Differentiating Instruction in Grades 3-5 with Open-Ended Problem Solving

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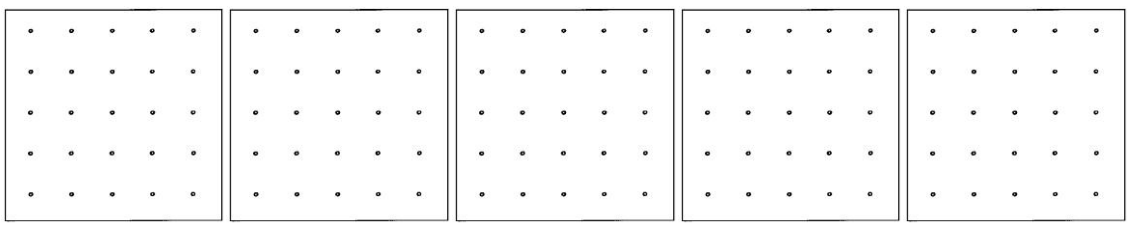
Using dot paper, make different figures that have an area of 6 square units. How many can you find?



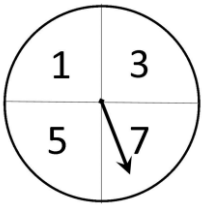
What is a Mathematical Problem?

A mathematical problem involves a situation with which the solver is not familiar. The means to the solution is not immediately obvious. A typical "word problem" is not necessarily a mathematical problem: I have six pounds of peanuts and want to put them into boxes holding 1½ pounds. How many boxes will I fill?" may be a true mathematical problem when first encountered by a fifth grader, but not when it is the tenth example in a list of similar exercises where the whole number is always divided by the fraction.

Traditional vs. Open-ended Mathematical Problems

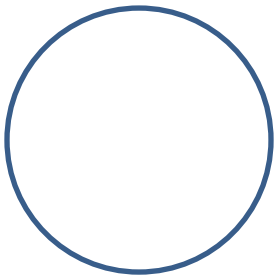
We can classify mathematical problems in two ways: **traditional or open-ended**. In the **traditional** problem we are given a set of constraints or conditions and we apply strategies to obtain a single desired result. For example,

Problem 1a: Are the outcomes equally likely? To spin a 1 or to spin a 5? To spin a 5 or 7? (Scott Foresman, Grade 3, p. 493)



An **open-ended** problem has an additional dimension – more than one solution is possible.

Problem 1b: Design a spinner that contains the numbers 1,3,5,7. Draw your spinner so it would be more likely to spin a 5.



Occasionally, this definition of open-ended is broadened to include problems for which different approaches or strategies lead to the correct single result. It is the approach that is open-ended. However, if the teacher defines one approach as "best", then the open-endedness is lost. (Shimada, p.1)

Problem 2. Open-ended approach: Calculate mentally: 29+42+31. Do your calculation in several ways.

Problem 3a. Traditional: Complete the input/output table and write the rule

Input	Output
1	3
2	5
3	7
4	…

This would be an open-ended approach to problem 3a:

Problem 3b: Directions to teacher: Think of a numerical rule (output = 2 times input+ 1).

Make a T chart on the board. Ask a student to give a numerical input. Write the input on the chart and write the appropriate output beside it. Do the same process with other students. When a student thinks he or she has discovered the rule, ask him to raise his hand, but not to give the rule out loud. Let him give the output for the next input. Continue until several students have raised their hands. Ask one of those students to state the rule.

Example: Martha gives an input of "2". You write 2 5;
Brad says "5". You write 5 11;
Eling says "14". You write 14 29;
Martha raises her hand.
Henry gives an input of 9. Martha writes 9 19; etc.

