

High-Leverage Actions Ensure All Your Students Are Common Core Ready

Diane J. Briars
President-Elect, NCTM
djbmath@comcast.net

NCTM 2013 Las Vegas Regional Conference



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

FYI

Electronic copies of slides are available by
request

djbmath@comcast.net



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Today's Goals

- Analyze CCSS content knowledge and mathematical practices for middle grades and high school students.
- Identify high-leverage actions and instructional practices to help students meet these expectations.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

High-Leverage Actions

Research-informed actions that produce the greatest benefits for your efforts.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Reactions to CCSS



Andy Isaacs, 2011

How Familiar Are You with CCSS-M?

*Rate your knowledge
on a scale of
6 (high) to
1 (low)*



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Key Features of CCSS-M

- **Focus:** Focus strongly where the standards focus.
- **Coherence:** Think across grades, and link to major topics
- **Rigor:** In major topics, pursue **conceptual understanding**, procedural skill and fluency, and application



Curriculum Standards, Not Assessment Standards

**Define, evaluate, and compare functions.
(8.F)**

1. **Understand** that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.



Dimensions of Mathematical Understanding in CCSS

Skill-algorithm understanding

from the rote application of an algorithm through the selection and comparison of algorithms to the invention of new algorithms (calculators and computers included)

Property-proof understanding

from the rote justification of a property through the derivation of properties to the proofs of new properties

Use-application understanding

from the rote application of mathematics in the real world through the use of mathematical models to the invention of new models

Representation-metaphor understanding

from the rote representations of mathematical ideas through the analysis of such representations to the invention of new representations

Vocabulary

Problem Solving

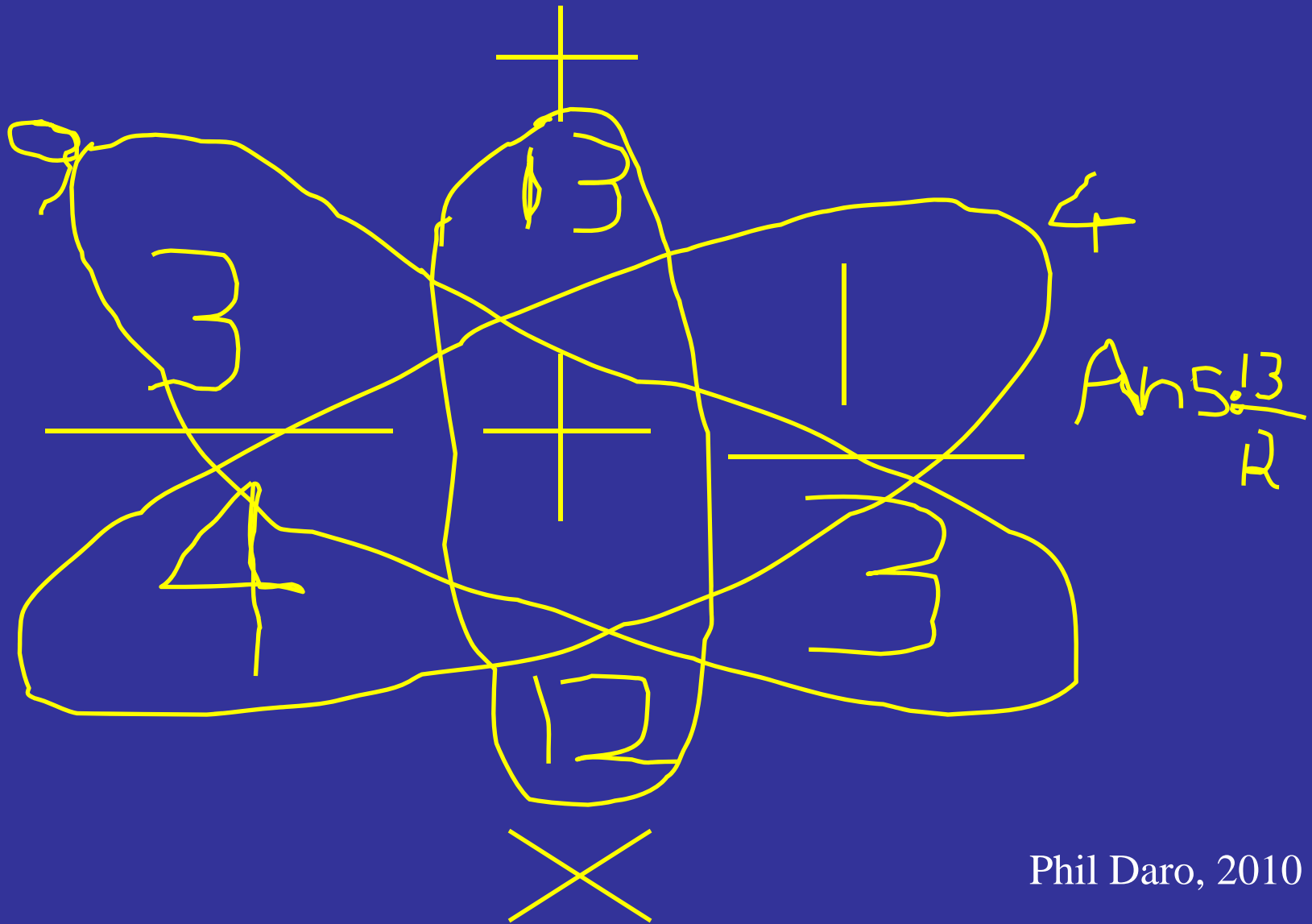


NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Zalman Usiskin, 2012

$$\frac{3}{4} + \frac{1}{3}$$

Phil Daro, 2010



Phil Daro, 2010

Other “Butterflies”?



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Key Instructional Shift

From emphasis on:
How to get answers

To emphasis on:
Understanding mathematics



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

High-Leverage Action 1

Teach for *mathematical understanding*, not *answer-getting*.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Consider This Situation:



?

=



\$.002

.002¢



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

www.videos/RealPlayer Downloads/verizon operator gets math lesson - YouTube.flv

Verizon Call

- What is the caller's issue?
- What mathematics are the Verizon employees able to do? What do they not understand?
- Take a moment and then share with a “shoulder partner”.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

The Outcome



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Verizon Call

- What would your middle school or high school students do in this situation?
- What does this have to do with CCSS?



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Common Core State Standards for Mathematics

Two type of standards:

- **Standards for Mathematical Practice**
- Standards for Mathematical Content



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Standards for Mathematical Practice

“The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.”

