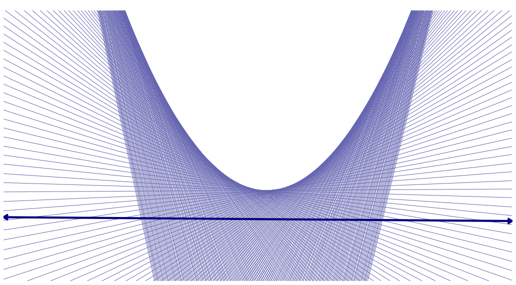
The Conic Sections:

From Paper Folding, to Sketches, to Equations

Terry Coes
Rocky Hill School
East Greenwich, RI
LCoes@aol.com



NCTM Annual Conference
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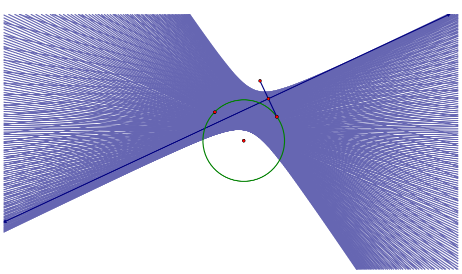
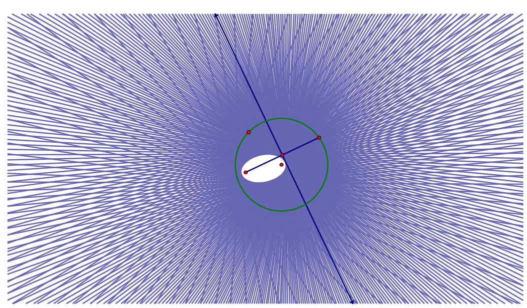
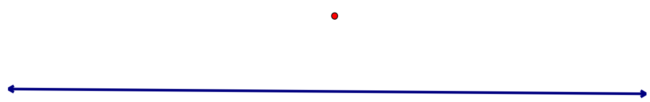


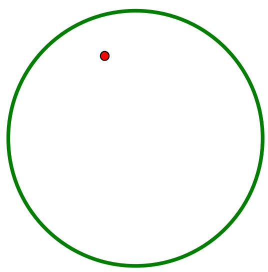




To make the tangents to a **parabola**:

Draw a line and a point off the line.

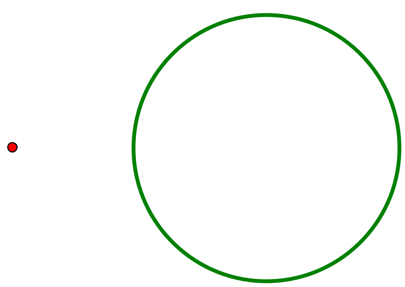
Fold numerous different points on the line to the point, and crease the paper.



To make the tangents to an **ellipse**:

Draw a circle and a point **inside** the circle.

Fold numerous different points on the circle to the point, and crease the paper.