Common Core Standards
Mathematical Practices (Grades 3-5)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NCTM Principles and Standards

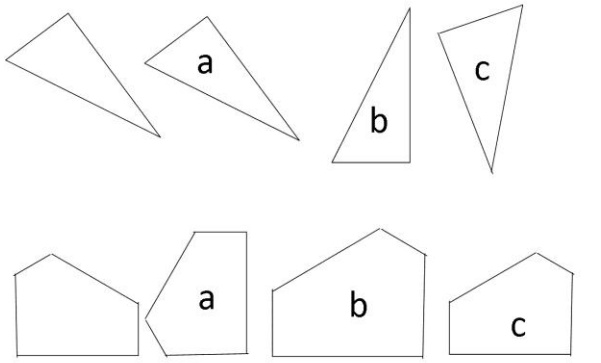
(NCTM, 2000) says that mathematics in grades 3-5 "must be active and intellectually stimulating and must help students make sense of mathematics." Students will lose interest when "learning merely becomes a process of mimicking and memorizing". (p. 143)

Goals of This Workshop

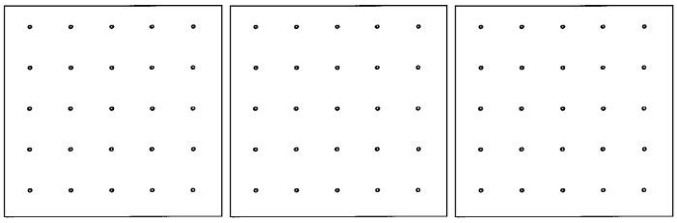
In this workshop we will solve some more-opened ended problems together and then we will examine some traditional textbook problems and find ways to change them to open-ended problems, reflecting on the processes used to reach the solution(s). The problems have been chosen from the content areas of number and operations, algebra, geometry, measurement, and data analysis and probability. Then, we will write scoring guides for a few of the problems.

Solving More Open-Ended Problems

Problem 4a: Traditional: Which polygons are congruent to the first polygon? (Scott Foresman, Grade 4, p. 221)



Problem 4b: Open-ended: Using your geoboard, make three congruent triangles (trapezoids, parallelograms). Use a different color rubber band for each.



Extension: Find the area of each figure. What conclusion can you make about the area of congruent figures?

Problem 5a: Traditional: How many lines of symmetry does the design have?

Problem 5b: Open-ended: Use your pattern blocks to help you solve this problem.

You must create a symmetrical design on a bookmark for the Math Fair:

- Create a symmetrical design on the bookmark by tracing around **at least** 3 different pattern block shapes. Each shape may be used more than one time.
- Draw the line of symmetry on your bookmark.
- Explain how you know the design on your bookmark is symmetrical.

(Open-Ended Released Item from Grade 4, 1999 Missouri Assessment Test)

Problem 6a: Traditional: You can find patterns Involving the counting numbers when they are written in an array. Look at the 8-by-8 array. Do you see any patterns? A diagonal has one pattern. The numbers are 1, 10, 19, 28, 37, 46, 55, 64. (Scott Foresman, Grade 5, p. 286)

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

Problem 6b: Open-ended: Look at a 100s chart. Find as many patterns as you can. Describe them in words and numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

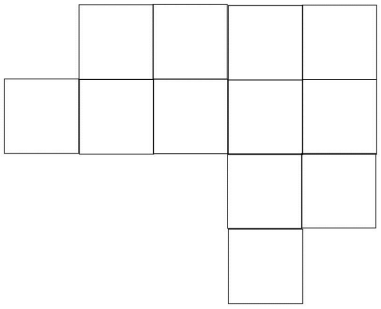
Rewriting Traditional Textbook Problems

Historically textbook problems are of the traditional type, but can often be changed to open-ended. Here are some problems taken directly from textbooks. We will practice rewriting them into open-ended problems.

Problem 7: Su Lin has 7 nickels in his pocket. How much money is that? (Scott Foresman, Grade 3, p. 315)

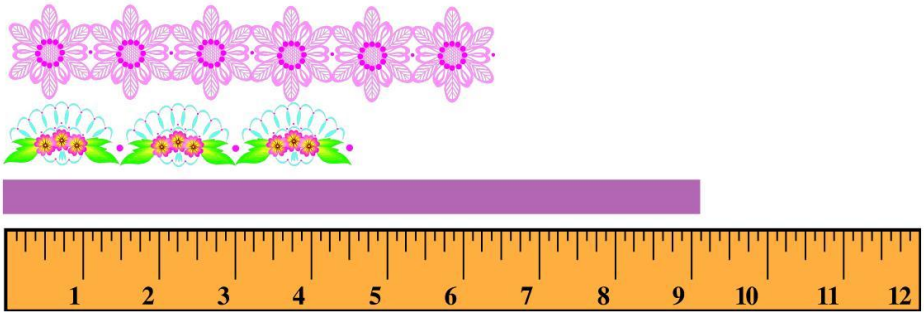
Problem 8: : How much money would you spend if you bought a hot dog, an orange, chips, and milk?

