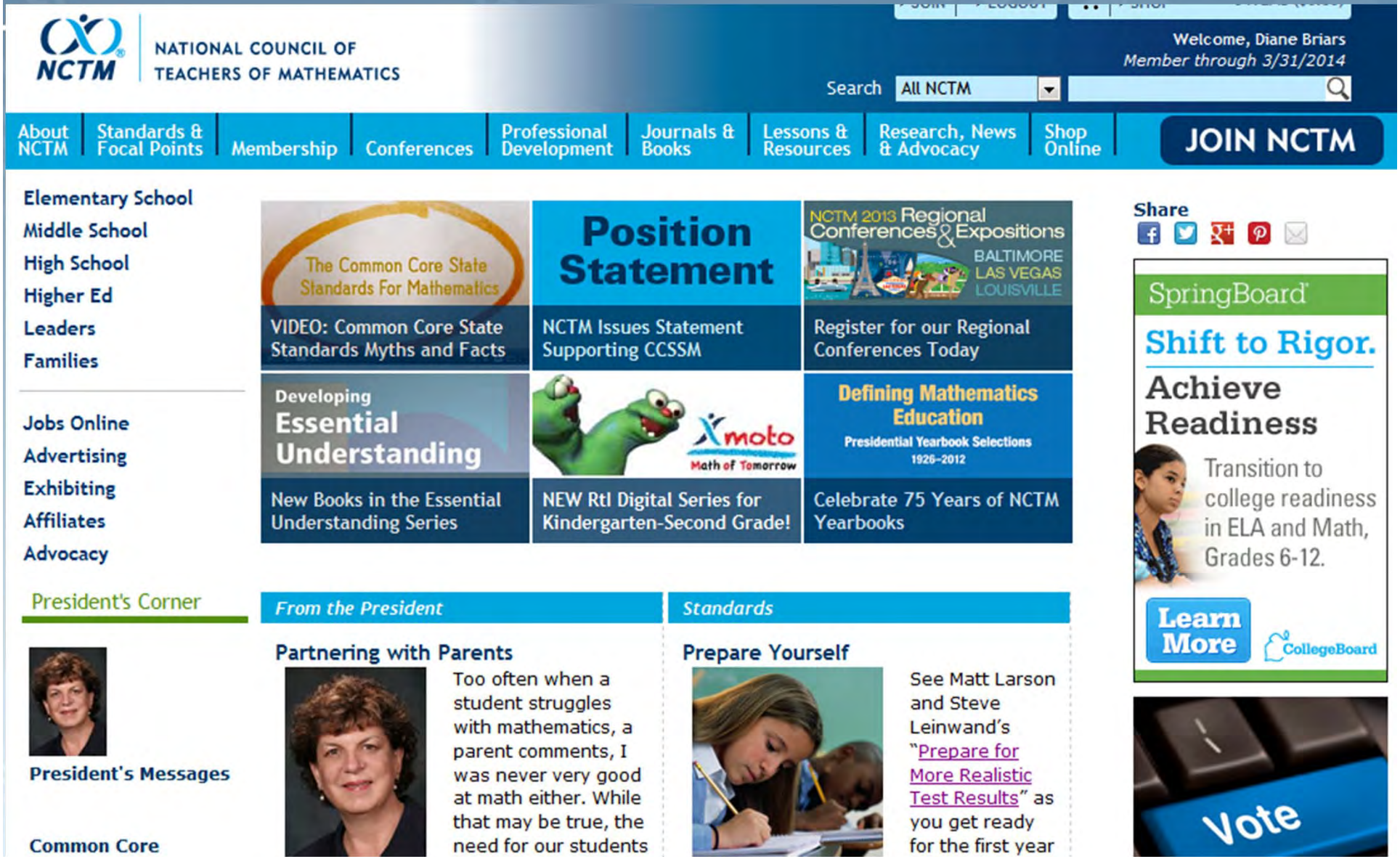
# High-Leverage Actions to Ensure Your Students are CCSS-Ready

1. Teach for ***mathematical understanding***, not ***answer-getting***.
2. Provide regular opportunities for students to develop proficiency in the Standards for Mathematical Practice.
3. Regularly incorporate high cognitive demand tasks into your instruction and assessment.
4. Analyze the CCSS-M expectations—practices and content--in the original CCSS-M document, in the CCSS-M progression documents, and prototype assessment tasks.
5. Work in grade-level/course collaborative teams

# RESOURCES

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# NCTM: www.nctm.org



# NCTM: Illuminations

ILLUMINATIONS
Resources for Teaching Math

Activities | Lessons | Standards | Web Links

NCTM Resourc

## Activities

Explore our library of 108 online activities that help to make math come alive in the classroom or at home

## Lessons

View our collection of 607 lessons for preK-12 math educators

## Standards

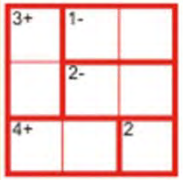
Learn about NCTM's *Principles and Standards for School Mathematics*

## Web Links

Check out hundreds of exemplary online resources, as identified by an editorial panel

***Highlighted Activity***

Daily KenKen®
**for Tuesday, October 1**




KenKen is a logic puzzle that involves arithmetic. Similar to SuDoku, KenKen will exercise your mind as you fill in the blanks. Pick your poison — easy or hard, only addition or all four operations.

Solutions posted the following day... check back tomorrow!

Game On!


When's th
Nation®?
Race to c
with **Dirt**

Check out all 12 games!