(CCSS, 2010)



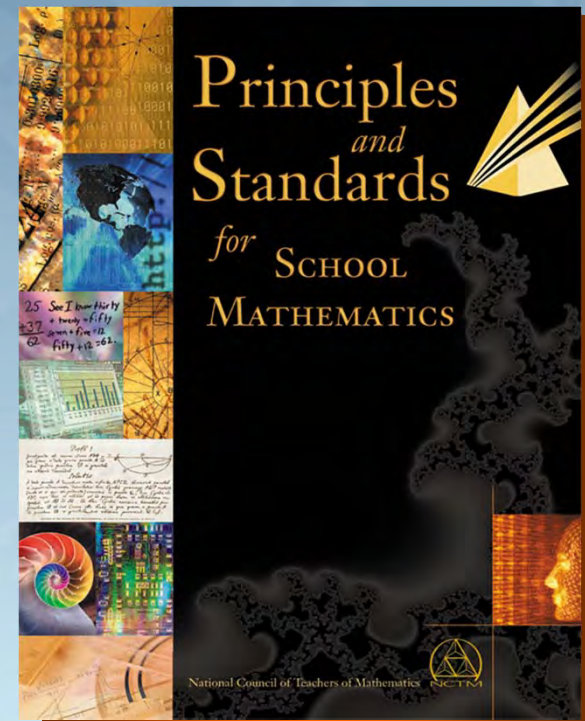
NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Underlying Frameworks

National Council of Teachers of Mathematics

5 **Process** Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations



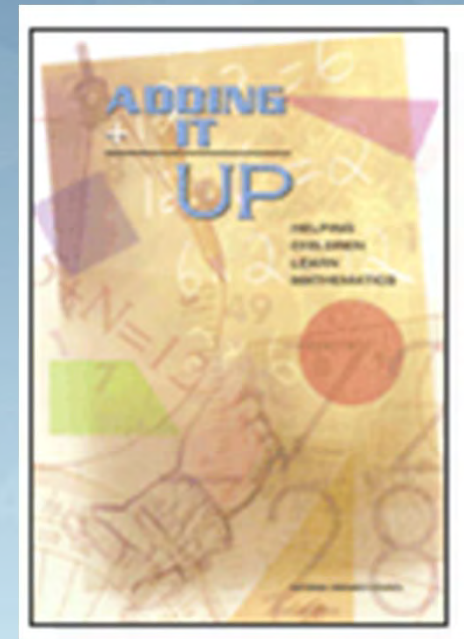
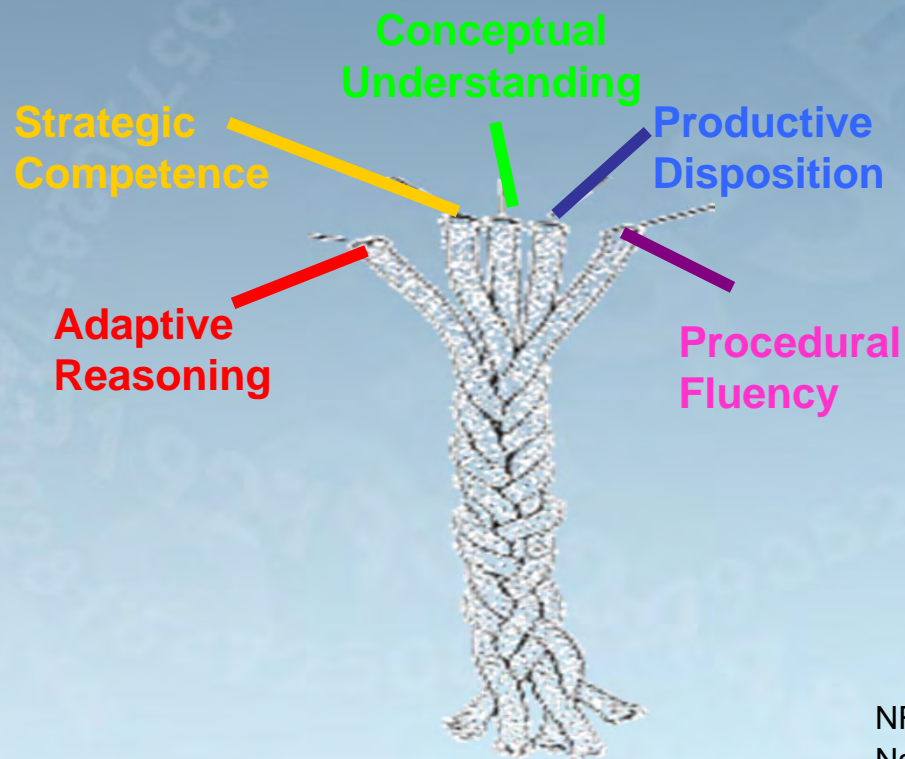
NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Underlying Frameworks

Strands of Mathematical Proficiency



NRC (2001). *Adding It Up*. Washington, D.C.: National Academies Press.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Strands of Mathematical Proficiency

- ***Conceptual Understanding*** – comprehension of mathematical concepts, operations, and relations
- ***Procedural Fluency*** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- ***Strategic Competence*** – ability to formulate, represent, and solve mathematical problems
- ***Adaptive Reasoning*** – capacity for logical thought, reflection, explanation, and justification
- ***Productive Disposition*** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.



Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

Reasoning and explaining

4. Model with mathematics

5. Use appropriate tools strategically

Modeling and using tools

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Seeing structure and generalizing

Verizon Call

- Which mathematical *practices* are needed to complete the task?
- In which mathematical *practices* were the Verizon operators lacking proficiency?



SMP 2:

Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations.

They bring two complementary abilities to bear on problems involving quantitative relationships:

- the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
- the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved.

Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.



High-Leveral Action 2. Develop Students' Proficiency in the Standards for Mathematical Practice

- Analyze the Standards for Mathematical Practice
- Use the Mathematical Practice as the primary vehicle for learning the Content Standards
- Provide opportunities for students to develop the Standards for Mathematical Practice as “habits of mind” (ways of thinking about mathematics that are rich, challenging, and useful) throughout the development of the Content Standards



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Grade 6: Expressions and Equations

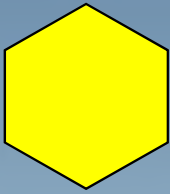
Apply and extend previous understandings of arithmetic to algebraic expressions.

6.EE.3. Apply the properties of operations to generate equivalent expressions.

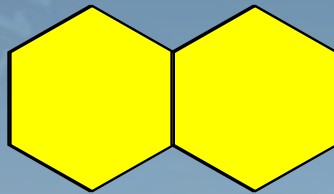
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).



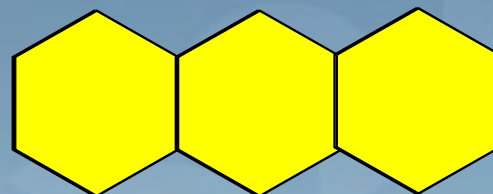
Hexagon Trains



Train 1



Train 2



Train 3

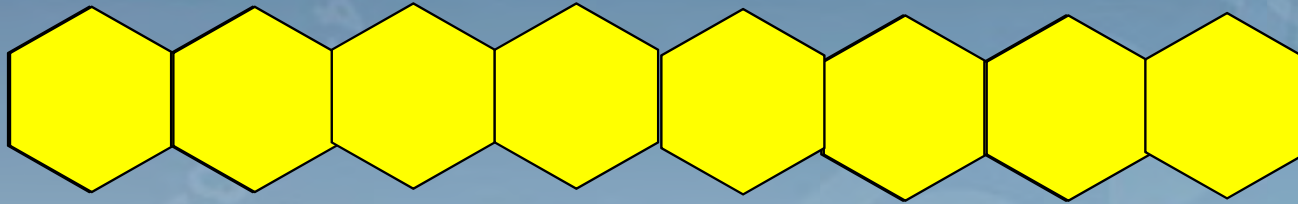
- Compute the perimeter for the first four trains.
- Determine the perimeter for the tenth train without constructing it.
- Write a description /expression that could be used to compute the perimeter of any train in the pattern.
- Find as many different ways as you can to represent the perimeter of any train.