Problem 9: Draw a figure that is congruent to the one below.



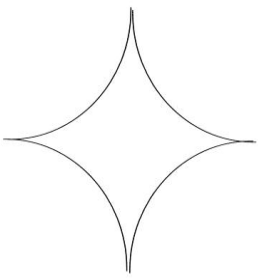
Problem 10: The product of two numbers is 20. The sum of the numbers is 12. What are the numbers?

Problem 11: Use a ruler to find the length of each piece of trim. (ScottForesman, Gr. 4, p. 365)

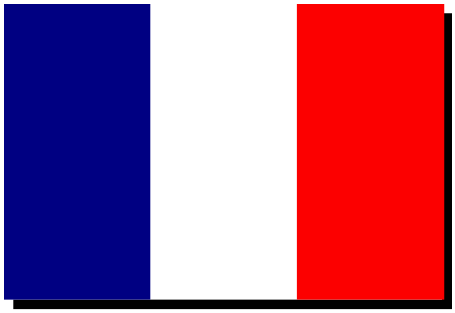


Problem 12: A paisley rectangular shawl made in Scotland has a perimeter of 76 inches. The scarf is 13 inches wide. How long is the scarf? (Scott Foresman, Grade 4, p.125, #13)

Problem 13: How many lines of symmetry does this figure have? (Scott Foresman, Grade 4, p.240)



Problem 14: What fraction of France's flag is red? What part is not red? (Scott Foresman, Grade 4, p.355)



Problem 15: Write the next four numbers in the pattern: 0.05, 0.1, 0.15, … Describe the rule. (Scott Foresman, Grade 5, p.287)

Problem 16:

a. Connie has 250 telephone books to deliver. The books are packaged 18 to a carton. How many cartons does Connie need to complete the order?

b. If 250 names are printed in an office directory and each page holds 18 names, how many pages will be filled?

c. There were 250 directories to be packed. As many full cartons of 18 directories as possible were packed. How many directories were left over?
(Scott Foresman, Grade 5, p.p. 164-165.)

Advantages and Disadvantages of Open-Ended Problems

Open-ended problems require "start up" decision making and organization of thoughts and strategies. Being able to select from different problem forms allows for more variety and challenge. Teachers also discover more about their students' thinking – how they interpret and organize information and the strategies they use.

Some disadvantages also occur. Solutions are difficult to evaluate and even experienced teachers do not always agree on the scoring of them. Furthermore, problems must be carefully stated. The wording must be devised in such a way for the student to understand what is expected. This will improve with practice, both on the part of the student and of the teacher.

Scoring Guides/ Rubrics

Benefits to Students	Benefits to Teachers
	Know ahead how problem will be evaluated. Improved students responses. Consistent scoring.
Self assessment Student construction Practice for state exam	

Drawbacks	Possible Solutions
Time consuming to construct	Use only on selected problems Use a generic rubric Use a commercially prepared rubric
Difficult to construct	Be flexible Revise and refine for future use Work with peers, share feedback Practice, practice, practice
Time consuming to grade	Streamline components, criteria Develop an easy format Practice, practice, practice Weigh the benefits

Constructing a Scoring Guide/Rubric

1. Determine important components of solution.
2. Determine continuum of responses for a component.
3. Determine weight/points of components.

Open-ended problem: (Give each student two congruent right triangles where the height is a little more than twice the base.)

Directions: Make as many different shapes as you can by joining congruent sides of the two triangles. Draw a picture of each shape, identify it by name, and describe it by writing as many properties as you can. (Claus, 1989)

Solution: Six different shapes are possible – a rectangle, a kite, two different triangles, and two different parallelograms.

