

## Second Annual Math Contest

### Anacostia Indians vs Chavez Eagles

“Once a MATHlete, Always a MATHlete”

1. Ken Ken—Time limit 10 minutes

(Do with a teammate if you want to.)

9+		6×		
1	24×		1-	
2÷			11+	24×
12+	3			
		2÷		

This page do by yourself—10 minutes

2. The last decade, the 2000's, had the distinction of having years with nothing but even digits every other year. 2000, 2002, 2004, 2006, and 2008 all had even digits and no odd digits.

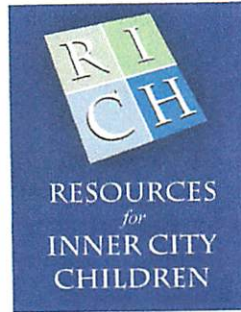
Before 2000, when was the previous year that all the digits were even?

3. A rectangle has perimeter 22. What is the largest possible area if the side lengths are integers?

Work in pairs if necessary. 10 minutes

4. In a 40 minute period, 25 students want to check their Facebook account, but there are only 10 computers available. If each of the students go on line for the same amount of time, what is the most amount of time each student can have on the computer?

5. Each day for four days in a row, you flip a fair coin. If heads, you will have chocolate ice cream; if tails, vanilla ice cream. What is the probability you get eat more chocolate than vanilla?



## Third Annual Math Contest

Cesar Chavez Eagles vs.

Anacostia Indians

**General Rules: The Four Highest Scorers from Each Team  
Will be Scored In Each Round**

There will be gift cards for every winning team member  
and a cash prize for each team's top scorer.

Pizza will be served at 4 pm at the end of the last (fourth)  
round.

### First Round: Ken Ken

Complete as many of the Ken Ken puzzles as possible. There are three.  
One point for your team for each of the 4x4 puzzles completed  
correctly, and 2 additional points if you can also complete the 5x5  
puzzle.

<b>6×</b>		<b>4</b>	<b>2÷</b>
<b>3-</b>	<b>6+</b>		
	<b>2÷</b>		<b>3</b>
<b>3</b>		<b>3-</b>	

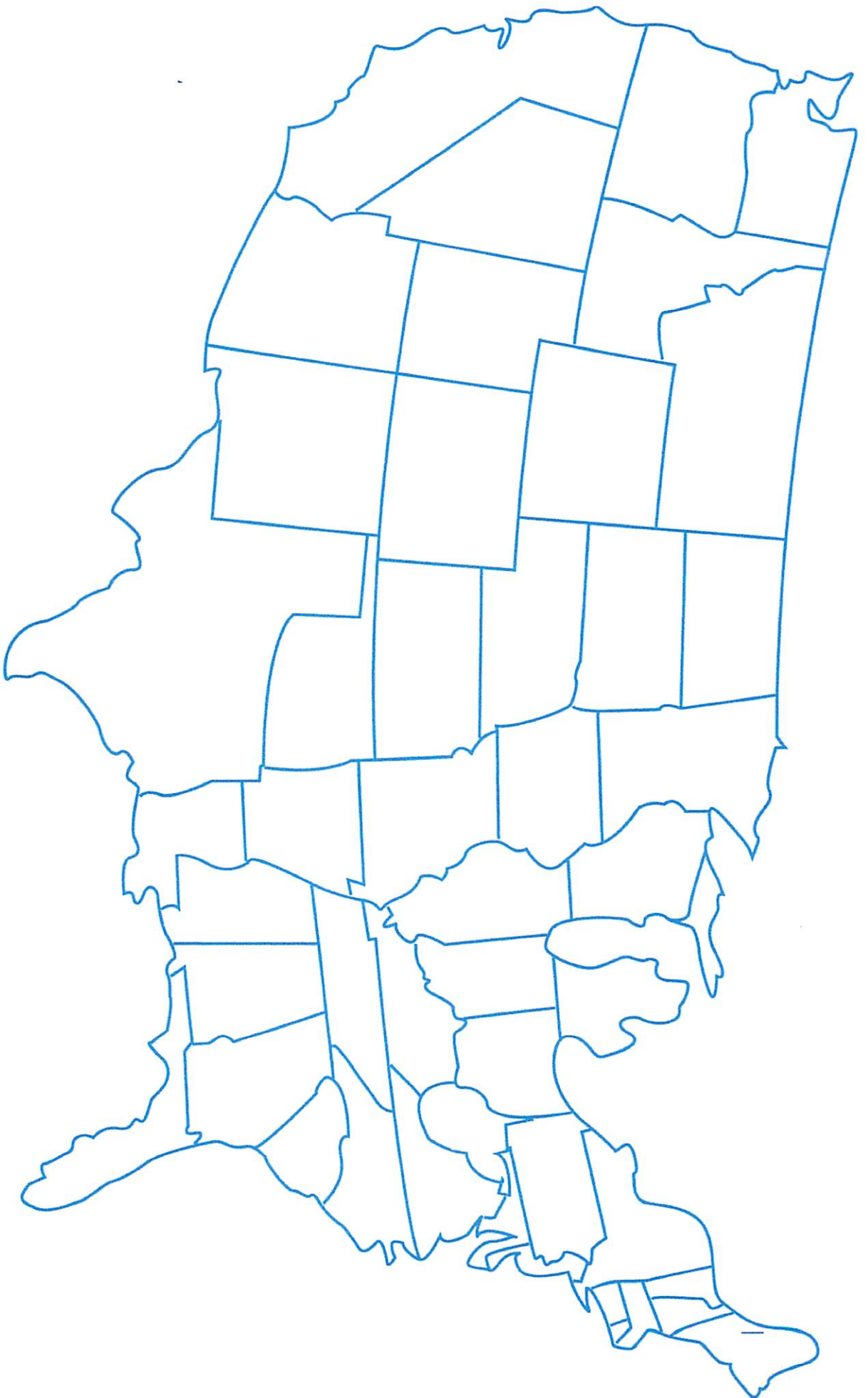
[www.kenken.com](http://www.kenken.com)

