Teaching and Learning Computation of Fractions Through Story Problems

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MENTAL MATH . . . FRACTIONS

$\frac{1}{4} + \frac{2}{4} =$	$\frac{10}{12} - \frac{1}{2} =$
$\frac{2}{3} + \frac{4}{8} =$	$\frac{3}{7} \times \frac{3}{5} =$
$\frac{3}{4} - \frac{1}{4} =$	$\frac{2}{3} \div \frac{1}{4} =$

MENTAL MATH . . . FRACTIONS

$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$	$\frac{10}{12} - \frac{1}{2} = \frac{1}{3}$
$\frac{2}{3} + \frac{4}{8} = 1\frac{1}{6}$	$\frac{3}{7} \text{ X } \frac{3}{5} = \frac{9}{35}$
$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$	$\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3}$

"Fractions are one of the most difficult of the elementary school math topics"

Mazzocco and Devlin 2008

- Let’s look at the progression of fractions from 3rd to 5th grade.
- What do you notice?

THE COMMON CORE STANDARDS

- 3.NF.1 Understand a fraction 1/*b as the quantity formed by 1 part when a* whole is partitioned into *b equal parts; understand a fraction a/b as* the quantity formed by *a parts of size 1/b.*

- 3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

  a. Represent a fraction 1/*b on a number line diagram by defining the* interval from 0 to 1 as the whole and partitioning it into *b equal* parts. Recognize that each part has size 1/*b and that the endpoint* of the part based at 0 locates the number 1/*b on the number line.*

  b. Represent a fraction *a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number* line.

THE COMMON CORE STANDARDS

4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b as a multiple of 1/b. For example, use* a *visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

b. Understand a multiple of *a/b as a multiple of 1/b, and use this* understanding to multiply a fraction by a whole number. *For example,* use a *visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

THE COMMON CORE STANDARDS

5.NF.4 **Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.**

a. Interpret the product (*a/b) × q as a parts of a partition of q* into *b equal parts; equivalently, as the result of a sequence of* operations *a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

b. **Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction** side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Many students have procedures for fractions but are not sure when to use the procedures

Name Ryan Date 1/13/14

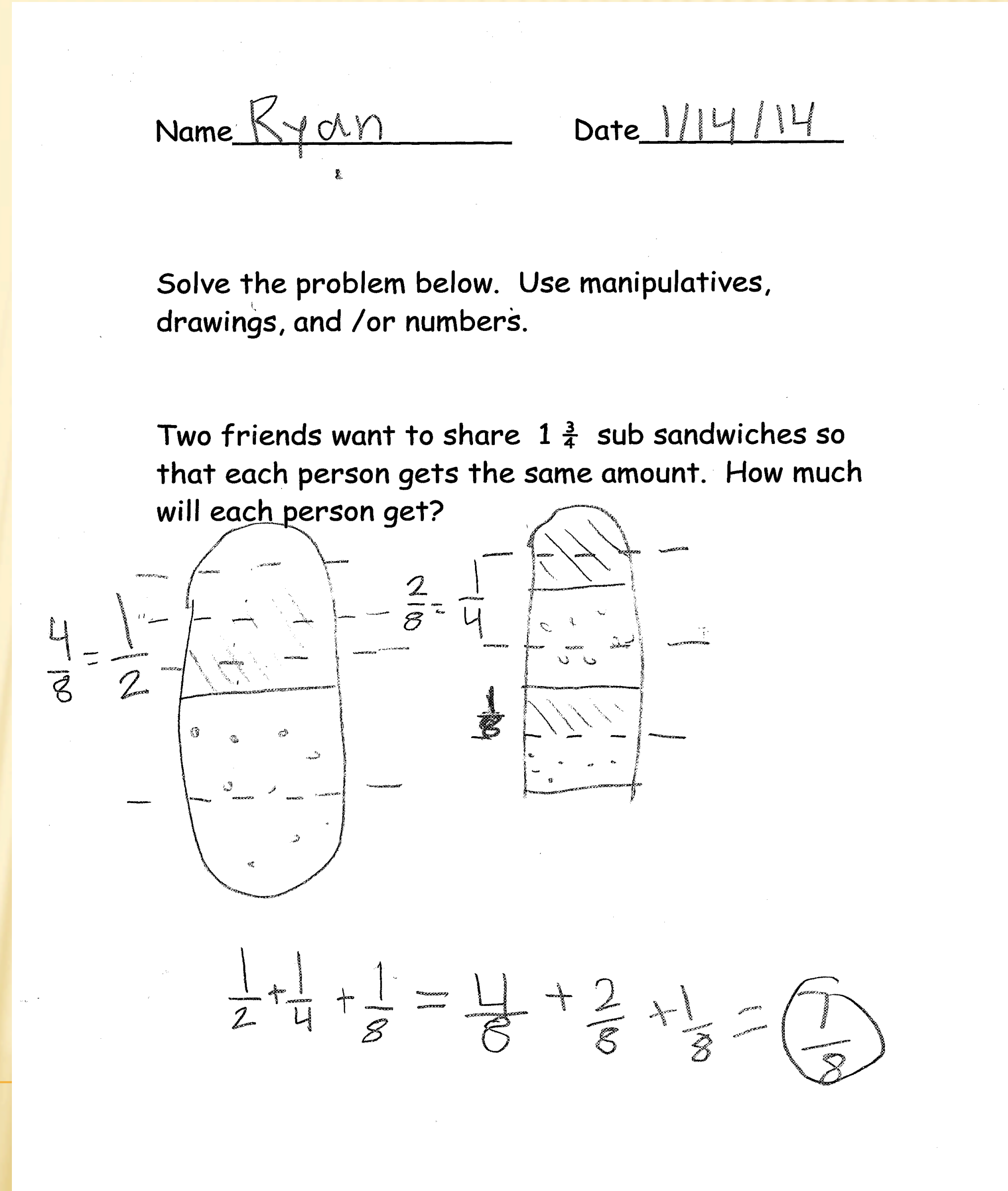
Add the fractions below. You may choose to use manipulatives, drawings and or numbers to solve the problem.

