"What if?" Questions in Real World Contexts: Sensitivity Analysis

S_4 is a sporting goods company manufacturing two styles of snowboards, *Layout* and *Laser*. The profit on each *Layout* is \$115 and on each *Laser*, \$154.50. There are two main steps in the manufacture of a snowboard: preparation and finishing. Each *Layout* requires 44.5 minutes of preparation and 41 minutes of finishing, while each *Laser* requires 50 minutes of preparation and 60 minutes of finishing. The company has 11,000 minutes of preparation time and 12,000 minutes of finishing time available each week. What weekly production rates of each type of snowboard maximize profit?
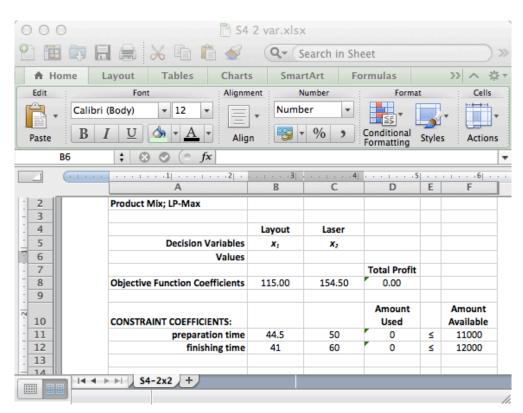
Algebraic Formulation

Let: x_1=the weekly production rate of *Layout* snowboards
x_2=the weekly production rate of *Laser* snowboards
P=the weekly profit

Maximize: $P=115x_1+154.5x_2$

Subject to: $44.5x_1+50x_2 \leq 11,000$ (preparation)
$41x_1+60x_2 \leq 12,000$ (finishing)
$x_1, x_2 \geq 0$ (non-negativity)

Spreadsheet Formulation

1. **1.** What is the optimal solution?

2. **2.** Which constraints are "binding"?

3. **3.** What does it mean to say that a constraint is binding?

4. **4.** What does "slack" mean?

5. **5.** What, if anything, would happen to the optimal solution if the unit profit on Laser snowboards increased to $164.50?   … decreased to $134.50?

6. **6.** What happens if the unit profit on Laser snowboards increased to $169.50?

The table below contains the unit profits per board for four snowboards that S_4 can produce, the time in minutes for each board for five different production processes, and the total time available for each process.

	Types of Snowboard				
	Layout	Laser	Warrior	Feather	
Unit Profit	$115.00	$154.50	$142.00	$150.00	
Time in Minutes to Complete Tasks:					Total Minutes Available
Shaping	24.5	28	22	23.5	6,000
Cutting	10	8	7	7.6	2,000
Silk-Screening	10	14	10	18	3,000
Assembling	27	38	36	38	8,000
Final Touches	14	22	15	17	4,000

Algebraic Formulation

Let: x_1=the weekly production rate of *Layout* snowboards
x_2=the weekly production rate of *Laser* snowboards
x_3=the weekly production rate of *Warrior* snowboards
x_4=the weekly production rate of *Feather* snowboards
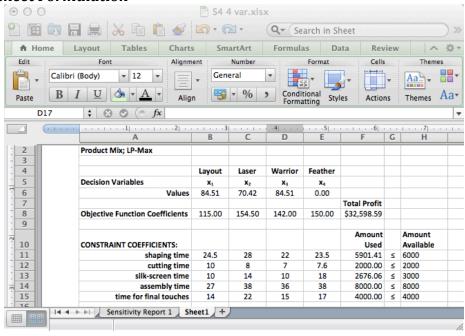P=the weekly profit

Maximize: $P=115x_1+154.5x_2+142x_3+150x_4$

Subject to: $24.5x_1+28x_2+22x_3+23.5x_4 \leq 6,000$ (shaping)
$10x_1+8x_2+7x_3+7.6x_4 \leq 2,000$ (cutting)
$10x_1+14x_2+10x_3+18x_4 \leq 3,000$ (silk-screening)
$27x_1+38x_2+36x_3+38x_4 \leq 8,000$ (assembling)
$14x_1+22x_2+15x_3+17x_4 \leq 4,000$ (final touches)
$x_1, x_2, x_3, x_4 \geq 0$ (non-negativity)

Spreadsheet Formulation

7. What is the optimal solution to the expanded problem?

8. Which of the constraints are binding?

9. If there are any non-binding constraints, how much slack is there?

10. What is the allowable increase on the unit profit of the Feather board?

11. What, if anything, will happen if we increase the unit profit on the Feather board by $1?

12. What is the allowable decrease on the unit profit of the Feather board?
What does that really mean?
Why does it make sense?

13. What, if anything, would happen if we could get 1 more minute of cutting time?
… of shaping time?

14. Three of the decision variables have a reduced cost of zero, and one has a reduced cost of -$0.90. In the context of the problem, how is the decision variable with the non-zero reduced cost different from the others?

15. If you wanted to force the production of 10 Feather boards, how would you have to change the mathematical model?

16. What, if anything, happens if we force the production of 10 Feather boards?