## Second Round. The Map Problem. Collaboration With Teammates Is Allowed

For centuries, cartographers and mathematicians conjectured on the least number of colors needed to make a map. That is, if you were to color in every state and make sure no adjacent states had the same color, how many colors were necessary?

This question is in two parts.

1. How many colors is the fewest number needed to color a map so that no adjacent state has the same color? (3 points/ team)
2. Color your map as much as possible using as few colors as possible. For much of the map of the United States, you can actually use fewer colors than the answer to #1. (2 points for a well-colored map, an extra 2 points for the team with the best colored map)



Third Round. Grab Bag. Team  
Collaboration Is Allowed.

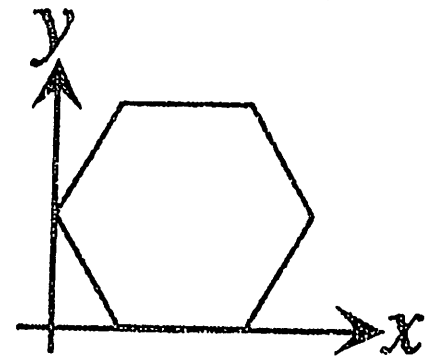
Do any of the following problems for  
three points each.

First problem.

What is the only integer  $x$  for which  
 $x^2 - 100x + 2500$  is not positive?



A regular hexagon, drawn as shown, has one vertex on the  $y$ -axis and one side on the  $x$ -axis. If the  $x$ -coordinate of the hexagon's rightmost vertex is 4, what is the hexagon's perimeter?



Third grab bag problem.

For how many integers  $x$ ,  $0 < x < 100$ , is  $\log_8 x$  a rational number?

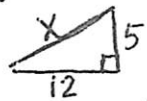
(That is, what powers of 8, like 64 or 2, give you an exponent that is rational?)

A rational number is any number that can be expressed as a simple fraction or decimal, like  $1/3$  or  $.7$  or  $5$ .

# Valentine Math Facts Riddle

**Directions:** Solve each math problem. Then write the letter of the problem in the blank above the answer to solve the Valentine riddle.

U. *Number of digits on a Simpson's hand*

I.  \_\_\_\_\_

T. *Fourscore and \_\_\_\_\_ years ago*

E. *A homonym*

R.  $2^3$  \_\_\_\_\_

M.  $\sqrt{121}$  \_\_\_\_\_

L.  $\log_2 64$  \_\_\_\_\_

H. *A lucky number*

W.  $3\frac{1}{2} + 3$  \_\_\_\_\_

A.  $\frac{12^4}{12^3}$  \_\_\_\_\_

O.  $\sqrt{\sqrt{81}}$  \_\_\_\_\_

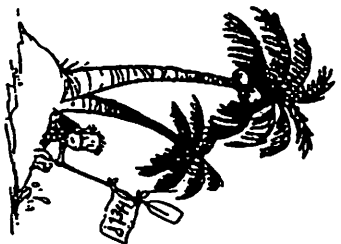
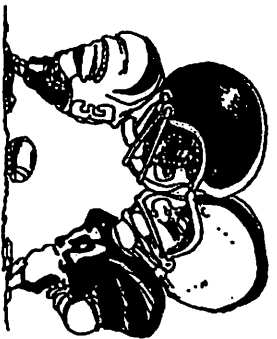
V. *Your age? One less than a perfect square*