$1/2 + 1/4 + 1/8 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{3}{14}$

Explain how you solved the above problem.

I added the numerators and got 3. I added the demominaters and got 14.

Story problems help students build understanding of fractions so that the procedures make sense.



Name Ryan Date 1/14/14

Solve the problem below. Use manipulatives, drawings, and /or numbers.

Two friends want to share $1\frac{3}{4}$ sub sandwiches so that each person gets the same amount. How much will each person get?

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8} = \frac{7}{8}$

RIBBON PROBLEM

I had a string of ribbon. The ribbon is cut into strips that are ¼ of a foot. There are 3 pieces of the ribbon. How long was the original piece of ribbon?

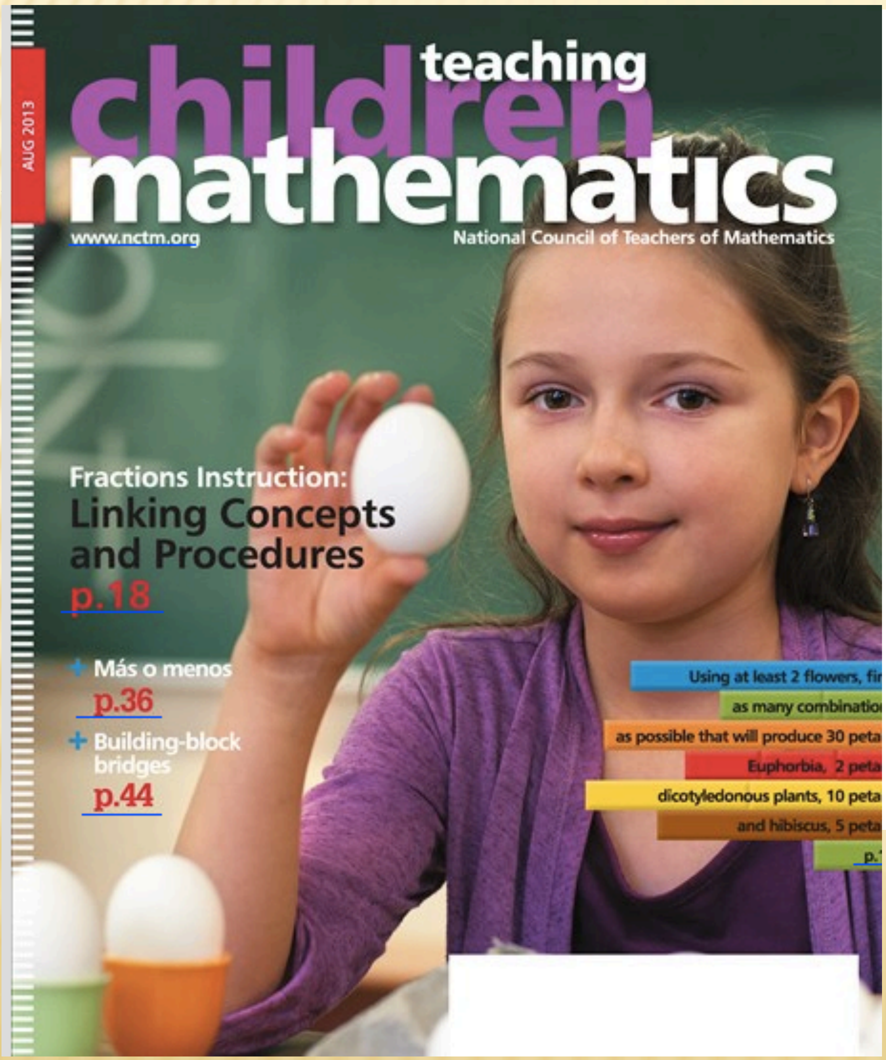
I had a string of ribbon. The ribbon is cut into strips that are 1/8 of a foot. There are 6 pieces of the ribbon. How long was the original piece of ribbon?

