Presentation #78, Herrmann

Number, Shape and Symmetry: Ideas to Inspire Teachers and Students

In this presentation, participants will explore four activities involving number, shape, and symmetry to see how these topics can be used to develop deep mathematical ideas and introduce proof in mathematics. These four activities uncover the connection and interplay between number and symmetry. Participants will experience aspects of mathematical inquiry by using a variety of strategies in solving challenging problems that have multiple solutions and lead to interesting generalizations.

Through two essential ideas of mathematics, number theory and geometry, participants will explore real and deep mathematics through accessible topics. Linking different parts of mathematics can be particularly effective in demonstrating to students that the spectrum of mathematical ideas is not separated into discrete parts. Using four engaging examples, participants will see how foundational mathematical ideas can come to life for students through carefully crafted problem-solving activities

The Triangle Game

Draw an equilateral triangle. On the midpoint of each side, draw in a dot, and also draw a dot at each vertex. Place each of the numbers 1 through 6 on one of the dots. Do this any way you like. For example, let's start by putting 1 at the top and following around the dots clockwise. If you sum the numbers along each side, you get 1 + 2 + 3= 6 on the right, 3 + 4 + 5 = 12 across the bottom, and 5 + 6 + 1 = 12 on the left. The object of this Triangle Game is to place the six numbers on the dots in such a way that the sum along each of the three sides is the same. What sums can you get as solutions? How many different ways are there to get each solution?

Four Numbers Game

Choose four numbers and place them at the corners of a square. At the midpoint of each edge, write the absolute value of the difference of the two adjacent numbers. This produces a new list of four numbers, written on a smaller square. What happens when this process is repeated?

One's Digit Arithmetic

Explore a number system that builds on familiar arithmetic operations by considering addition and multiplication using the one's digit of numbers. For example, in ODA, the result of adding two positive integers is just the one's digit of the result. We say that $8 + 7 \equiv 5$ in Ones Digit Arithmetic. If we multiply in Ones Digit Arithmetic, $8 \bullet 7 \equiv 6$. This simple introduction to modular arithmetic is a rich source of exploratory questions about number theory.

Symmetry Motions of Plane Figures

Using an equilateral triangle, we will explore its symmetry properties. By working with compositions of reflections and rotations, we can create a model for non-commutative multiplication. Extensions to other regular polygons and polyhedra, and to the use of permutations will be cited.

Symmetry Motions of Infinite Patterns

Frieze patterns, one dimensional infinite patterns, and tessellations have symmetry properties in addition to those of plane figures. We will discuss how using an infinite repeating pattern changes the discussion to include translation symmetry and glide reflection symmetry. How many different symmetry pattern types are there?

References

Herrmann, Diane and Sally, Jr., Paul J. *Number, Shape, & Symmetry*. Boca Raton:CRC Press, 2013.

Sally, Judith D. and Sally, Jr., Paul J. *TriMathlon*. Natick, MA: A K Peters, 2003.