What Mathematical Practices Did You Use?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Hexagon Trains



- Explain what each student was thinking to find the perimeter of the n^{th} train.
- Connect your explanation to the picture of the tables.

Terri: $1 + 4n + 1$

Tim: $1 + 2(2n) + 1$

Jerry: $5 + 4(n - 2) + 5$

Linda: Multiply n times 6, then subtract $n-1$ times 2.



What Mathematical Practices Did You Use?

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Grade 7: The Number System

7.NS.1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

What instructional and/or assessment tasks would you use?



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Properties of Integer Addition and Subtraction

Decide if the statements below are true or false. Then, justify your answer mathematically; that is, explain your reasoning in a way that will convince someone else that you are correct.

1. The sum of a negative integer and a positive integer is always positive.
2. The sum of two negative integers is always negative.
3. The difference between two negative integers is always positive.



Properties of Integer Addition and Subtraction

Ms. Lora is discussing properties of arithmetic with integers with students, asking them to say whether a statement is true or false and provide some reasoning to justify their conclusion.

1. For the statement "The sum of a negative integer and a positive integer is always positive." Keisha says "This is false. The sum *can* be positive, like $10 + -3 = 7$. But, it can also be negative. For example, $-9 + 3$ is -6 ."

Is Keisha's reasoning correct? Explain why you think so.



Properties of Integer Addition and Subtraction

2. For the statement "The sum of two negative integers is always negative." Mike says, "This is true. I tried lots of examples, like $-3 + -2$, $-10 + -27$, and even ones with big numbers, like $-2,000 + -5,000$. All the sums were negative. So this must be true."

Did Mike prove that the sum of two negative integers is always negative? Explain why you think so.



Properties of Integer Addition and Subtraction

3. For the same statement "The sum of two negative integers is always negative." Dev says, "I agree with Mike that the statement is true, but I don't think giving examples is good enough to prove that it is always true. I wonder if I could use the number line to show that when you add two negative numbers together, the sum is always negative?"

Is Dev's reasoning correct? Explain why you think so.

How could Dev use a number line to prove that the sum of two negative integers is always negative?



Properties of Integer Addition and Subtraction

Which mathematical *practices* are needed to complete this task?



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Engaging Students in the Standards for Mathematical Practice

SMP 3. Construct viable arguments and critique the reasoning of others

- Make conjectures
- Justify their conclusions and communicate them to others
- **Recognize and use counterexamples**
- Compare the effectiveness of two plausible arguments
- Listen and respond to the arguments of others for sense making and clarity

SMP 8: Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated
- Look both for general methods and for shortcuts.
- Maintain oversight of the process of solving a problem, while attending to the details.
- Continually evaluate the reasonableness of intermediate results.



Promoting Proficiency in the Standards for Mathematical Practice and Conceptual Understanding

“Not all tasks are created equal, and *different tasks will provoke different levels and kinds of student thinking.*”

Stein, Smith, Henningsen, & Silver, 2000

“*The level and kind of thinking in which students engage determines what they will learn.*”

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Higher-Level Tasks

- Doing mathematics
 - Hexagon Train Task
- Procedures with connections
 - Using a 10 x 10 grid, identify the decimal and percent equivalents of $\frac{3}{5}$.
 - Explain how the graph of $y = -3x^2 + 7$ compares to the graph of $y = x^2$. Sketch the graph of $y = -3x^2 + 7$.



Lower-Level Tasks

- Memorization
 - What are the decimal equivalents for the fractions $\frac{1}{2}$ and $\frac{1}{4}$?
 - What is $\sin 30^\circ$, $\cos 30^\circ$ and $\tan 30^\circ$?
- Procedures without connections
 - Convert the fraction $\frac{3}{8}$ to a decimal.
 - A rectangular carpet is 12 feet long and 9 feet wide. What is the area of the carpet in square feet?
 - Factor $x^2 - 3x - 10$



Research Tells Us

Learners should:

- Acquire conceptual knowledge as well as skills to enable them to organize their knowledge, transfer knowledge to new situations, and acquire new knowledge.
- Engage with challenging tasks that involve active meaning-making



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Core Implementation Issue

Do *all* students have the opportunity to engage in mathematical tasks that promote students' attainment of the mathematical practices on a regular basis?



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Opportunities for *all* students to engage in challenging tasks?

- Examine tasks in your instructional materials:
 - Higher cognitive demand?
 - Lower cognitive demand?
- Where are the challenging tasks?
- Do *all* students have the opportunity to grapple with challenging tasks?
- Examine the tasks in your assessments:
 - Higher cognitive demand?
 - Lower cognitive demand?



High-Leverage Action 3

Regularly incorporate high cognitive demand tasks into your instruction and assessment.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Common Core State Standards for Mathematics

Two type of standards:

- Standards for Mathematical Practice
- **Standards for Mathematical Content**



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Key Advances in Content Grades 6 – 12

- Ratios and Proportional Relationships
- The Number System
- Expressions and Equations
- Functions
- Modeling



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Strategies for Solving Proportions

If 2 pounds of beans cost \$5, how much will 15 pounds of beans cost?



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

If 2 pounds of beans cost \$5, how much will 15 pounds of beans cost?

Method 1

pounds	2	4	6	8	10	12	14	1	15
dollars	5	10	15	20	25	30	35	2.50	37.50

Method 2

	$\div 2$	$\cdot 15$	
pounds	2	1	15
dollars	5	2.50	37.50
	$\div 2$	$\cdot 15$	

Method 3

	$\cdot \frac{15}{2}$	
pounds	2	15
dollars	5	37.50
	$\cdot \frac{15}{2}$	



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

If 2 pounds of beans cost \$5, how much will 15 pounds of beans cost?

CCSS de-emphasizes means/extremes as solution method

$$\frac{2}{5} = \frac{15}{x}$$

$$2x = 5 \cdot 15$$

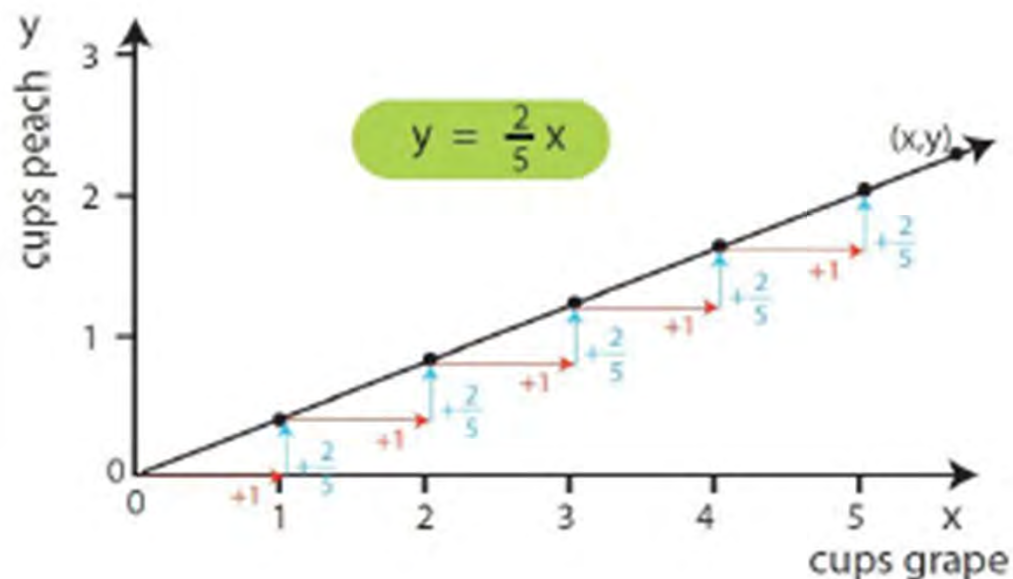


Correspondence among a table, graph, and equation of a proportional relationship

For every 5 cups grape juice, mix in 2 cups peach juice.