### Mobile Apps for PreK-2!


What's that you say? You already love the Concentration game on Illuminations? Well, now it's available as a mobile app (iOS, Android). Now you can play on your phone, your iPad, or your Andriod tablet, anywhere you want!

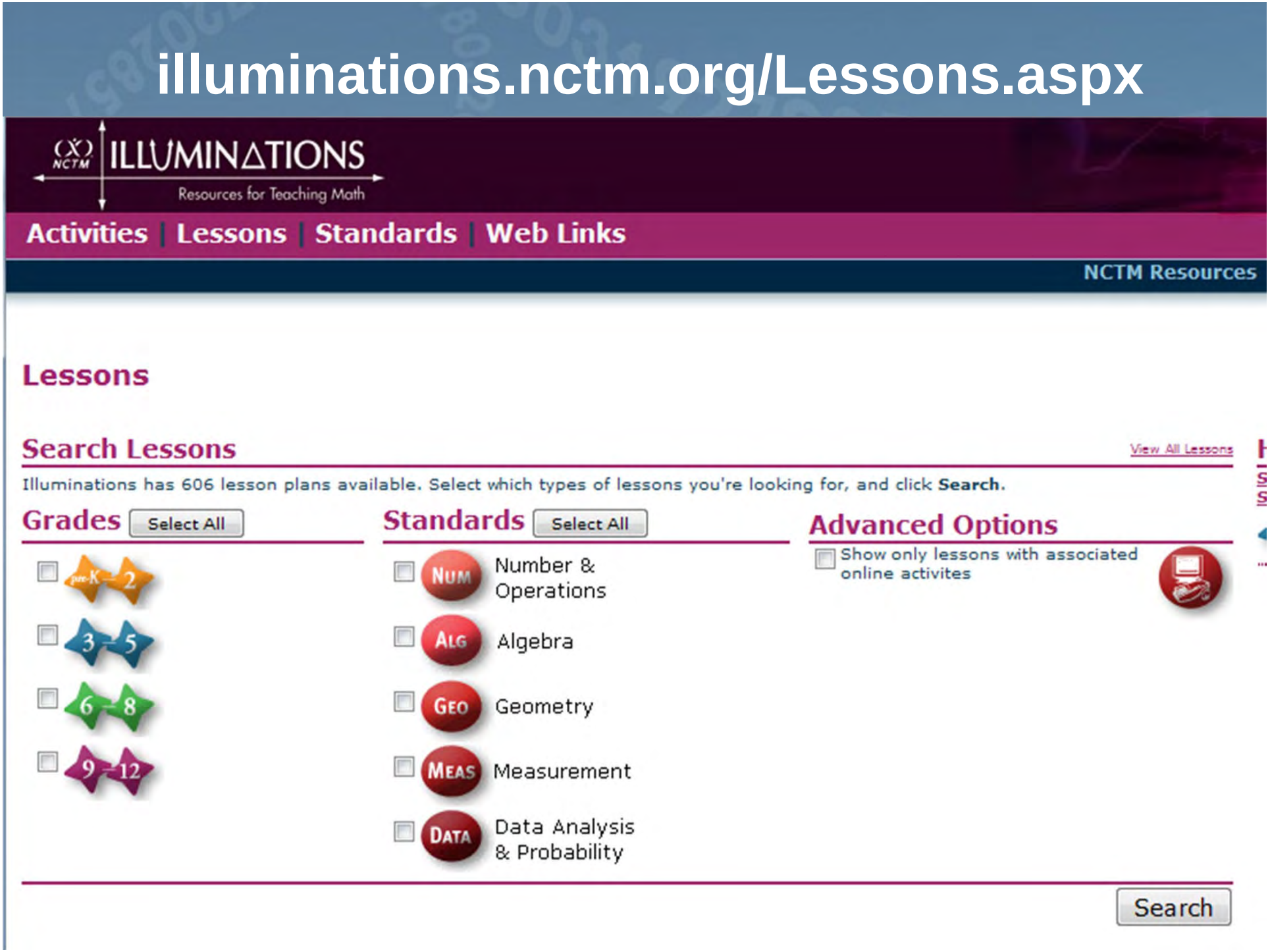
But you'd rather have an app that teaches counting, subitizing, and more? Yeah, we've got an app for that, too. **Okta's Rescue** is also available for mobile devices (iOS, Android).