SPECIFIC RUBRIC, MAKING GEOMETRY SHAPES

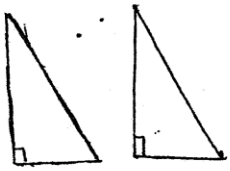
Exemplary (6 pts.)	Satisfactory (4 pts.)	Needs Improvement (2 pts.)	Unsatisfactory (0 pts.)
Creates 6 shapes	4-5 shapes	2-3 shapes	0-1 shapes
Identifies names of 6 shapes	4-5, includes all four categories	2-3, includes two or three categories	0-1 shapes
Clearly defines two properties of each of the four categories(triangles, parallelograms, kite, rectangle)	Defines one property for each of the four categories	Defines one property for each shape.	Properties are missing.

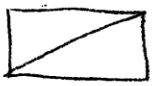
GENERIC RUBRIC, MAKING GEOMETRY SHAPES

	Exemplary (2 pts.)	Needs Improvement (1 pt)	Unsatisfactory (0 pts.)
SOLUTION	Accurate	Partially accurate	Inaccurate
	Complete	Partially complete	Incomplete
	Clear	Somewhat unclear	Unclear
	Exemplary (4 pts.)	Needs Improvement (2-3 pts.)	Unsatisfactory (0-1 pts)
EXPLANATION	Accurate	Partially accurate	Inaccurate
	Complete	Partially complete	Incomplete
	Clear	Somewhat unclear	Unclear

Making Shapes Name(s) Sample 1

You have been given two congruent right triangles. Your job is to make as many different shapes as you can by joining congruent sides of the two triangles. Draw a picture of each shape, identify it by name, and describe it by writing as many properties as you can.

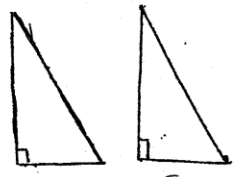


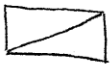
Picture of shape	Name	Properties
1. 	rectangle	4 right angles 2 sets of parallel lines. Symmetrical in 2 ways
2. 	Triangle	acute Symmetrical in 3 ways
3. 	Rhumbus	Not symetrical Parellel

2

Making Shapes Name(s) Sample 2

You have been given two congruent right triangles. Your job is to make as many different shapes as you can by joining congruent sides of the two triangles. Draw a picture of each shape, identify it by name, and describe it by writing as many properties as you can.



Picture of shape	Name	Properties
1. 	Triangle	There is a line of Semmetry and there is three sides.
2. 	Kite	There is a line of symmetry and it has four sides.
3. 	Triangle	They have the Simle size sides.
4. 	Parallelagram	The Diamond has four Sides, it is Semmetrical.
5. 	rectangle.	The rectangle has four sides.

Explanation Rubric

Question _____3_____ Score _____/ 10

Elements to be Graded	Exemplary		Needs Improvement		Seek Help
	2		1		0
Position or Solution	Position or solution is • accurate • complete • clear		Position or solution is • slightly inaccurate • somewhat incomplete • somewhat unclear		Position or solution is • inaccurate • incomplete • unclear
	4	3	2	1	0
Explanation	Written explanation • is accurate • is complete • is clear • is well organized • shows thorough understanding of concept • includes relevant example when applicable		Written explanation • is slightly inaccurate • is somewhat incomplete • is somewhat unclear • is somewhat unorganized • shows partial understanding of concept • includes no relevant example when applicable		Written explanation • is inaccurate • is incomplete • is unclear • shows little or no understanding of concept • includes no relevant example when applicable
	4	3	2	1	0
Diagram	Diagram, table, or graph is • accurate • complete • clear • well organized • directly and correctly related to the problem		Diagram, table, or graph is • slightly inaccurate • somewhat incomplete • somewhat unclear • somewhat unorganized • somewhat related to the problem		Diagram, table, or graph is • inaccurate • incomplete • unclear • not organized • not related to the problem

Comments:

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Resources

Claus A. (1989). Fifth graders investigate geometry. In P. Trafton & A. Schulte (eds.), New Directions for Elementary School Mathematics, 1989 NCTM Yearbook, (p. 57-58). Reston, Va. NCTM.

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Missouri Assessment Program, 1999. Released Items ande Rubrics, Elementary Mathematics. www.dese.state.mo.us Go to programs and services, a – z, choose c, curriculum services, released items.

Principles and Standards for School Mathematics (2000). Reston, Va. National Council of Teachers of Mathematics.

Scott, Foresman and Company . (1996). Exploring Mathematics, Grades 3,4, and 5.

Shimada S. (1997). The significance of the open-ended approach. In J. Becker & S. Shimada (Eds.), The Open Ended Approach (pp. 1-9), Reston Va: National Council of Teachers of Mathematics.

Some Solutions for Problems (Rewritten as open-ended problems)

#1b Largest section on spinner should be for 5.

#2. $29 + 42 + 31 = (20 + 40 + 30) + (9 + 2 + 1) = 90 + 12 = 102$
$= (29+31)+42 = (30+30)+40+2=102$
$= (30 + 41) + 31 = 71+31 = 70+30+2=102$

#8. $3.50-$0.40= $3.10 to spend.

Menu 1: Hamburger	$1.50	Menu 2: Hot Dog	$1.25
Apple	.50	Orange	.60
Cocoa	.55	Milk	.80
Chips	.45	Ice Cream	.40
	$3.10		$3.05

#10.
1x20	1+20=21
2x10	2+10=12
4x5	4 +5=9

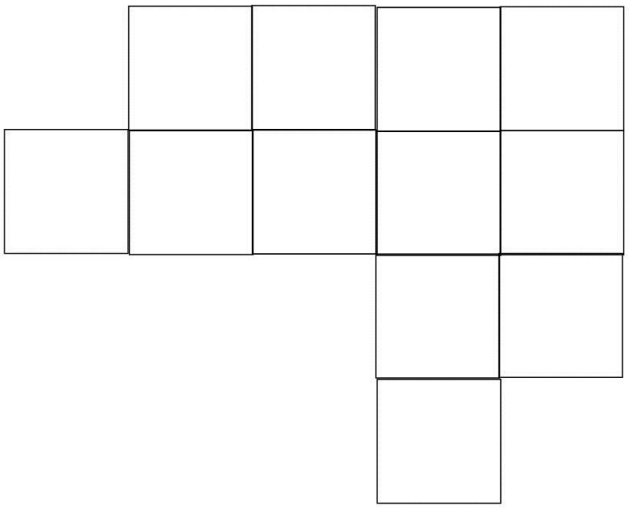
Possible Open-Ended Problems for 7-16

#7. Su Lin has 35 cents in his pocket. What are some possible combinations of coins that he could have? (Could also put some conditions on his change, e.g. he has no pennies, he has exactly two dimes, etc.)

#8 Aaron has $3.50. He wants to buy lunch. He must choose one meat, one fruit, one drink, and one treat (chips, candy, ice-cream, or cookie). He must have 40 cents left for his bus ride home. Find two menus that he could choose. Show the calculation for each menu and show all conditions are met.

Extensions: Find a menu if (a) Aaron wants the cookie; (b) Aaron wants both the hamburger and the milk.

#9. Find different ways to cut the figure into two congruent pieces:



#10. The product of two numbers is 20. Find all possible sums of the numbers.

#11. Find objects in our classroom that measure 3½ in., ¾ in, and 5 ¼ in.

#12. Michael is designing a rectangular scarf that has a perimeter of 76 inches. The scarf must be at least 13 inches long. Find the dimensions of three different scarves that Michael could design.

#13. Using your geoboard (dot paper), make several figures that have exactly (a) one line of symmetry, (b) two lines of symmetry, (c) four lines of symmetry, (d) no lines of symmetry. Draw your figures on dot paper and organize what you discovered into a chart.

#14. You are asked to design a flag for your classroom. You can choose from four different colors (Red, Blue, Green, Yellow). The class has decided that ½ of the flag should be blue. You can choose the fraction of the remaining three colors. Sketch the flag you designed and write what fraction of each color you used:

Picture	Fraction of Blue	Red	Green	Yellow
	½			

#15. You decide on a number pattern and write the first five numbers in your pattern. Pass your paper to your partner and let him or her write the next five numbers. Then both of you should try to write the rule for the pattern. Compare your rule with your partner's rule. Do you agree?

#16. Directions to teacher: Write 250 18 = 13 R 16 on the board.

Directions to students: Write a story problem where

a. the quotient is the answer;
b. the remainder is the answer;
c. the quotient is rounded up one unit;