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$

The table shows a constant rate of change. Red arrows on the left indicate an increase of +1 in x for each row. Blue arrows on the right indicate an increase of $+\frac{2}{5}$ in y for each row.



On the graph: For each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

When you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.

When you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.

When you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.

When you go x units to the right, you go up $x \cdot \frac{2}{5}$ units.

Starting from $(0, 0)$, to get to a point (x, y) on the graph, go x units to the right, so go up $x \cdot \frac{2}{5}$ units.

Therefore $y = x \cdot \frac{2}{5}$ $y = \frac{2}{5}x$

Ratios & Proportional Relationships

Key Advances:

- Emphasize understanding unit rates associated with ratios.
- Expect students to represent proportional relationships by tables, equations, and graphs, and understand informally that that unit rate indicates the steepness of the graph of the line (informal introduction to slope).
- Expect students to solve problems involving proportional relationships using various methods, such as equivalent ratios and unit rates.



Number and Operations in Base Ten

Conceptual Understanding Facilitates Skill Acquisition:

1. Develop conceptual understanding building on students' informal knowledge
2. Develop informal strategies to solve problems
3. Refine informal strategies to develop fluency with standard procedures (algorithms)



Numbers and Operations in Base Ten

1	Use place value understanding and properties of operations to add and subtract.
2	Use place value understanding and properties of operations to add and subtract.
3	Use place value understanding and properties of operations to perform multi-digit arithmetic. <i>A range of algorithms may be used.</i>
4	Use place value understanding and properties of operations to perform multi-digit arithmetic. <i>Fluently add and subtract multi-digit whole numbers using the standard algorithm.</i>
5	Perform operations with multi-digit whole numbers and with decimals to hundredths. <i>Fluently multiply multi-digit whole numbers using the standard algorithm.</i>
6	Compute fluently with multi-digit numbers and find common factors and multiples. <i>Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</i>

What is Meant by “Standard Algorithm?”

“In mathematics, an algorithm is defined by its steps and not by the way those steps are recorded in writing. With this in mind, **minor variations in methods of recording standard algorithms are acceptable.**”

(Fuson & Beckmann, 2013; NBT, p13)



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Multiplication Algorithms

$$\begin{array}{r} 1 \leftarrow \text{From } 30 \times 4 = 120. \\ 2 \text{ The 1 is 1 hundred,} \\ 94 \text{ not 1 ten.} \\ \times 36 \\ \hline 564 \\ 1 \text{ } \\ 282 \\ \hline 3384 \end{array}$$

$$\begin{array}{r} 94 = 90 + 4 \\ \times 36 = 30 + 6 \\ \hline 6 \times 4 = 24 \\ 6 \times 90 = 540 \\ 30 \times 4 = 120 \\ 30 \times 90 = 2700 \\ \hline 3384 \end{array}$$

Multiplication Algorithms

Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

Multiplication Algorithms

Computation of 36×94 : Ways to record general methods

Showing the partial products

$$\begin{array}{r} 94 \\ \times 36 \\ \hline 24 \\ 540 \\ 120 \\ 2700 \\ \hline 3384 \end{array}$$

thinking:

- 6×4
- 6×9 tens
- 3 tens $\times 4$
- 3 tens $\times 9$ tens

Recording the carries below for correct place value placement

$$\begin{array}{r} 94 \\ \times 36 \\ \hline \overset{5}{} \overset{2}{} 44 \\ \boxed{2} \boxed{1} \\ 720 \\ \hline 3384 \end{array}$$

0 because we are multiplying by 3 tens in this row

Multiplication Algorithms

Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

“Rewrite” vs. “Simplify”

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$p^{-3/5} = \frac{1}{p^{3/5}} = \frac{\sqrt[5]{p^2}}{p}$$

The Number System

Key Advances:

- Algorithms: Standard multiplication algorithm; proficiency with standard division algorithm delayed until grade 6.
- Emphasize on number line models, e.g., absolute value introduced in Grade 6 with negative numbers.
- Bi-directionality of expectations: use rational numbers to solve problems and create problems that can be modeled by particular operations.
- Emphasis on properties, especially distributive property.
- “Rewriting” instead of “simplifying”; writing fractions in lowest terms not a major emphasis.



Expressions and Equations

Key Advances:

- Using variables and algebraic expressions and equations to describe situations and solve problems in Grade 6.
- Solve equations of the form $px + q = r$ and use the distributive property in Grade 7.
- In Grade 8:
 - All students work with radicals and integer exponents
 - Understand connections between proportional relationships, lines, and linear equations.
 - Analyze and solve linear equations and pairs of linear equations.



Functions

Key Advances:

- In Grade 8:
 - Concept of *function* is introduced
 - Explore and compare functional relationships represented in different ways, with emphasis on linear functions.
 - Use functions to model relationships between quantities.
- Build as well as interpret functions
 - Build functions that model relationships between two quantities
 - Build new functions from existing functions

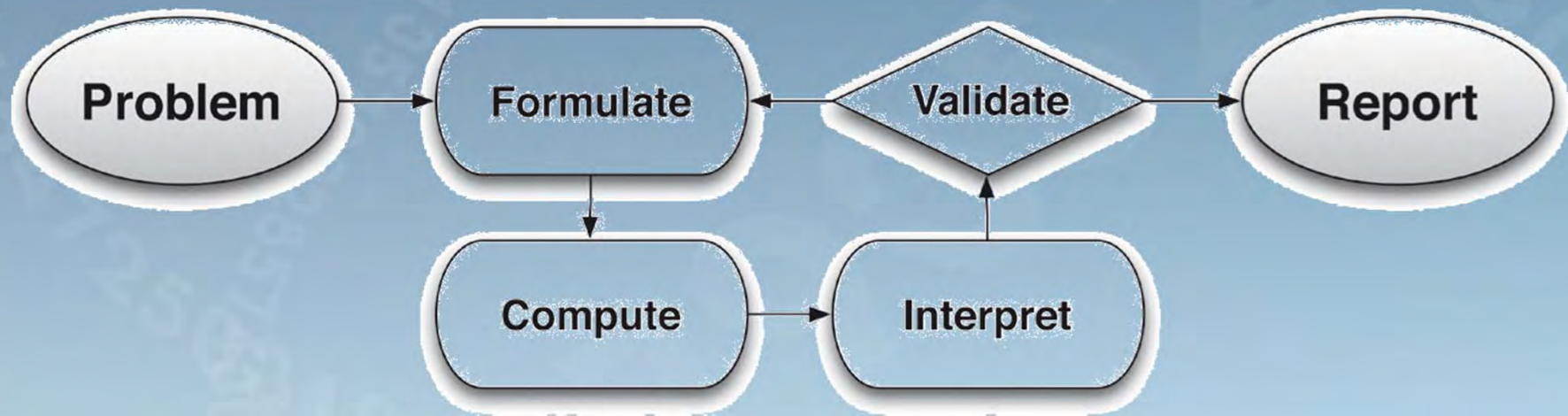


Modeling

- Practice K-12
- Content Conceptual Category HS Standards embedded in other categories (★)
- Examples of models:
 - Equations: Writing total cost as a product of unit price and number bought
 - Geometric shape to represent physical object



CCSS Modeling Cycle



Examples of Situations to be Modeled

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.