### Video: Serving Up Classroom Fun!

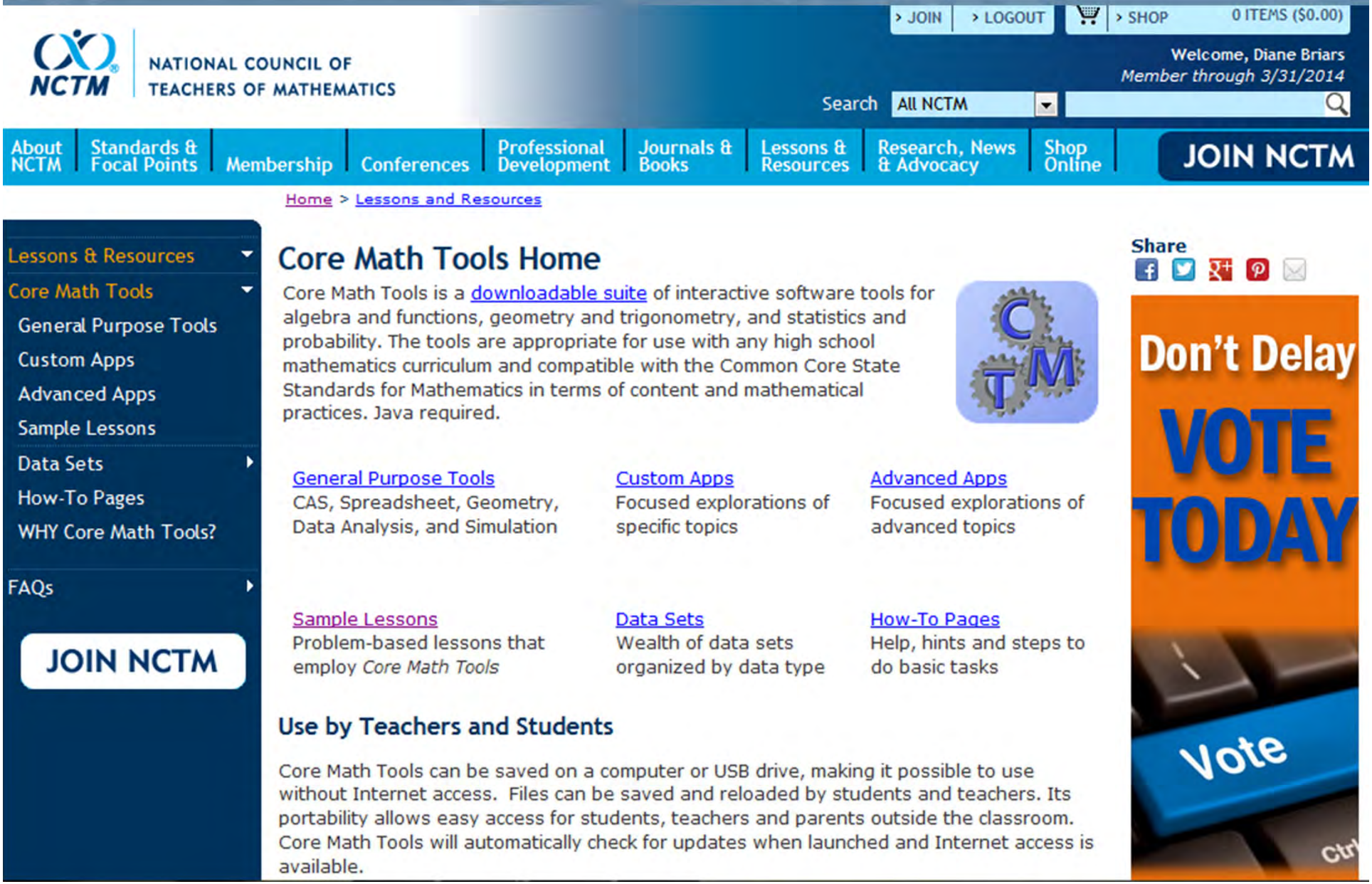
Laurie Penney, a teacher at Willard School in Concord, Mass., was a participant at the 2011 Illuminations Summer Institute. As part of th Institute, she recently helped to develop Illuminations's newest online activity, Quotier Café.

Be the first to know about new Illuminations resources, ne and events by subscribing to our monthly e-newsletter Bri Ideas. Subscribe today!

# illuminations.nctm.org/Lessons.aspx

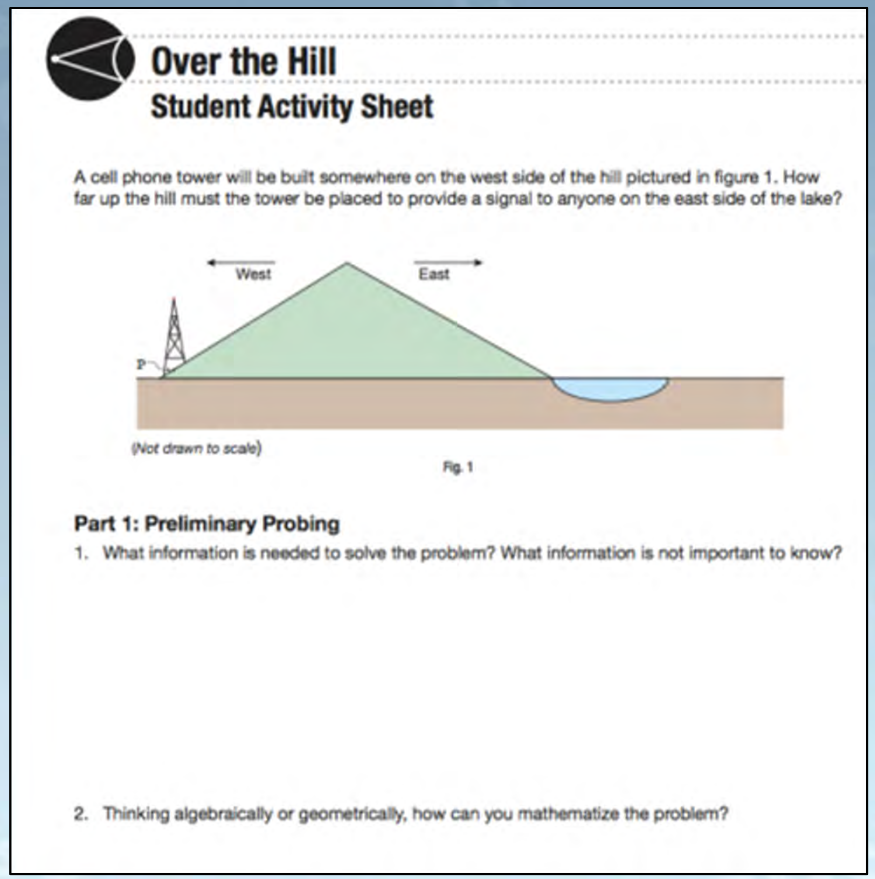


# NCTM: Core Math Tools



# NCTM's High School Reasoning and Sense-Making Task Library (High School)

http://www.nctm.org/rsmtasks



**Over the Hill**

**Student Activity Sheet**

A cell phone tower will be built somewhere on the west side of the hill pictured in figure 1. How far up the hill must the tower be placed to provide a signal to anyone on the east side of the lake?