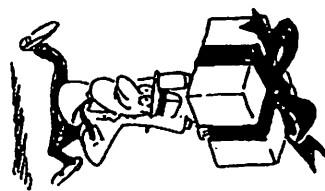
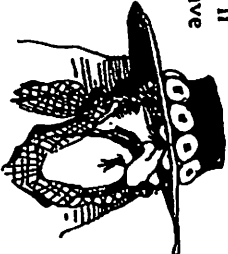
Y.  $(\sin 30^\circ) * 10$  \_\_\_\_\_

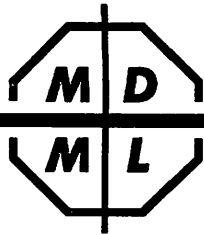


What did the painter say to his Valentine?

“  
 13      6   3   15   2      5   3   4      9   13   20   7  
 12   6   6      11   5      12   8   20  
 !”

1. $1^{2005} + 1^{2005} =$ A) 14010 B) 2 <sup>1</sup> C) 2 <sup>2005</sup> D) 2 <sup>4010</sup>		1.
2. From $n$ piles of 12 coconuts each, I am able to make <u>2</u> piles of 3 coconuts each. A) $n+3$ B) $n+4$ C) $3n$ D) $4n$		2.
3. $x^{400} + x^{100} =$ A) $x^{500}$ B) $x^{300}$ C) $x^4$ D) 4		3.
4. $(-1)^1 + (-1)^2 + (-1)^3 + \dots + (-1)^{98} + (-1)^{99} =$ A) 1 B) 0 C) -1 D) -99		4.
5. If $x^2 - y^2 = 10$ , and $x + y = 10$ , then $x - y =$ A) 1 B) -1 C) 10 D) -10		5.
6. The total value of $2x$ nickels and $x$ dimes is 60¢ when $x =$ A) 6 B) 4 C) 3 D) 2		6.
7. The least common multiple of 2, 4, and 8 is A) 2 B) 8 C) 16 D) 64		7.
8. $2 = \sqrt{8} + \underline{2}$ A) 4 B) $\sqrt{8}$ C) $\sqrt{4}$ D) $\sqrt{2}$		8.
9. There are 6 more football players wearing dark helmets than wearing light ones. The ratio of dark helmets to light is 2:1. The number of light helmets is A) 2 B) 3 C) 6 D) 12		9.
10. The graph of <u>2</u> is parallel to the graph of $2x + y = -3$ . A) $2x + y = 3$ B) $2x + 4y = 6$ C) $2x - y = 3$ D) $x + 2y = -3$		10.
11. Of 5 consecutive integers whose average is $x$ , the smallest is A) $x-2$ B) $x-3$ C) $x-4$ D) $x-5$		11.

12. Of 5 consecutive even integers whose average is $x$ , the smallest is A) $x-2$ B) $x-3$ C) $x-4$ D) $x-5$		12.
13. The greatest common factor of $2^{2004}$ and $2^{2005}$ is A) 1 B) 2 C) $2^{2004}$ D) $2^{2005}$		13.
14. I ran away with a big prize when I was the 7th caller to know that the slope of every horizontal line is A) 0 B) 1 C) -1 D) nonexistent		14.
15. If 10% of $a$ is $b$ , then $a =$ A) $0.1b$ B) $b$ C) $9b$ D) $10b$		15.
16. For which of the following is $n^n$ the square of an integer? A) $n = 3$ B) $n = 5$ C) $n = 6$ D) $n = 7$		16.
17. If $k = \underline{2}$ then the two roots of $x^2 + 4x + k = 0$ are equal. A) 1 B) 2 C) 3 D) 4		17.
18. Jesse has worn the same hat for $d$ years. If he wears it for 12 more years, he will have worn this hat for $d^2$ years. For how many years has Jesse worn this hat? A) 4 B) 6 C) 8 D) 12		18.
19. $ x  +  -x  =$ A) 0 B) $ x $ C) $- x $ D) $2 x $		19.
20. Circle C's center is $(0,0)$ , and the length of C's radius is 5. Which of the following are the coordinates of a point on C? A) $(0,5)$ B) $(-5,-5)$ C) $(-10,0)$ D) $(5,5)$		20.
21. For primes $a$ and $b$ , if $a > b$ , then $ab$ has <u>2</u> unequal positive factors. A) 4 B) 3 C) 2 D) 1		21.
22. The product of <u>2</u> and $x^{100}$ has the same value as $(-x)^{100}$ . A) 100 B) 1 C) -1 D) -100		22.



# MARYLAND MATHEMATICS LEAGUE

P.O. Box 298, West Friendship, Maryland 21794-0298

All official participants must take this contest at the same time.

Contest Number 2 Any calculator without a QWERTY keyboard is allowed. Answers must be exact or have 4 (or more) significant digits, correctly rounded. November 12, 2013

Name \_\_\_\_\_ Teacher \_\_\_\_\_ Grade Level \_\_\_\_\_ Score \_\_\_\_\_

Time Limit: 30 minutes

NEXT CONTEST: DEC. 3, 2013

Answer Column

2-1. When written as 02-03-04, the date Feb. 3, 2004 consists of three consecutive integers whose sum is a perfect square. Writing your answer as MM-DD-YY, what is the first date after 02-03-04 that consists of three consecutive integers whose sum is a perfect square?

2-1.

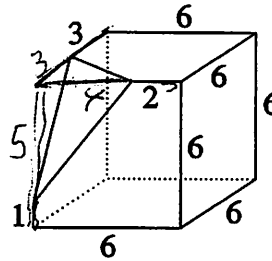
2-2. If  $a \neq b$ , but  $a^2 + a = b^2 + b$ , what is the value of  $a + b$ ?

2-2.

2-3. What is the only odd prime factor of  $2^{67} + 2^{71}$ ?

2-3.

2-4. The solid shown at the right was formed by making straight-line cuts through a corner of a cube of edge-length 6, then removing the corner. The distances of the three vertices of the soon-to-be-removed corner to the vertex of the cube nearest that corner were 1, 2, and 3, as shown. What is the volume of the solid remaining after the corner is removed?



2-4.

2-5. I wrote a list of 100 positive integers whose sum and product are equal. Of the integers on my list, at most how many can be a 1?



2-5.

2-6. In a certain quadrilateral, the three shortest sides are congruent, and both diagonals are as long as the longest side. What is the degree-measure of the largest angle of this quadrilateral?

2-6.

EIGHTEEN books of past contests, *Grades 4, 5, & 6* (Vols. 1, 2, 3, 4, 5, 6), *Grades 7 & 8* (Vols. 1, 2, 3, 4, 5, 6), and *HS* (Vols. 1, 2, 3, 4, 5, 6), are available, for \$12.95 each volume (\$15.95 Canadian), from Math League Press, P.O. Box 17, Teraft, NJ 07670-0017.



# Moody's Mega Math Challenge® 2013

A contest for high school students

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M3Challenge.siam.org



## Waste Not, Want Not: Putting Recyclables in Their Place

Plastics are embedded in a myriad of modern-day products, from pens, cell phones, and storage containers to car parts, artificial limbs, and medical instruments; unfortunately, there are long-term costs associated with these advances. Plastics do not biodegrade easily. There is a region of the Northern Pacific Ocean, estimated to be roughly the size of Texas, where plastics collect to form an island and cause serious environmental impact. While this is an international problem, in the U.S. we also worry about plastics that end up in landfills and may stay there for hundreds of years. To gain some perspective on the severity of the problem, the first plastic bottle was introduced in 1975 and now, according to some sources, roughly 50 million plastic water bottles end up in U.S. landfills every day.

The United States Environmental Protection Agency (EPA) has asked your team to use mathematical modeling to investigate this problem.

**How big is the problem?** Create a model for the amount of plastic that ends up in landfills in the United States. Predict the production rate of plastic waste over time and predict the amount of plastic waste present in landfills 10 years from today.

**Making the right choice on a local scale.** Plastics aren't the only problem. So many of the materials we dispose of can be recycled. Develop a mathematical model that a city can use to determine which recycling methods it should adopt. You may consider, but are not limited to:

- providing locations where one can drop off pre-sorted recyclables
- providing single-stream curbside recycling
- providing single-stream curbside recycling in addition to having residents pay for each container of garbage collected

Your model should be developed independent of current recycling practices in the city and should include some information about the city of interest and some information about the recycling method. Demonstrate how your model works by applying it to each of the following cities: Fargo, North Dakota; Price, Utah; Wichita, Kansas.

**How does this extend to the national scale?** Now that you have applied your model to cities of varying sizes and geographic locations, consider ways that your model can inform the EPA about the feasibility of recycling guidelines and/or standards to govern all states and townships in the U.S. What recommendations does your model support? Cite any data used to support your conclusions.

Submit your findings in the form of a report for the EPA.

The following references may help you get started:

<http://www.epa.gov/epawaste/nonhaz/municipal/index.htm>

[http://15gyres.org/what\\_is\\_the\\_issue/the\\_problem/](http://15gyres.org/what_is_the_issue/the_problem/)

Moody's Mega Math Challenge supports  
Mathematics of Planet Earth (MPE2013).  
[www.mpe2013.org](http://www.mpe2013.org)