What tasks would you use to assess students' proficiency with this standard?

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*



TV Sales-Part A (PARCC Grade 7)

A store is advertising a sale with 10% off all items in the store. Sales tax is 5%.

A 32-inch television is regularly priced at \$295.00. What is the total price of the television, including sales tax, if it was purchased on sale? Fill in the blank to complete the sentence. Round your answer to the nearest cent.



The total cost of the television is \$.

Submit Answer



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

TV Sales-Part B (PARCC Grade 7)



Write your answers to the following problem in your answer booklet.

A store is advertising a sale with 10% off all items in the store. Sales tax is 5%.

Adam and Brandi are customers discussing how the discount and tax will be calculated.

Here is Adam's process for finding the total cost for any item in the store.

- Take 10% off the original price.
- Then, add the sales tax to the discounted price.

Adam represents his process as:

$$T = \underbrace{0.9p}_{\text{sale price}} + \underbrace{0.05(0.9p)}_{\text{sales tax}}$$

Here is Brandi's process for finding the total cost for any item in the store.

- Determine the original price of the item, including sales tax.
- Then, take 10% off.

Brandi represents her process as:

$$T = \underbrace{1.05p}_{\text{T.V. price plus tax}} - \underbrace{0.10(1.05p)}_{\text{10% off discount}}$$

In both equations, T represents the total cost of the television and p represents the regular price.

Are they both correct? Use the properties of operations to justify your answer.

TV Sales-Part B (PARCC Grade 7)

Adam's Process

$$T = \underbrace{0.9p}_{\text{sale price}} + \underbrace{0.05(0.9p)}_{\text{sales tax}}$$

$$\begin{aligned} T &= (1)0.9p + 0.05(0.9p) \\ &= (1 + 0.05)(0.9p) \\ &= (1.05)(0.9p) \\ &= (1.05)(0.9)p \end{aligned}$$

Brandi's Process

$$T = \underbrace{1.05p}_{\text{T.V. price plus tax}} - \underbrace{0.10(1.05p)}_{\text{10\% off discount}}$$

$$\begin{aligned} T &= (1)(1.05p) - (0.10)(1.05p) \\ &= (1 - 0.10)(1.05p) \\ &= (0.9)(1.05p) \\ &= (1.05)(0.9)p \end{aligned}$$



Tasks Clarify Expectations

- Range of content
- Depth of knowledge
- Type of reasoning and evidence of it
- Types of applications



Tasks Clarify Expectations

PARCC

“[The prototypes] are designed to shine a light on important elements of the CCSS”

SBAC

“The sample items and tasks illustrate the knowledge and skills students will be expected to demonstrate on the Smarter Balanced assessments, giving educators clear benchmarks to inform their instruction.”



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Analyze PARCC & SBAC Prototypes

- Compare and contrast them to typical assessment items.
- To what extent do they:
 - Assess conceptual understanding as well as procedural skills?
 - Include higher cognitive demand tasks? Evidence?
 - Assess the Standards for Mathematical Practice? Evidence?
- What are the characteristics of tasks that assess conceptual understanding and/or the Standards for Mathematical Practice?



Analyze PARCC & SBAC Prototypes

View actual prototypes at:

PARCC:

<http://www.parcconline.org>

SBAC:

<http://smarterbalanced.org>



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

High-Leverage Action 4

Analyze the CCSS-M expectations—practices and content--in the original CCSS-M document, in the CCSS-M progression documents, and prototype assessment tasks.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

[Common Core State Standards for Mathematics] represent **a significant departure from *what* mathematics is currently taught in most classrooms and *how* it is taught.** Developing teachers' capacity to enact these new standards in ways that support the intended student learning outcomes will require considerable changes in mathematics instruction in our nation's classrooms. Such changes are likely to occur only through **sustained and focused professional development opportunities** for those who teach mathematics.



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Sztajn, Marrongelle, & Smith, 2011

High-Leverage Action 5

Work in Collaborative Teams

- Work in grade level/course and cross-grade level/course teams to develop a common understanding of CCSS content standards
- Analyze progressions of CCSS standards across grades
- Understand and agree upon common unit goals
- Develop common unit assessments and analyze the results
- Develop a plan to transition from current course scope and sequence to CCSS scope and sequence



High-Leverage Actions to Ensure Your Students are CCSS-Ready

1. Teach for *mathematical understanding*, not *answer-getting*.
2. Provide regular opportunities for students to develop proficiency in the Standards for Mathematical Practice.
3. Regularly incorporate high cognitive demand tasks into your instruction and assessment.
4. Analyze the CCSS-M expectations—practices and content--in the original CCSS-M document, in the CCSS-M progression documents, and prototype assessment tasks.
5. Work in grade-level/course collaborative teams



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

RESOURCES



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

NCTM: www.nctm.org



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Welcome, Diane Briars
Member through 3/31/2014

Search

- About NCTM
- Standards & Focal Points
- Membership
- Conferences
- Professional Development
- Journals & Books
- Lessons & Resources
- Research, News & Advocacy
- Shop Online

JOIN NCTM

- Elementary School
- Middle School
- High School
- Higher Ed
- Leaders
- Families

- Jobs Online
- Advertising
- Exhibiting
- Affiliates
- Advocacy



VIDEO: Common Core State Standards Myths and Facts



New Books in the Essential Understanding Series



NCTM Issues Statement Supporting CCSSM



NEW RtI Digital Series for Kindergarten-Second Grade!



Register for our Regional Conferences Today



Celebrate 75 Years of NCTM Yearbooks

Share

SpringBoard
Shift to Rigor.

Achieve Readiness

Transition to college readiness in ELA and Math, Grades 6-12.

Learn More

President's Corner



President's Messages

Common Core

From the President

Partnering with Parents



Too often when a student struggles with mathematics, a parent comments, I was never very good at math either. While that may be true, the need for our students

Standards

Prepare Yourself



See Matt Larson and Steve Leinwand's "[Prepare for More Realistic Test Results](#)" as you get ready for the first year



NCTM: Illuminations



Activities | Lessons | Standards | Web Links

NCTM Resource

Activities

Explore our library of 108 online activities that help to make math come alive in the classroom or at home

Lessons

View our collection of 607 lessons for preK-12 math educators

Standards

Learn about NCTM's *Principles and Standards for School Mathematics*

Web Links

Check out hundreds of exemplary online resources, as identified by an editorial panel

Highlighted Activity

Daily KenKen® for Tuesday, October 1



3+	1-	
	2-	
4+		2

KenKen is a logic puzzle that involves arithmetic. Similar to SuDoku, KenKen will exercise your mind as you fill in the blanks. Pick your poison — easy or hard, only addition or all four operations.

Solutions posted the following day... check back tomorrow!

Game On!



When's the Nation®?
Race to c
with Dirt!

Check out all 12 games!

Mobile Apps for PreK-2!



What's that you say? You already love the [Concentration](#) game on Illuminations? Well, now it's available as a mobile app ([iOS](#), [Android](#)). Now you can play on your phone, your iPad, or your Android tablet, anywhere you want!

But you'd rather have an app that teaches counting, subitizing, and more? Yeah, we've got an app for that, too. [Okta's Rescue](#) is also available for mobile devices ([iOS](#), [Android](#)).

Video: Serving Up Classroom Fun!



Laurie Penney, a teacher at Willard School in Concord, Mass., was a participant at the 2011 Illuminations Summer Institute. As part of the Institute, she recently helped to develop Illuminations's newest online activity, [Quotier Café](#).

Be the first to know about new Illuminations resources, news, and events by subscribing to our monthly e-newsletter [Bright Ideas](#). [Subscribe today!](#)

illuminations.nctm.org/Lessons.aspx



[Activities](#) | [Lessons](#) | [Standards](#) | [Web Links](#)

NCTM Resources

Lessons

Search Lessons






[View All Lessons](#)

Illustrations has 606 lesson plans available. Select which types of lessons you're looking for, and click **Search**.

Grades

- 
- 
- 
- 

Standards

-  Number & Operations
-  Algebra
-  Geometry
-  Measurement
-  Data Analysis & Probability

Advanced Options

- Show only lessons with associated online activities



NCTM: Core Math Tools



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

> JOIN > LOGOUT > SHOP 0 ITEMS (\$0.00)

Welcome, Diane Briars
Member through 3/31/2014

Search All NCTM

- About NCTM
- Standards & Focal Points
- Membership
- Conferences
- Professional Development
- Journals & Books
- Lessons & Resources
- Research, News & Advocacy
- Shop Online
- JOIN NCTM**

[Home](#) > [Lessons and Resources](#)

- Lessons & Resources
- Core Math Tools
 - General Purpose Tools
 - Custom Apps
 - Advanced Apps
 - Sample Lessons
- Data Sets
- How-To Pages
- WHY Core Math Tools?

FAQs

JOIN NCTM

Core Math Tools Home

Core Math Tools is a [downloadable suite](#) of interactive software tools for algebra and functions, geometry and trigonometry, and statistics and probability. The tools are appropriate for use with any high school mathematics curriculum and compatible with the Common Core State Standards for Mathematics in terms of content and mathematical practices. Java required.



Share

[General Purpose Tools](#)
CAS, Spreadsheet, Geometry,
Data Analysis, and Simulation

[Custom Apps](#)
Focused explorations of
specific topics

[Advanced Apps](#)
Focused explorations of
advanced topics

[Sample Lessons](#)
Problem-based lessons that
employ *Core Math Tools*

[Data Sets](#)
Wealth of data sets
organized by data type

[How-To Pages](#)
Help, hints and steps to
do basic tasks


Use by Teachers and Students

Core Math Tools can be saved on a computer or USB drive, making it possible to use without Internet access. Files can be saved and reloaded by students and teachers. Its portability allows easy access for students, teachers and parents outside the classroom. Core Math Tools will automatically check for updates when launched and Internet access is available.



NCTM's High School Reasoning and Sense-Making Task Library (High School)

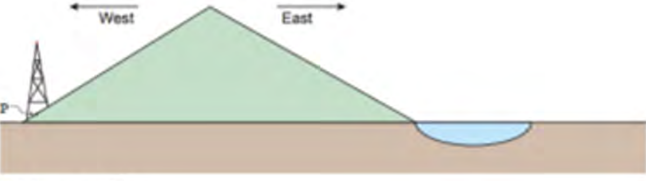
<http://www.nctm.org/rsmtasks>



Over the Hill

Student Activity Sheet

A cell phone tower will be built somewhere on the west side of the hill pictured in figure 1. How far up the hill must the tower be placed to provide a signal to anyone on the east side of the lake?



(Not drawn to scale)

Fig. 1

Part 1: Preliminary Probing

1. What information is needed to solve the problem? What information is not important to know?
2. Thinking algebraically or geometrically, how can you mathematize the problem?

While linking the task directly with NCTM's Focus in *High School Mathematics: Reasoning and Sense Making*, NCTM's *Principles and Standards for School Mathematics*, and the Common Core State Standards, each item addresses:

- **Task Design:** *what the task is asking students to do* (see Task Purpose, Task Overview, Focus on Reasoning and Sense Making, Focus on Mathematical Content, Materials and Technology, Assessment and the Student Activity sheet)
- **Teaching Design:** *how teachers might facilitate reasoning and sense making* (see Use in the Classroom)
- **Student Engagement:** *what student might actually do in the classroom* (see Focus on Student Thinking)



Student Explorations in Mathematics (SEM) (Grades 5-10)



NBA Salaries: A Statistical Dunk

LeBron James's NBA salary is over \$16 million a year. How does his salary compare with others' on his team or other NBA players' salaries? Examining two measures of central tendency, **mean** and **median**, gives insight about the salaries of NBA players and will be more informative to individuals who are investigating the data or those anticipating entering the NBA as players. This activity will help you explore these two measures of central tendency and will promote your understanding of data

interpretation and analysis. The investigation lists the salaries of the thirteen highest paid players on three teams: the Miami Heat, the New York Knicks, and the Los Angeles Lakers. Let's examine the data.

TEACHER VERSION • NBA Salaries: A Statistical Dunk



Miami Heat Thirteen Highest NBA Player Salaries for 2011–2012		New York Knicks Thirteen Highest NBA Player Salaries for 2011–2012		Los Angeles Lakers Thirteen Highest NBA Player Salaries for 2011–2012	
Name	Salary	Name	Salary	Name	Salary
Bosh, Chris	\$16,022,500	Anthony, Carmelo	\$18,518,514	Bryant, Kobe	\$25,244,493
James, LeBron	16,022,500	Stoudamire, Amar'e	18,217,705	Gasol, Pau	18,714,150
Wade, Dwyane	15,691,000	Chandler, Tyson	13,107,837	Bynum, Andrew	14,900,000
Miller, Mike	5,400,000	Balkman, Rivaldo	1,675,000	Peace, Metta World	6,790,640
Chalmers, Mario	4,000,000	Shumpert, Iman	1,563,120	Sessions, Ramon	4,257,973
Haslem, Udonis	3,780,000	Smith, J. R.	1,411,290	Blake, Steve	4,000,000
Anthony, Joel	3,600,000	Bibby, Mike	1,352,181	Hill, Jordan	3,630,000
Battier, Shane	3,000,000	Davis, Baron	1,352,181	McRoberts, Josh	3,000,000
Jones, James	1,500,000	Jeffries, Jared	1,229,255	Barnes, Matt	1,906,200
Howard, Juwan	1,352,181	Douglas, Tony	1,145,640	Murphy, Troy	1,352,181
Curry, Eddy	1,229,255	Fields, Landry	762,195	Ebanks, Devin	762,195
Cole, Norris	1,035,000	Lin, Jeremy	762,195	Goudelock, Andrew	473,604
Pittman, Dexter	762,195	Hamelison, Josh	473,604	Morris, Darius	473,604
Salary sum	\$73,392,000	Salary sum	\$61,570,000	Salary sum	\$85,490,000

- Published in January, March, May, September, and November each year
- Each issue develops a single mathematical theme/concept so that 5th graders can understand first 1-2 pages and HS students will be challenged by the last page.
- Content and style is intended to interest students in the power and beauty of math and to introduce teachers to some of the challenging areas of math that are within the reach of their students.