(Not drawn to scale)

Fig. 1

**Part 1: Preliminary Probing**

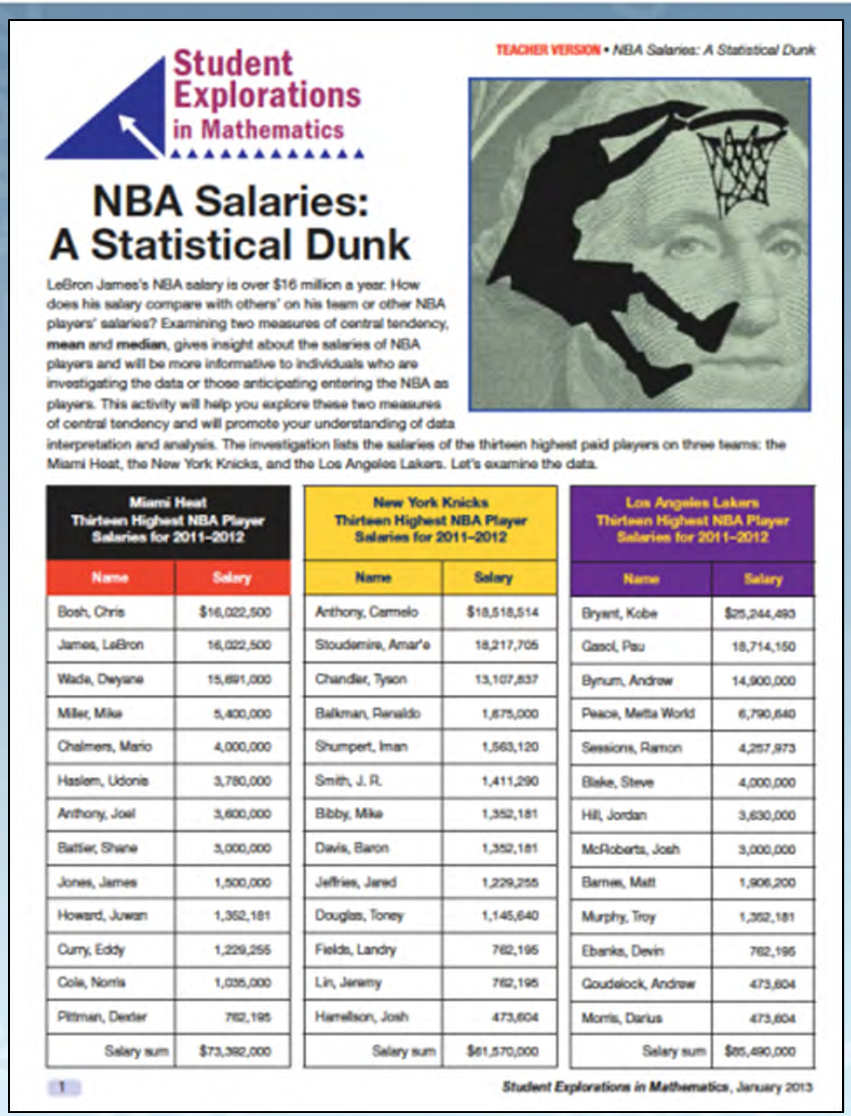
1. What information is needed to solve the problem? What information is not important to know?

2. Thinking algebraically or geometrically, how can you mathematize the problem?

While linking the task directly with NCTM's Focus in *High School Mathematics: Reasoning and Sense Making*, NCTM's *Principles and Standards for School Mathematics*, and the Common Core State Standards, each item addresses:

- **Task Design:** *what the task is asking students to do* (see Task Purpose, Task Overview, Focus on Reasoning and Sense Making, Focus on Mathematical Content, Materials and Technology, Assessment and the Student Activity sheet)
- **Teaching Design:** *how teachers might facilitate reasoning and sense making* (see Use in the Classroom)
- **Student Engagement:** *what student might actually do in the classroom* (see Focus on Student Thinking)

# Student Explorations in Mathematics (SEM)
## (Grades 5-10)



Student Explorations in Mathematics

TEACHER VERSION • *NBA Salaries: A Statistical Dunk*

### NBA Salaries: A Statistical Dunk

LeBron James's NBA salary is over $16 million a year. How does his salary compare with others' on his team or other NBA players' salaries? Examining two measures of central tendency, **mean** and **median**, gives insight about the salaries of NBA players and will be more informative to individuals who are investigating the data or those anticipating entering the NBA as players. This activity will help you explore these two measures of central tendency and will promote your understanding of data interpretation and analysis. The investigation lists the salaries of the thirteen highest paid players on three teams: the Miami Heat, the New York Knicks, and the Los Angeles Lakers. Let's examine the data.

**Miami Heat Thirteen Highest NBA Player Salaries for 2011–2012**

| Name | Salary |
|---|---|
| Bosh, Chris | $16,022,500 |
| James, LeBron | 16,022,500 |
| Wade, Dwyane | 15,691,000 |
| Miller, Mike | 5,400,000 |
| Chalmers, Mario | 4,000,000 |
| Haslem, Udonis | 3,780,000 |
| Anthony, Joel | 3,600,000 |
| Battier, Shane | 3,000,000 |
| Jones, James | 1,500,000 |
| Howard, Juwan | 1,352,181 |
| Curry, Eddy | 1,229,255 |
| Cole, Norris | 1,035,000 |
| Pittman, Dexter | 762,195 |
| Salary sum | $73,392,000 |

**New York Knicks Thirteen Highest NBA Player Salaries for 2011–2012**

| Name | Salary |
|---|---|
| Anthony, Carmelo | $18,518,514 |
| Stoudemire, Amar'e | 18,217,705 |
| Chandler, Tyson | 13,107,837 |
| Balkman, Ronaldo | 1,675,000 |
| Shumpert, Iman | 1,563,120 |
| Smith, J. R. | 1,411,290 |
| Bibby, Mike | 1,352,181 |
| Davis, Baron | 1,352,181 |
| Jeffries, Jared | 1,229,255 |
| Douglas, Toney | 1,145,640 |
| Fields, Landry | 762,195 |
| Lin, Jeremy | 762,195 |
| Hamilton, Josh | 473,604 |
| Salary sum | $61,570,000 |

**Los Angeles Lakers Thirteen Highest NBA Player Salaries for 2011–2012**

| Name | Salary |
|---|---|
| Bryant, Kobe | $25,244,493 |
| Gasol, Pau | 18,714,150 |
| Bynum, Andrew | 14,900,000 |
| Peace, Metta World | 6,790,640 |
| Sessions, Ramon | 4,257,973 |
| Blake, Steve | 4,000,000 |
| Hill, Jordan | 3,630,000 |
| McRoberts, Josh | 3,000,000 |
| Barnes, Matt | 1,906,200 |
| Murphy, Troy | 1,352,181 |
| Ebanks, Devin | 762,195 |
| Goudelock, Andrew | 473,604 |
| Morris, Darius | 473,604 |
| Salary sum | $85,490,000 |

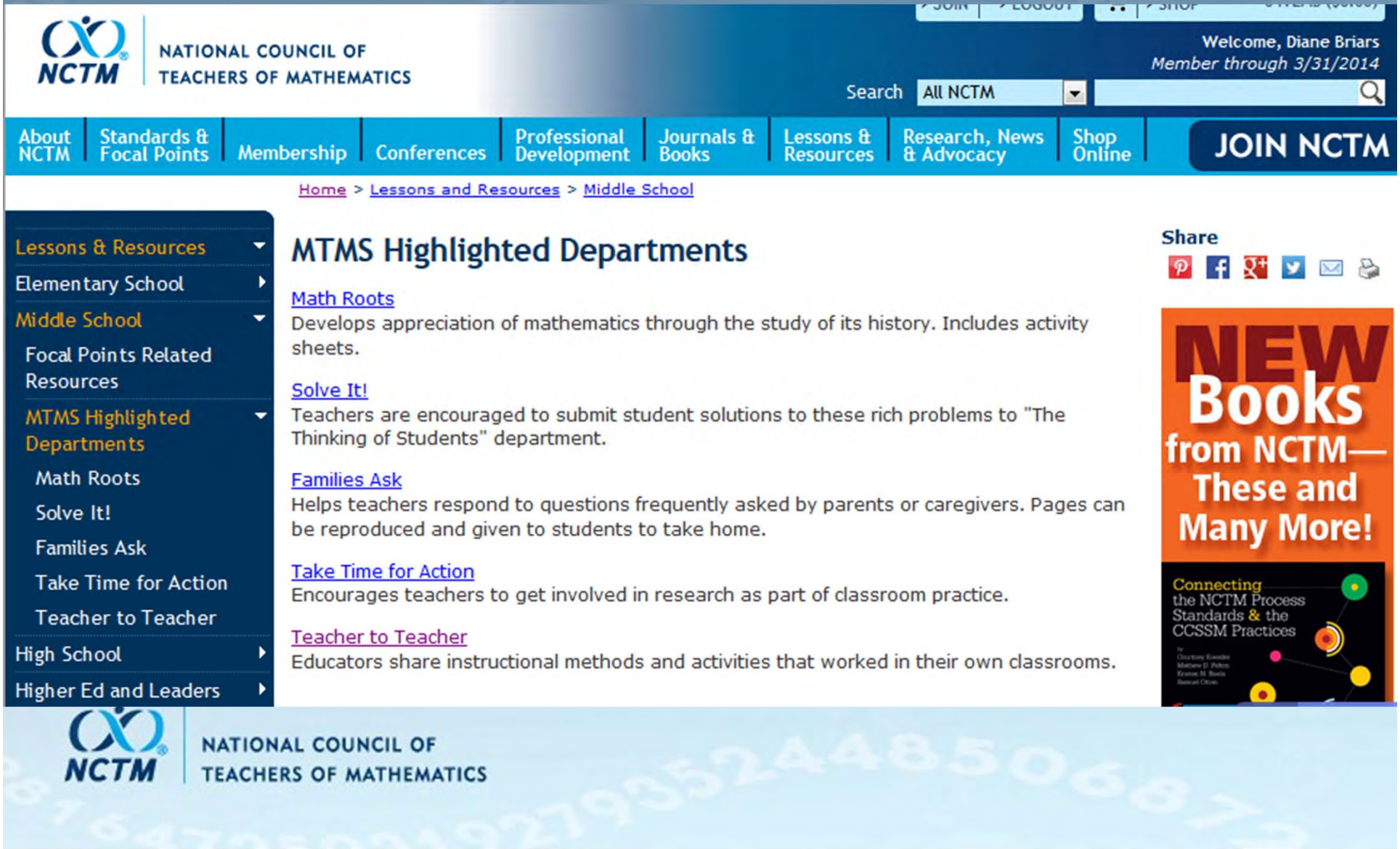
1

*Student Explorations in Mathematics*, January 2013

- Published in January, March, May, September, and November each year
- Each issue develops a single mathematical theme/concept so that 5th graders can understand first 1-2 pages and HS students will be challenged by the last page.
- Content and style is intended to interest students in the power and beauty of math and to introduce teachers to some of the challenging areas of math that are within the reach of their students.

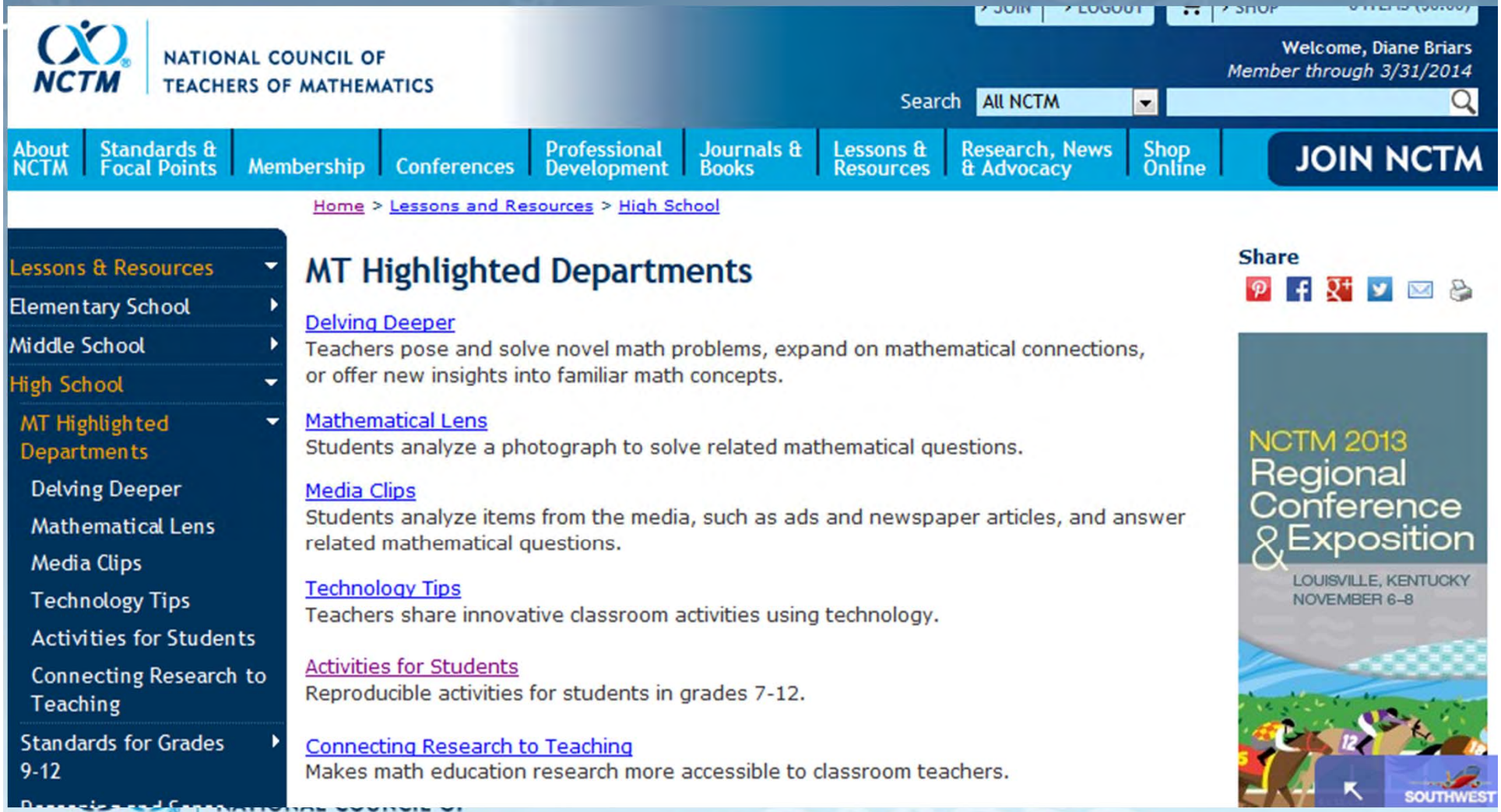
# MTMS Highlighted Departments

## http://www.nctm.org/resources/content.aspx?id=8976



# MT Highlighted Departments

## http://www.nctm.org/resources/content.aspx?id=8960



# MT Highlighted Department: Activities for Students

# NCTM Publications



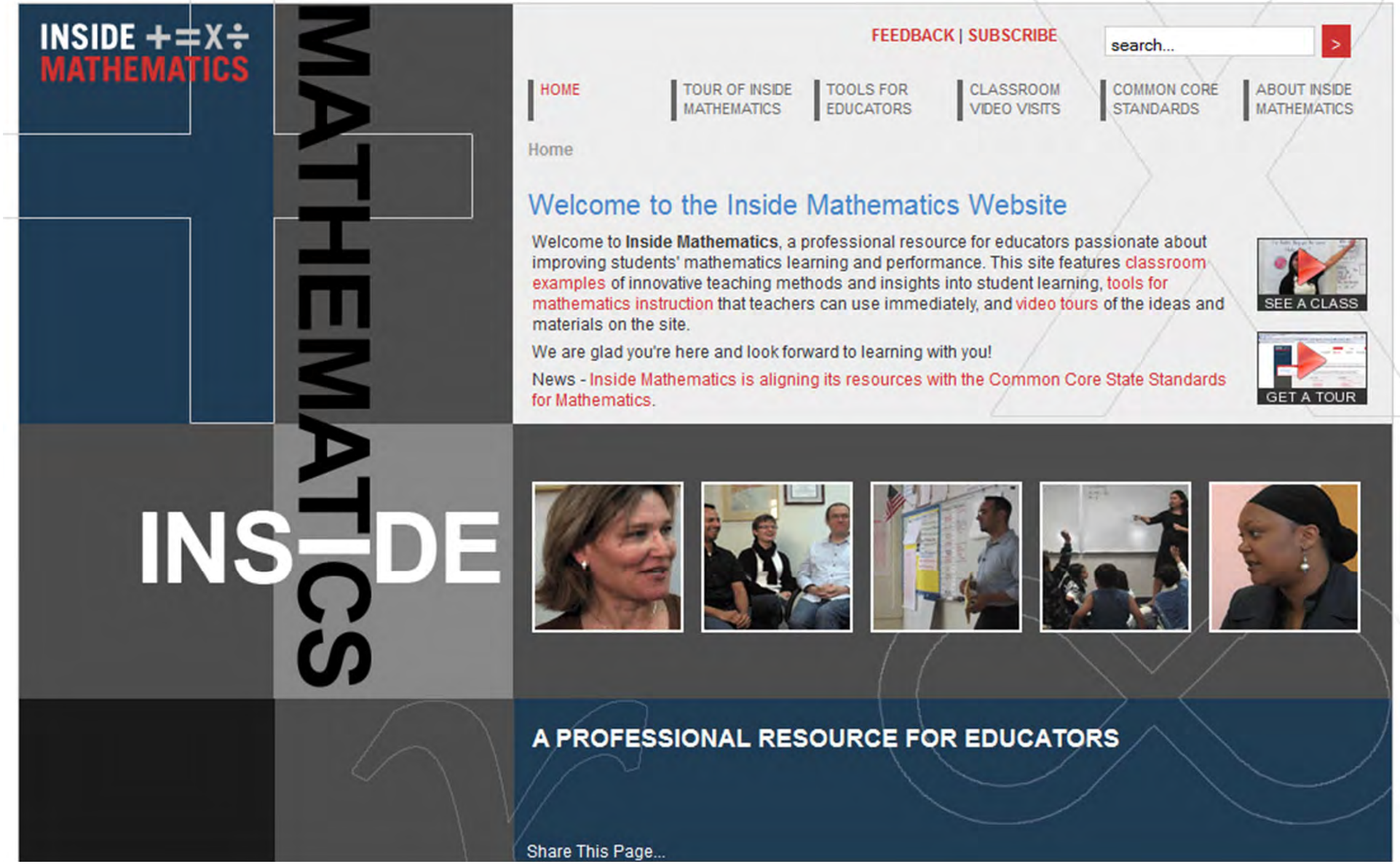
NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# Collaborative Team Tools

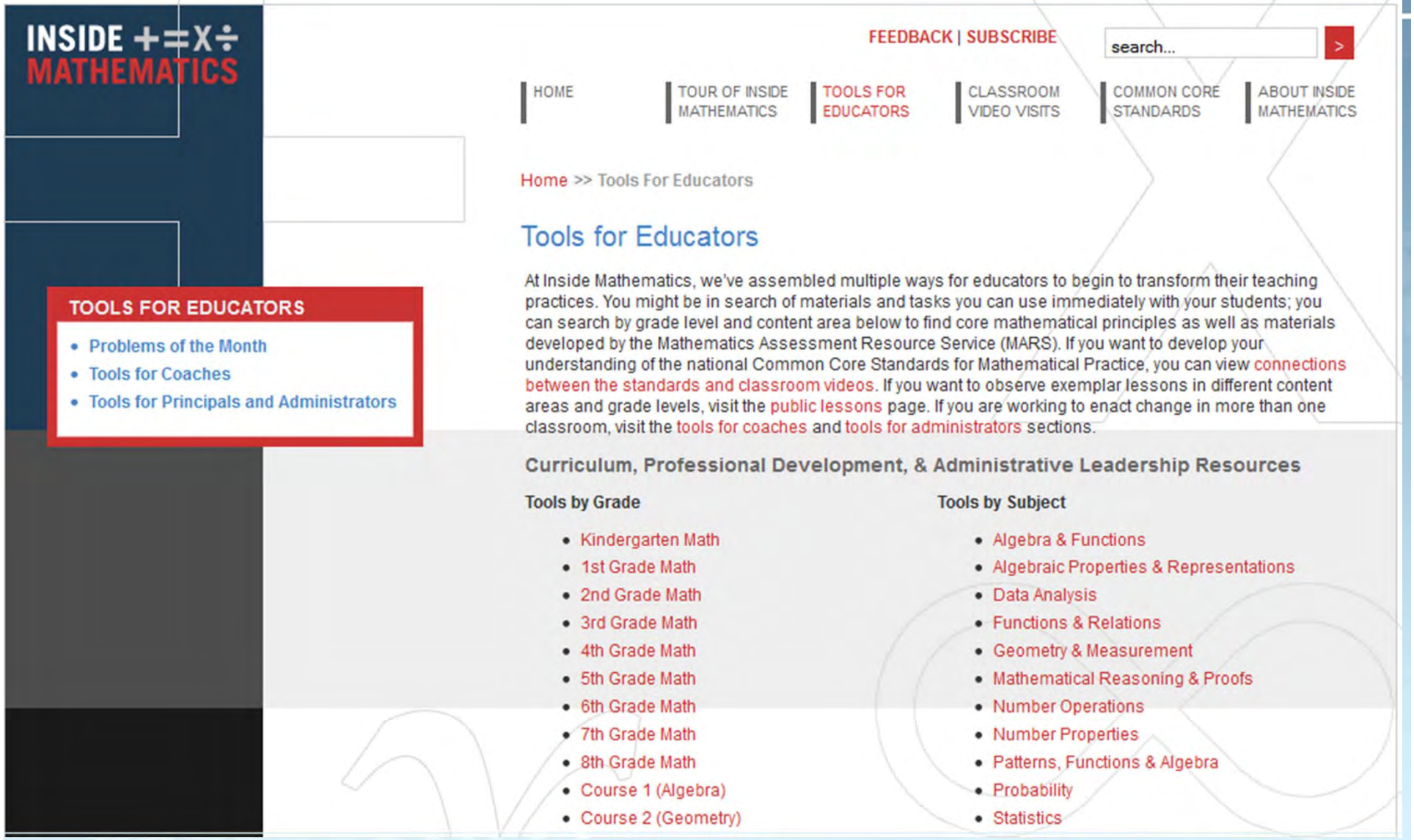

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Available at nctm.org

# Insidemathematics.org



# Insidemathematics.org



# Mathematics Assessment Project (MAP)
# http://map.mathshell.org.uk/materials

Mathematics Assessment Project
ASSESSING 21ST CENTURY MATH
Welcome to the Mathematics Assessment Project

MARS Mathematics Assessment Resource Service

Home | MAP Overview | Lessons | Tasks | Tests | Professional Development | Standards | Instructions | Log In

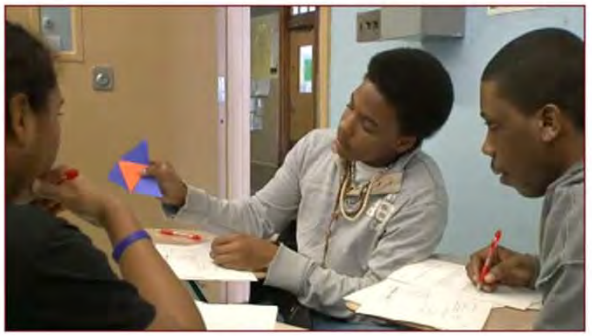
**MAP Home**

- Project goals
- Products
- The Team
- What's on this site?
- Who can use the MAP materials?

### The Mathematics Assessment Project

*"And I'm calling on our nation's governors and state education chiefs to develop standards and assessments that don't simply measure whether students can fill in a bubble on a test, but whether they possess 21st Century skills like problem solving and critical thinking and entrepreneurship and creativity."*

President Obama, 1 March 2009.



### Project goals

The project is working to design and develop well-engineered assessment tools to support US schools in implementing the Common Core State Standards for Mathematics (CCSSM).

### Products

Tools for formative and summative assessment that make knowledge and reasoning visible, and help teachers to guide students in how to improve, and monitor their progress. These tools comprise:

- **Classroom Challenges:** lessons for formative assessment, some focused on developing math concepts, others on non-routine problem solving.
- **Professional Development Modules:** to help teachers with the new pedagogical challenges that formative assessment presents.

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

# http://mathpractices.edc.org/

Implementing the Mathematical Practice Standards

*6. Attend to precision.*


Login Sign up

Search

Home | About MPs | About Illustrations | Browse Illustrations | Use Illustrations | Links | Contact Us

## Need help understanding the mathematical practices?

Explore this site to learn more about the Common Core Standards for Mathematical Practice and how they can be connected to the content standards. Use our Illustrations, centered on student dialogues, to see the Mathematical Practices (MPs) in action.

See All Illustrations

## About Illustrations

Each Illustration of the Mathematical Practices (MPs) consists of a mathematics task; a student dialogue based on that task; information about grade level, standards, and the context for the dialogue; teacher reflection questions; a mathematical overview; and optional student materials. While the primary use of Illustrations is for teacher learning about the MPs, some components are designed for classroom use with students. Go to "Browse Illustrations" to find Illustrations for particular MPs.

## Spotlight on...

Mathematical Practice 8: Look for and express regularity in repeated reasoning.

Rectangles with the Same Numerical Area and Perimeter

In this Illustration students are trying to find all rectangles that

# http://www.illustrativemathematics.org/

Illustrative Mathematics

HOME
ILLUSTRATIONS
K-8 STANDARDS
HIGH SCHOOL STANDARDS
PRACTICE STANDARDS
FRACTIONS PROGRESSION
FREQUENTLY ASKED QUESTIONS
COMMUNITY
ABOUT US
TERMS OF USE

K-8 Standards

High School Standards

Practice Standards

Illustrative Mathematics provides guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the Common Core State Standards, and by publishing other tools that support implementation of the standards.

NCTM | TEACHERS OF MATHEMATICS

# Thank You!

djbmath@comcast.net

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS