**Stories provided by Grade 8 students for the number sentence -17 + 12 = ☐.**

**Dana:** Skylar is currently in debt with the bank. And, she owes them seventeen dollars. She gets twelve dollars in tips from waitressing one day. After that, she still owes the bank a little bit of money.

**Joseph:** There are 17 people in a house. They are hungry so they want cookies. I only made 12 cookies. That only feeds 12 people, of course. And then there are 5 people left that are hungry.

**Hailey:** There were negative seventeen markers and then twelve were added to the person's collection. They ended up with negative five markers.

**Isaac:** Carrie lost seventeen dollars in… She *owed* seventeen dollars to the bank. And, she got a certain amount of money, twelve dollars. She repaid the bank, but still the bank told her she owed five more dollars.

**Drake:** You owe someone seventeen chocolate bars. And, you give them give them twelve chocolate bars. And, now you owe them five chocolate bars.

**Wesley:** I wanted seventeen baseball cards and I got twelve. Now, I want so many more.

**Stories provided by Grade 8 students for the number sentence -2 – 3 = ☐.**

**Dana:** Dylan has to owe two dollars to his banking company. And, he borrows another three dollars. And in the end he owes an amount of money.

**Joseph:** Say you got a new pair of pants and you got two stains on them. My mom wouldn't be very happy. And, uh, you're playing football. So you would get another three stains on your pants, which would be a negative five, which would be a total of five stains on your pants.

**Hailey:** There were negative two baseballs and three were subtracted. And there were negative five left.

**Drake:** A racecar is behind the finish line two feet. And, goes back another three feet. And is now negative five feet behind the finish line.

**Isaac:** Lewis wasn't really good at keeping his homework coming in. He owed two assignments already. And later on in the day, he realized he still had three more assignment due. And in total he owed five assignments.

**Wesley:** You lost two pennies and then you lost three more. And now you have a total of so many lost.

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**Summary of Conventional (C) and Unconventional Contexts (U)**

| | 18 + □ = 5 | -17 + 12 = □ | -14 + -7 = □ | -5 + □ = -15 | 8 – 20 = □ | 5 – □ = 17 | -2 – 3 = □ | □ – -20 = 6 | -10 – -22 = □ | -5+ □ = 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| Joseph | U | U | U | U | C | U | U | U | U | Not Asked |
| Hailey | U | U | U | U | U | U | U | U | U | Not Asked |
| Drake | C | U | C | C | C | C | U | C | U | Not Asked |
| Dana | U | C | C | C | C | C | C | C | C | U |
| Isaac | U | C | C | U | U | C | U | C | C | U |
| Wesley | C | U | U | U | U | No Story Provided | U | U | U | U |

**Summary of Consistent (C) and Inconsistent (I)**

| | 18 + □ = 5 | -17 + 12 = □ | -14 + -7 = □ | -5 + □ = -15 | 8 – 20 = □ | 5 – □ = 17 | -2 – 3 = □ | □ – -20 = 6 | -10 – -22 = □ | -5+ □ = 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| Joseph | I | C | C | C | I | I | I | I | I | Not Asked |
| Hailey | I | C | C | C | C | C | C | C | C | Not Asked |
| Drake | I | C | I | I | C | I | C | I | I | Not Asked |
| Dana | I | C | C | C | C | I | C | I | I | I |
| Isaac | I | C | C | C | I | I | I | I | I | C |
| Wesley | I | C | C | I | C | No Story Provided | C | I | I | I |

**Drake, Grade 8:**
Positive integers are easier to understand because you can add *tangible* things. Whereas, money, you have money, although with like owing you don't *have* it. You have it like a thought thing. It's kind of mental. You can like literally take away so many apples or slices of pie from someone and you can still have apples or slices of pie left. And, the other person would still end up having some. Whereas, negatives, if you have someone and you take something away from them and they don't have any, you can still keep taking more. But, you don't really have anything. You still won't. So, it's easier to do the positives because there are more things to think about in real-life that apply to it.

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**Summary of Conceptual Models**

| | 18 + □ = 5 | -17 + 12 = □ | -14 + -7 = □ | -5 + □ = -15 | 8 – 20 = □ | 5 – □ = 17 | -2 – 3 = □ | □ – -20 = 6 | -10 – -22 = □ | -5+ □ = 21 |
|---|---|---|---|---|---|---|---|---|---|---|
| Joseph | C | B | T | R | B | Rule | Unsure | Unsure | Unsure | Not Asked |
| Hailey | Rule | Rule | Rule | Rule | Rule | Rule | Rule | Rule | Rule | Not Asked |
| Drake | B | B | B | B | T | B | T | T | B | Not Asked |
| Dana | Unsure | B | B | B | B | B | B | B | B | B |
| Isaac | Unsure | B | B | B | Unsure | B | B | B | B | C |
| Wesley | B | B | B | B | Rule | No Story Provided | B | Rule | B | B |

## Conceptual Models of Negative Integers

### Bookkeeping

A conceptual model of "bookkeeping" is used if negative numbers are used in a way to describe losses and gains. An example of the bookkeeping conceptual model is the borrowing and gaining of money. However, the bookkeeping conceptual represents a gain and loss of anything, not necessarily limited to money. For example, gains and losses can be conceptualized with "the owing and gaining of candy bars" or "wanting and receiving of baseball cards." The zero in this model represents neither a gain nor a loss of anything. An example of a bookkeeping conceptual model is illustrated in the following story:

> Lewis wasn't really good at keeping his homework coming in. He owed two assignments already. And later on in the day, he realized he still had three more assignment due. And in total he owed five assignments. (-2 – 3 = -5)

### Counterbalance

A conceptual of "counterbalance" is used if the negative numbers in the instructional task seem to balance or "cancel" each other out. This is similar to the "Balanced Metric" idea and chip modeling concepts in the literature (e.g., Battista, 1983; Whitacre, Bishop, Lamb, Phillipp, Schappelle, & Lewis, 2012). The counterbalance conceptual model is often used with in the context of charges of electrons. The zero in the counterbalance conceptual model represents neutralization. Positive and negative numbers in counterbalance are not just opposites, but opposites that balance each other out to neutralize. A distinguishing element of the counterbalance model is that the quantities always remain in the counterbalance conceptual model, even when neutralized. An example of the counterbalance conceptual model can be seen in the following story:

> Joe did some bad things in the past. And, he's trying to even out the scales by doing good things. So far, he still has five bad things to re-pay for what he's done. And, he can think of 26 good deeds. He does the 26 and by the end of week he's evened out the scales and did more than he expected. He did 21 more good deeds. (-5 + 26 = 21)

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**Relativity**

A conceptual model of "relativity" is used if negative numbers are used in relative positions. For example, the conventional context of temperature provides a conceptual model of the negative numbers. Although a temperature might be -2 degrees Fahrenheit, the actual temperature is not negative two nor is -2 degrees Fahrenheit really opposite of the temperature 2 degrees Fahrenheit. Rather, it is a measure with referent to 0 degree Fahrenheit. With relativity the zero is not actually zero. The zero is a selected point of comparison, sometime arbitrarily chosen. With the conceptual model of "relativity" scales, like numbers lines or coordinate planes and axes, and number can be shifted or selected for convenience. An example of the counterbalance conceptual model can be seen in the following story:

> Consider a baseball game. Suppose you are down five runs in the first inning and you end up losing by fifteen runs. You would have to have to be down ten runs in the other innings to be down by fifteen runs at the end of the game. (-5 + -10 = -15)

**Translation**

A conceptual model of "translation" is used if negative numbers are treated as vectors or translations. With the translations conceptual model, the negative is used to shift any kind of mathematical objects (e.g., a number, a point, a curve). The translation conceptual model often emerges from the contexts of travelling or moving about a linear model, coordinate plane, or three-dimensional space. The zero in this conceptual model is a zero vector, or a translation of no movement. Or, similar to relativity, the zero can represent a point of relativity with the positive and negative number represents a translation in one direction or another from the relative zero. An example of a translation conceptual model is provided next:

> Let's say, you are going to your family's house for Christmas and you're travelling down the road… the numbers would be the miles and you accidentally turned in the wrong direction. And so the further and further away would be the larger the negative number. So, first you take a right and go negative fourteen miles away. And then, you take another right and go negative seven miles away. So total you are negative twenty-one miles away. (-14 + -7 = -21)

Addition with Integers Lesson

Pete and Allyson both wrote stories to correspond with the following open number sentence:

$$-20 + 5 = \square$$

| Pete's Story and Number Sentence | Allyson's Story and Number Sentence |
|---|---|
| I want 20 baseball cards and I got five baseball cards, and now I still want 15 baseball cards. | I lost 20 pencils. My mom bought me 5 pencils the next day. |

1. What do you think about Pete and Allyson's stories?

2. Is there anything you would like to modify in either story?

3. Explain Pete's thinking.

4. Explain Allyson's thinking.

5. Do you agree with, Pete or Allyson? Both of them? Or, neither of them? Why?

6. Why do you disagree with Pete or Allyson, or neither?

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7. Using ________________’s reasoning, solve the following number sentences.

    a. -10 + 3 = ☐

    b. 10 + 3 = ☐

    c. 10 + -3 = ☐

    d. -10 + -3 = ☐

8. Solve the following number sentences and explain your reasoning.

    a. -5 + 7 = ☐

    b. 5 + 7 = ☐

    c. 5 + -7 = ☐

    d. -5 + -7 = ☐

9. Tell a story that matches each of the number sentences above.

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