MTMS Highlighted Departments

<http://www.nctm.org/resources/content.aspx?id=8976>



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Welcome, Diane Briars
Member through 3/31/2014

Search

About NCTM

Standards & Focal Points

Membership

Conferences

Professional Development

Journals & Books

Lessons & Resources

Research, News & Advocacy

Shop Online

JOIN NCTM

[Home](#) > [Lessons and Resources](#) > [Middle School](#)

Lessons & Resources

Elementary School

Middle School

Focal Points Related Resources

MTMS Highlighted Departments

Math Roots

Solve It!

Families Ask

Take Time for Action

Teacher to Teacher

High School

Higher Ed and Leaders

MTMS Highlighted Departments

[Math Roots](#)

Develops appreciation of mathematics through the study of its history. Includes activity sheets.

[Solve It!](#)

Teachers are encouraged to submit student solutions to these rich problems to "The Thinking of Students" department.

[Families Ask](#)

Helps teachers respond to questions frequently asked by parents or caregivers. Pages can be reproduced and given to students to take home.

[Take Time for Action](#)

Encourages teachers to get involved in research as part of classroom practice.

[Teacher to Teacher](#)

Educators share instructional methods and activities that worked in their own classrooms.

Share



NEW Books
from NCTM—
These and Many More!

Connecting the NCTM Process Standards & the CCSSM Practices

By Dorothy Euxler, Melissa D. Pickett, Kristin H. Swick, Ronald Chase



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

MT Highlighted Departments

<http://www.nctm.org/resources/content.aspx?id=8960>



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

JOIN | LOGOUT | SHOP | OTHERS (0000)

Welcome, Diane Briars
Member through 3/31/2014

Search All NCTM

- About NCTM
- Standards & Focal Points
- Membership
- Conferences
- Professional Development
- Journals & Books
- Lessons & Resources
- Research, News & Advocacy
- Shop Online
- JOIN NCTM**

[Home](#) > [Lessons and Resources](#) > [High School](#)

- Lessons & Resources
- Elementary School
- Middle School
- High School
- MT Highlighted Departments**
- Delving Deeper
- Mathematical Lens
- Media Clips
- Technology Tips
- Activities for Students
- Connecting Research to Teaching
- Standards for Grades 9-12

MT Highlighted Departments

[Delving Deeper](#)

Teachers pose and solve novel math problems, expand on mathematical connections, or offer new insights into familiar math concepts.

[Mathematical Lens](#)

Students analyze a photograph to solve related mathematical questions.

[Media Clips](#)

Students analyze items from the media, such as ads and newspaper articles, and answer related mathematical questions.

[Technology Tips](#)

Teachers share innovative classroom activities using technology.

[Activities for Students](#)

Reproducible activities for students in grades 7-12.

[Connecting Research to Teaching](#)

Makes math education research more accessible to classroom teachers.

Share



MT Highlighted Department: Activities for Students



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Welcome, Diane Briars
Member through 3/31/2014

Search All NCTM



About
NCTM

Standards &
Focal Points

Membership

Conferences

Professional
Development

Journals &
Books

Lessons &
Resources

Research, News
& Advocacy

Shop
Online

JOIN NCTM

[Home](#) > [Lessons and Resources](#) > [High School](#)

Lessons & Resources

Elementary School

Middle School

High School

MT Highlighted
Departments

Delving Deeper

Mathematical Lens

Media Clips

Technology Tips

Activities for Students

Connecting Research to
Teaching

Standards for Grades
9-12

Reasoning and Sense
Making Task Library

Higher Ed and Leaders

Title I Teachers and

Activities for Students

[Algebra](#)

[Geometry](#)

[Trigonometry](#)

[Statistics and Data Analysis](#)

[Probability](#)

[Calculus](#)

[Number and Operations](#)

[Mathematical Connections](#)

[Everyday Math](#)

[Fractals](#)

[Logic](#)

[Graphs](#)

[Technology](#)

[Proportional Thinking](#)

Algebra

[What Did One Angle Say to the Other Angles?](#) (August 2008)

[Visualizing Summation Formulas](#) (August 2006)

[Biology as a Source for Algebra Equations: The Heart](#) (November 2005)

[Biology as a Source for Algebra Equations: Insects](#) (August 2005)

[Discovering and Exploring Mandelbrot Set Points with a Graphing Calculator](#) (August 2004)

Share



Don't Delay
**VOTE
TODAY**

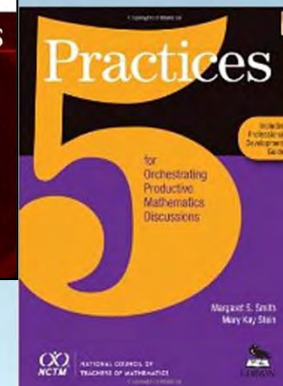
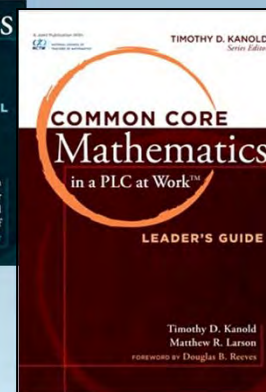
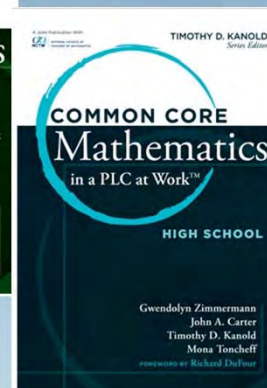
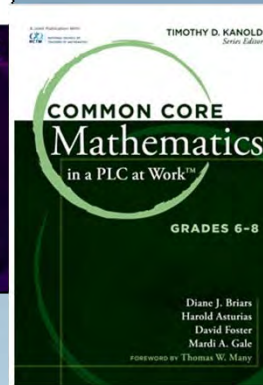
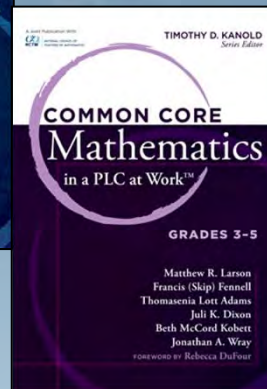
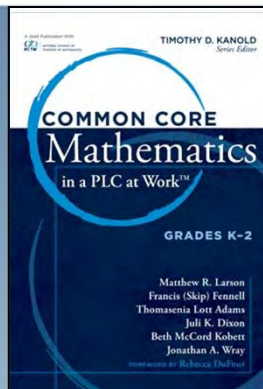


NCTM Publications



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Collaborative Team Tools



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Available at nctm.org

Insidemathematics.org

INSIDE $+=X\div$
MATHEMATICS

MATHEMATICS

INSIDE

[FEEDBACK](#) | [SUBSCRIBE](#)

search...