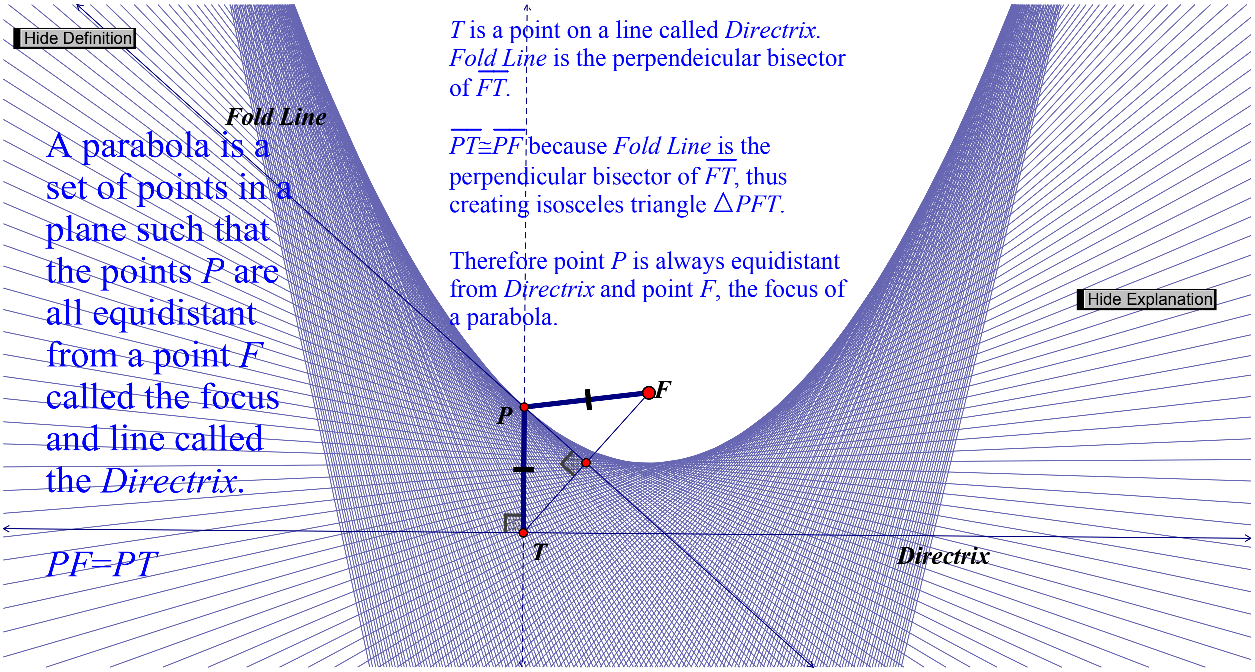


To make the tangents to a **hyperbola**:

Draw a circle and a point **outside** the circle.

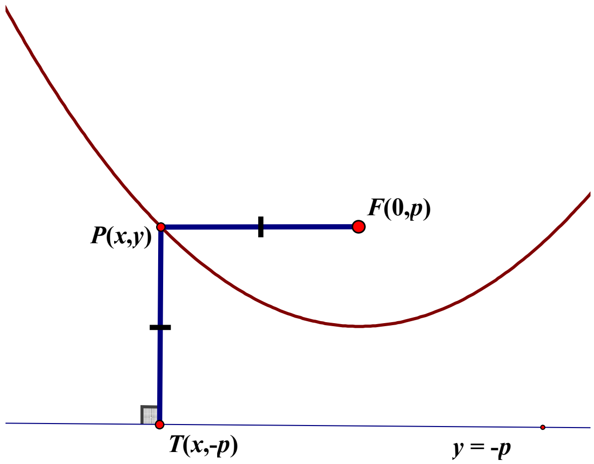
Fold numerous different points on the circle to the point, and crease the paper.

Parabola



When you fold point T (any point on the *Directrix*) to point F and crease the fold, the fold line will be tangent to the parabola.

The Parabola's Equation—one version, anyway



This parabola has a focus at F(0,p), a directrix y= -p, and its vertex at the origin (0,0)

By the definition

$$PF = PT$$
$$\sqrt{(x-0)^2+(y-p)^2} = \sqrt{(x-x)^2+(y--p)^2}$$

Squaring and expanding:

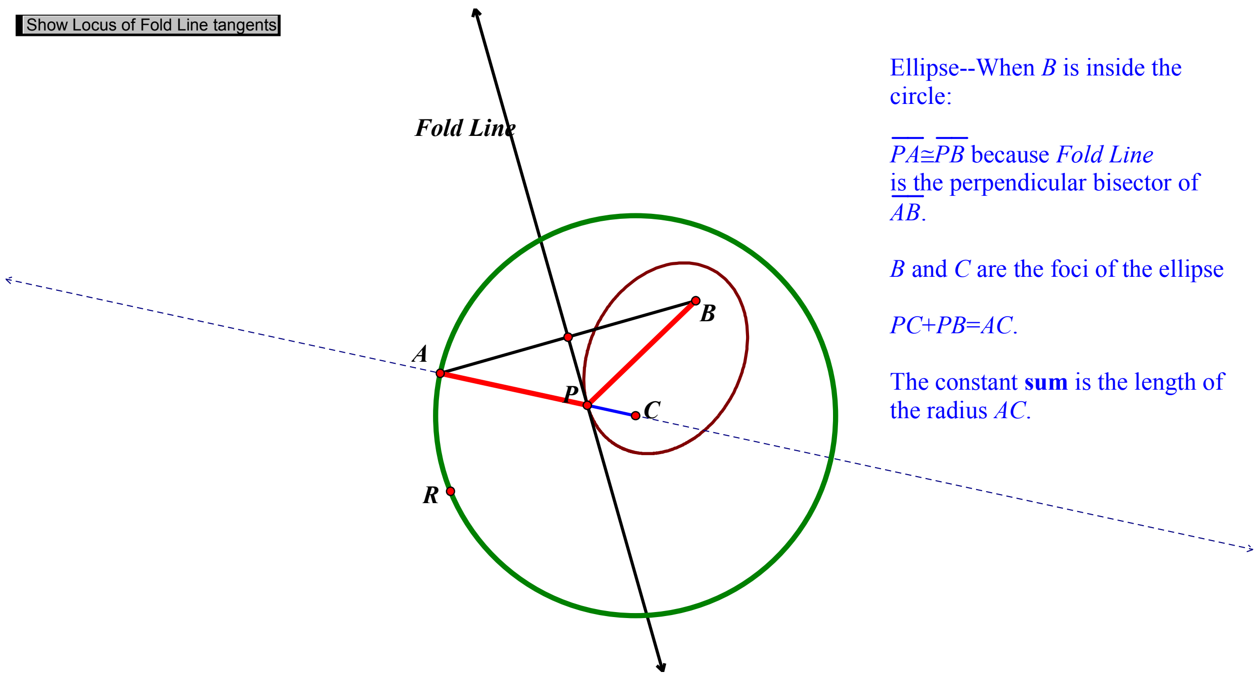
$$x^2+y^2-2py+p^2 = y^2+2py+p^2$$
$$x^2 = 4py$$

Move the vertex from the origin (0,0) to the point (h,k)

And the equation becomes

$$(x-h)^2 = 4p(y-k).$$

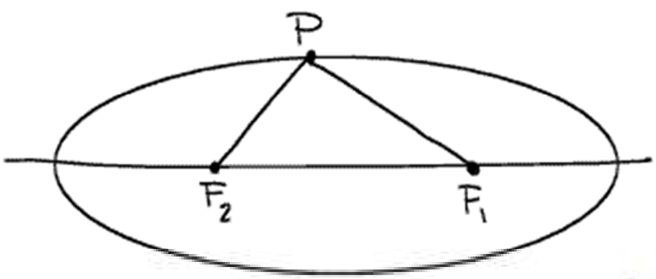
Fold point B (inside the circle) to point A on the circle.



With thanks to Ellie Sharpe, Rocky Hill class 0f 2009

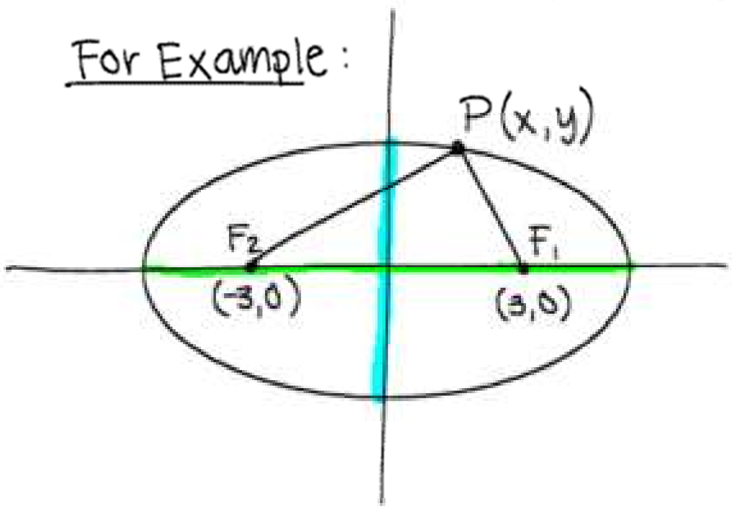
Class Notes - 4.24.08

Ellipse - a figure in a plane in which the sum of the distances of each point to 2 points called foci (plural of focus) is constant



$|\overline{PF_1}| + |\overline{PF_2}| = C$ ← C is always the same, all the way around the ellipse

For Example:



$|PF_1| + |PF_2| = \underline{10}$

= minor axis

= major axis

$$|PF_1| + |PF_2| = 10$$

distance formula

$$\sqrt{(x-3)^2+(y-0)^2} + \sqrt{(x+3)^2+(y-0)^2} = 10$$

$$\left(\sqrt{(x-3)^2+y^2}\right)^2 = \left(10 - \sqrt{(x+3)^2+y^2}\right)^2$$

$$(x-3)^2 + y^2 = 100 - 20\sqrt{(x+3)^2+y^2} + (x+3)^2 + y^2$$

$$x^2 - 6x + 9 + y^2 = 100 - 20\sqrt{(x+3)^2+y^2} + x^2 + 6x + 9 + y^2$$

$$-6x \qquad\qquad -6x$$

$$\frac{-12x = 100 - 20\sqrt{(x+3)^2+y^2}}{4}$$

$$-3x = 25 - 5\sqrt{(x+3)^2+y^2}$$

$$-25 \qquad -25$$

$$(-3x-25)^2 = \left(-5\sqrt{(x+3)^2+y^2}\right)^2$$

$$9x^2 + 150x + 625 = 25\left((x+3)^2 + y^2\right)$$

$$9x^2 + 150x + 625 = 25(x^2 + 6x + 9 + y^2)$$

$$9x^2 + 150x + 625 = 25x^2 + 150x + 225 + 25y^2$$

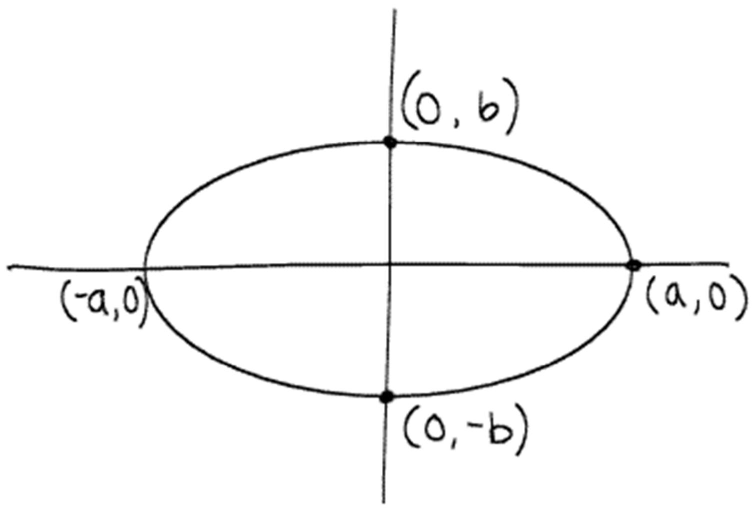
$$-9x^2 \qquad -225 \quad -9x^2 \qquad -225$$

$$\frac{400}{400} = \frac{16x^2 + 25y^2}{400}$$

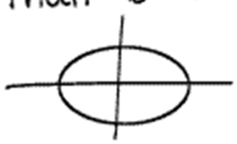
$$\boxed{1 = \frac{x^2}{25} + \frac{y^2}{16}}$$

Equation of an ELLIPSE

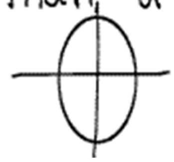
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



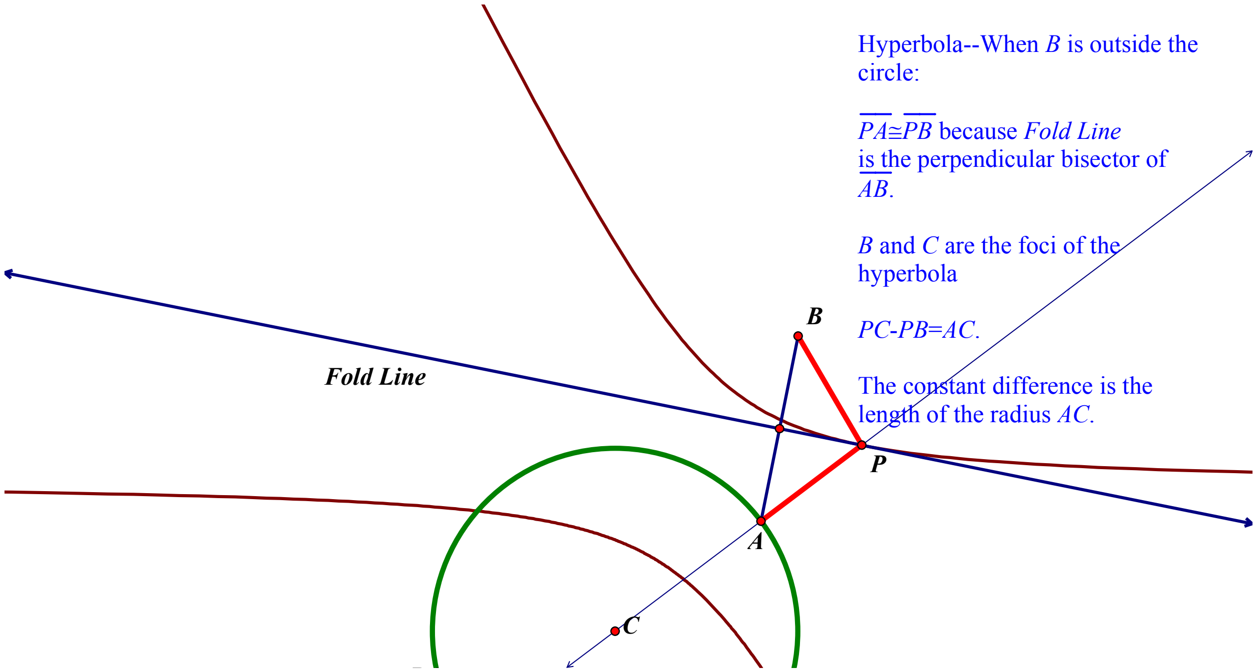
* if "a" is larger than "b":



* if "b" is larger than "a":



Fold a point B outside the circle to a point A on the circle.



Working with the standard equation of a hyperbola to understand asymptotes

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Solving for y:

$$\frac{x^2}{a^2} - 1 = \frac{y^2}{b^2}$$

$$\frac{x^2 - a^2}{a^2} = \frac{y^2}{b^2}$$

$$\frac{b^2}{a^2}(x^2 - a^2) = y^2$$

$$\pm\frac{b}{a}\sqrt{x^2 - a^2} = y$$

Since *a* is a constant, as x grows large, $\pm\frac{b}{a}\sqrt{x^2 - a^2} \to \pm\frac{b}{a}\sqrt{x^2} \to \pm\frac{b}{a}x$

So y approaches, but never equals $\pm\frac{b}{a}x$.