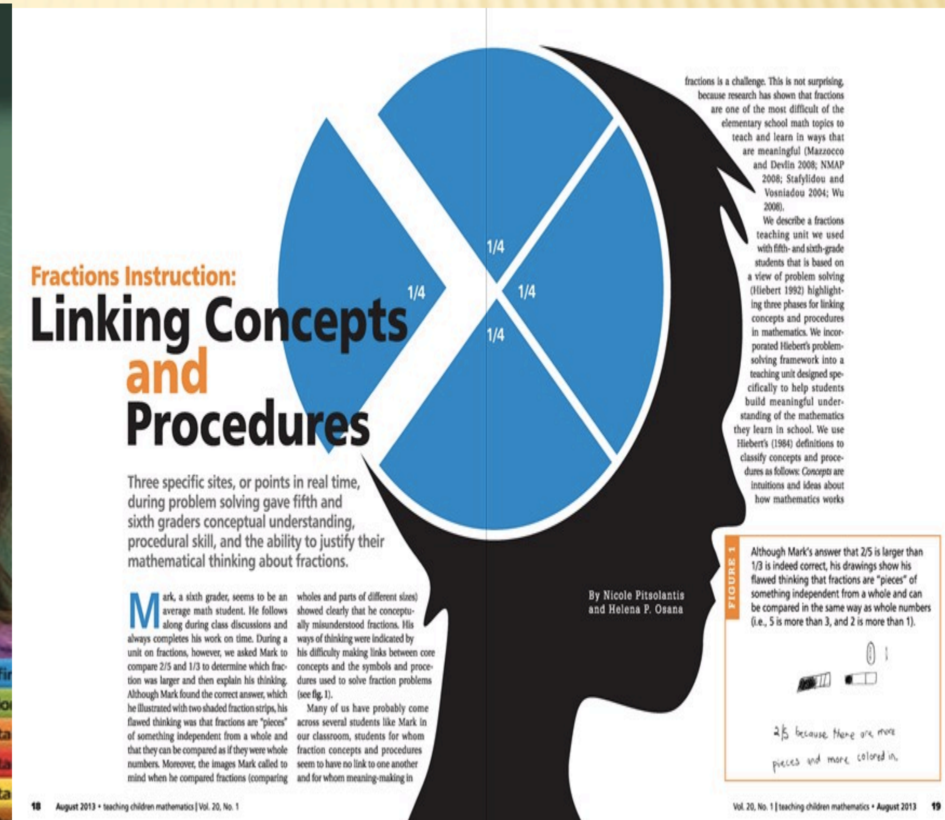
POSTER PROBLEM – SMARTER BALANCED

Connie created a square shaped poster. The length of each side of the poster is 1 ½ feet. What is the area of this poster in square feet?

Heather created a rectangular poster with the same area as Connie's. She has different dimensions than Connie's poster. What could be the dimensions of Heather's poster, in feet?

Research That Supports Developing Fraction Concepts with Problem Solving





Fractions Instruction:

Linking Concepts and Procedures

Three specific sites, or points in real time, during problem solving gave fifth and sixth graders conceptual understanding, procedural skill, and the ability to justify their mathematical thinking about fractions.

By Nicole Pitsolantis and Helena P. Osana

Mark, a sixth grader, seems to be an average math student. He follows along during class discussions and always completes his work on time. During a unit on fractions, however, we asked Mark to compare 2/5 and 1/3 to determine which fraction was larger and then explain his thinking. Although Mark found the correct answer, which he illustrated with two shaded fraction strips, his flawed thinking was that fractions are "pieces" of something independent from a whole and that they can be compared as if they were whole numbers. Moreover, the images Mark called to mind when he compared fractions (comparing wholes and parts of different sizes) showed clearly that he conceptually misunderstood fractions. His ways of thinking were indicated by his difficulty making links between core concepts and the symbols and procedures used to solve fraction problems (see fig. 1).

Many of us have probably come across several students like Mark in our classroom, students for whom fraction concepts and procedures seem to have no link to one another and for whom meaning-making in fractions is a challenge. This is not surprising, because research has shown that fractions are one of the most difficult of the elementary school math topics to teach and learn in ways that are meaningful (Mazzocco and Devlin 2008; NMAP 2008; Stafylidou and Vosniadou 2004; Wu 2008).

We describe a fractions teaching unit we used with fifth- and sixth-grade students that is based on a view of problem solving (Hiebert 1992) highlighting three phases for linking concepts and procedures in mathematics. We incorporated Hiebert's problem-solving framework into a teaching unit designed specifically to help students build meaningful understanding of the mathematics they learn in school. We use Hiebert's (1984) definitions to classify concepts and procedures as follows: *Concepts* are intuitions and ideas about how mathematics works

FIGURE 1

Although Mark's answer that 2/5 is larger than 1/3 is indeed correct, his drawings show his flawed thinking that fractions are "pieces" of something independent from a whole and can be compared in the same way as whole numbers (i.e., 5 is more than 3, and 2 is more than 1).

2/5 because there are more pieces and more colored in.

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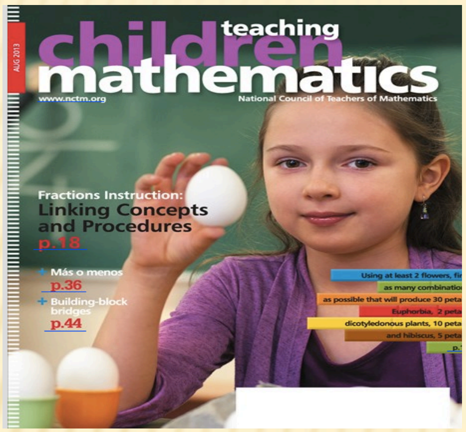
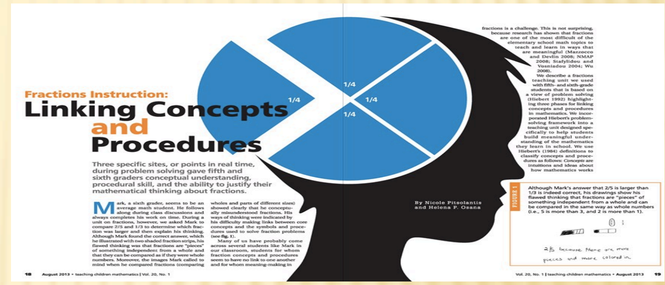
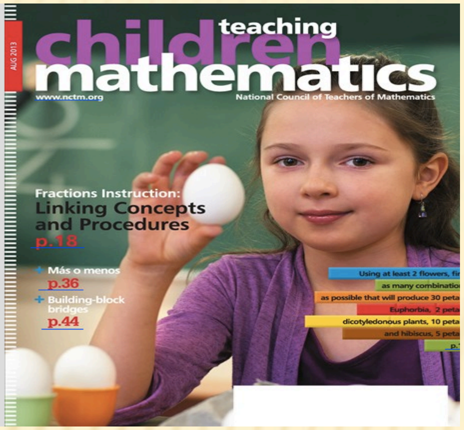


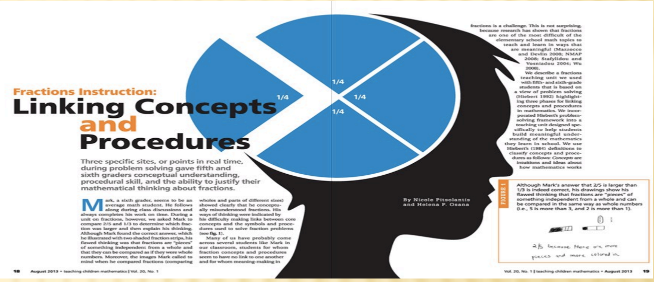


“ **Concepts** are intuitions and ideas about how mathematics works that make personal sense to students, who can acquire them from both everyday experiences and school instruction.

Procedures are efficient, step-by-step rules used to solve problems. Knowing procedures involves being able to use those rules, but it also involves being able to represent mathematical quantities using numbers.” (Hiebert 1984)

Research That Supports Developing Fraction Concepts with Problem Solving





Hiebert (1992) suggests. . .

Teachers should emphasize the links between concept and procedure during instruction. This will help students to see the underlying meanings of mathematical symbols and rules.

REMEMBER. . .

- Students learn best when math content is related to real world situations.
- Visual fraction models and/or equations help students to represent and solve the problem.
- Bridging the understanding of whole number computation to fractional computation will promote student understanding.