[HOME](#)

[TOUR OF INSIDE
MATHEMATICS](#)

[TOOLS FOR
EDUCATORS](#)

[CLASSROOM
VIDEO VISITS](#)

[COMMON CORE
STANDARDS](#)

[ABOUT INSIDE
MATHEMATICS](#)

Home

Welcome to the Inside Mathematics Website

Welcome to **Inside Mathematics**, a professional resource for educators passionate about improving students' mathematics learning and performance. This site features **classroom examples** of innovative teaching methods and insights into student learning, **tools for mathematics instruction** that teachers can use immediately, and **video tours** of the ideas and materials on the site.

We are glad you're here and look forward to learning with you!

News - **Inside Mathematics is aligning its resources with the Common Core State Standards for Mathematics.**



SEE A CLASS



GET A TOUR



A PROFESSIONAL RESOURCE FOR EDUCATORS

Share This Page...

Insidemathematics.org

INSIDE $+$ $=$ \times \div
MATHEMATICS

FEEDBACK | SUBSCRIBE

search...



HOME

TOUR OF INSIDE
MATHEMATICS

TOOLS FOR
EDUCATORS

CLASSROOM
VIDEO VISITS

COMMON CORE
STANDARDS

ABOUT INSIDE
MATHEMATICS

Home >> Tools For Educators

Tools for Educators

At Inside Mathematics, we've assembled multiple ways for educators to begin to transform their teaching practices. You might be in search of materials and tasks you can use immediately with your students; you can search by grade level and content area below to find core mathematical principles as well as materials developed by the Mathematics Assessment Resource Service (MARS). If you want to develop your understanding of the national Common Core Standards for Mathematical Practice, you can view [connections between the standards and classroom videos](#). If you want to observe exemplar lessons in different content areas and grade levels, visit the [public lessons](#) page. If you are working to enact change in more than one classroom, visit the [tools for coaches](#) and [tools for administrators](#) sections.

TOOLS FOR EDUCATORS

- [Problems of the Month](#)
- [Tools for Coaches](#)
- [Tools for Principals and Administrators](#)

Curriculum, Professional Development, & Administrative Leadership Resources

Tools by Grade

- [Kindergarten Math](#)
- [1st Grade Math](#)
- [2nd Grade Math](#)
- [3rd Grade Math](#)
- [4th Grade Math](#)
- [5th Grade Math](#)
- [6th Grade Math](#)
- [7th Grade Math](#)
- [8th Grade Math](#)
- [Course 1 \(Algebra\)](#)
- [Course 2 \(Geometry\)](#)

Tools by Subject

- [Algebra & Functions](#)
- [Algebraic Properties & Representations](#)
- [Data Analysis](#)
- [Functions & Relations](#)
- [Geometry & Measurement](#)
- [Mathematical Reasoning & Proofs](#)
- [Number Operations](#)
- [Number Properties](#)
- [Patterns, Functions & Algebra](#)
- [Probability](#)
- [Statistics](#)

Mathematics Assessment Project (MAP)

<http://map.mathshell.org.uk/materials>

Mathematics Assessment Project

ASSESSING 21ST CENTURY MATH

Welcome to the Mathematics Assessment Project

MARS Mathematics
Assessment
Resource
Service

[Home](#) [MAP Overview](#) [Lessons](#) [Tasks](#) [Tests](#) [Professional Development](#) [Standards](#) [Instructions](#) [Log In](#)



MAP Home

- ▶ Project goals
- ▶ Products
- ▶ The Team
- ▶ What's on this site?
- ▶ Who can use the MAP materials?



The Mathematics Assessment Project

"And I'm calling on our nation's governors and state education chiefs to develop standards and assessments that don't simply measure whether students can fill in a bubble on a test, but whether they possess 21st Century skills like problem solving and critical thinking and entrepreneurship and creativity."

President Obama, 1 March 2009.



Project goals

The project is working to design and develop well-engineered assessment tools to support US schools in implementing the [Common Core State Standards](#) for Mathematics (CCSSM).

Products

Tools for formative and summative assessment that make knowledge and reasoning visible, and help teachers to guide students in how to improve, and monitor their progress. These tools comprise:

- **Classroom Challenges:** lessons for formative assessment, some focused on developing math concepts, others on non-routine problem solving.
- **Professional Development Modules:** to help teachers with the new pedagogical challenges that formative assessment presents.

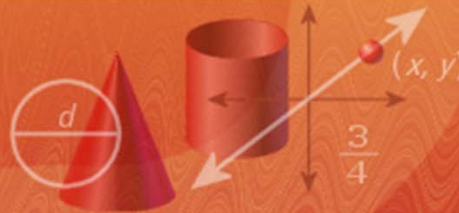


NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

<http://mathpractices.edc.org/>

Implementing the Mathematical Practice Standards

6. Attend to precision.



[Login](#) [Sign up](#)

[Home](#) [About MPs](#) [About Illustrations](#) [Browse Illustrations](#) [Use Illustrations](#) [Links](#) [Contact Us](#)

Need help understanding the mathematical practices?

Explore this site to learn more about the Common Core Standards for Mathematical Practice and how they can be connected to the content standards. Use our Illustrations, centered on student dialogues, to see the Mathematical Practices (MPs) in action.

[See All Illustrations](#)



About Illustrations

Each Illustration of the Mathematical Practices (MPs) consists of a mathematics task; a student dialogue based on that task; information about grade level, standards, and the context for the dialogue; teacher reflection questions; a mathematical overview; and optional student materials. While the primary use of Illustrations is for teacher learning about the MPs, some components are designed for classroom use with students. Go to "Browse Illustrations" to find Illustrations for particular MPs.

Spotlight on...

Mathematical Practice 8: Look for and express regularity in repeated reasoning.

[Rectangles with the Same Numerical Area and Perimeter](#)

In this Illustration students are trying to find all rectangles that have the same numerical area and perimeter. A table lists different

<http://www.illustrativemathematics.org/>



Illustrative Mathematics

HOME

ILLUSTRATIONS

K-8 STANDARDS

HIGH SCHOOL STANDARDS

PRACTICE STANDARDS

FRACTIONS PROGRESSION

FREQUENTLY ASKED QUESTIONS

COMMUNITY

ABOUT US

TERMS OF USE

K-8 Standards

High School
Standards

Practice
Standards



Illustrative Mathematics provides guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the Common Core State Standards, and by publishing other tools that support implementation of the standards.

NCTM

TEACHERS OF MATHEMATICS

Thank You!

djbmath@comcast.net



NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS