While you wait: Sample CCSS Assessment

Gasoline consumption (grade 6)

◀ About the task CCSSM Alignment Part a Part b **Part c** Scoring ▶

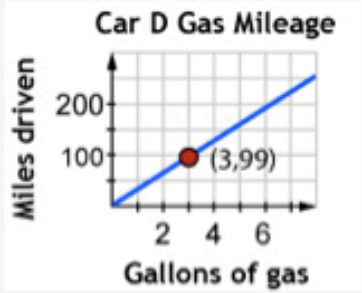
A car magazine is writing a story about four cars. For each car, they will report the number of miles driven for different amounts of gas.

The magazine received gas mileage information for cars from several companies.

Miles driven		360	480	
Gallons of gas	10	15		24

$D = 18g$ where D represents the distance traveled in miles, and g represents gallons of gas consumed.

Car C can travel 324 miles on a 12-gallon tank.



Krystal bought one of these cars. She drove 924 miles and used 28 gallons of gas. Based on her gas consumption, which car did she most likely buy?

- A
- Car B
- C
- Car D

http://www.ccsstoolbox.com/parcc/

The CCSS Mathematical Practices Come Alive: Focus on Proportional Reasoning

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The Standards for Mathematical Practice

Grouping the practice standards

1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

Reasoning and explaining

4. Model with mathematics

5. Use appropriate tools strategically

Modeling and using tools

7. Look for and make use of structure.

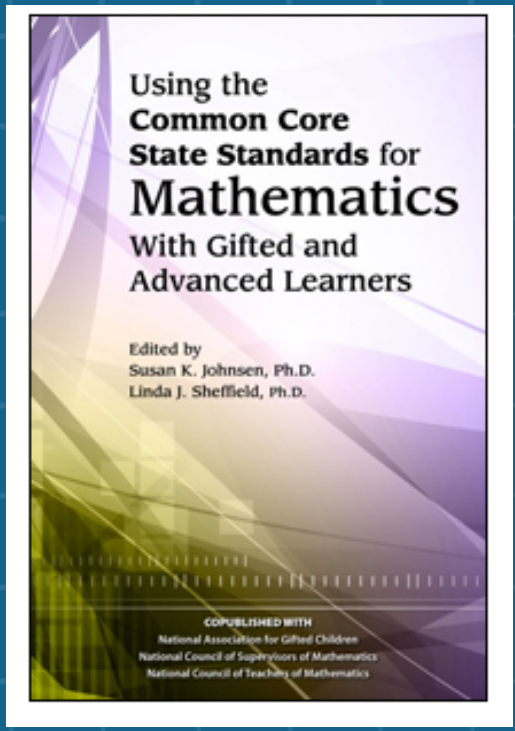
8. Look for and express regularity in repeated reasoning.

Seeing structure and generalizing

Proposed 9th Standard for Mathematical Practice

9. Solve problems in novel ways and pose new mathematical questions of interest to investigate.

- **NAGC/NCTM/NCSM, 2012: *Using the Common Core State Standards in Mathematics with Advanced and Gifted Learners***



Take a Look at Sample Assessments

Gasoline consumption (grade 6)

About the task | CCSSM Alignment | Part a | Part b | **Part c** | Scoring

A car magazine is writing a story about four cars. For each car, they will report the number of miles driven for different amounts of gas.

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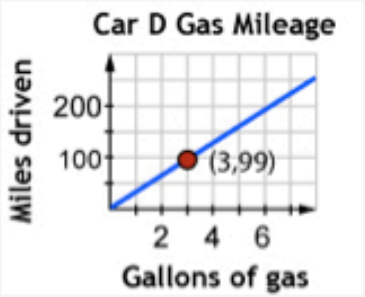


Krystal bought one of these cars. She drove 924 miles and used 28 gallons of gas. Based on her gas consumption, which car did she most likely buy?

- A
- Car B
- C
- Car D

Make sense of problems.

Reason abstractly and quantitatively.

Model with mathematics.

Use appropriate tools

http://www.ccsstoolbox.com/parcc/

The new standards and the consortia assessments of those standards **fully integrate** content with **higher-order thinking.**

Herman and Linn, 2014

- *Proportional reasoning is both the capstone of elementary mathematics and the cornerstone of higher mathematics.*

Lesh, Post & Behr, 1988

What is proportional reasoning?

Conceptual understanding of ratio relationships includes:

- **Learning to attend to two quantities simultaneously**
- **Forming a multiplicative comparison of two quantities**
- **Forming a composed unit, and iterating and partitioning the unit**
- **Creating a family of equivalent ratios by iterating and partitioning or by using multiplication and division**

NCTM Research Clip for Ratio and Proportion, posted March 11, 2013 www.nctm.org

Use ratio and rate reasoning

Jason and Jill are running. They both start at the same place and end at the same place. Jill runs faster than Jason.

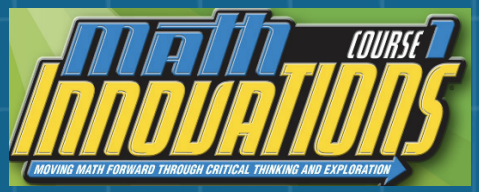
1. Who reached the end first?
2. What if Jason reached the finish line before Jill? How could this be? Write down as many reasons as you can think of.
3. What assumptions did you make to answer Question 1?

Reason abstractly and quantitatively.

Pose new problems to invstigate.

Solve problems in novel ways

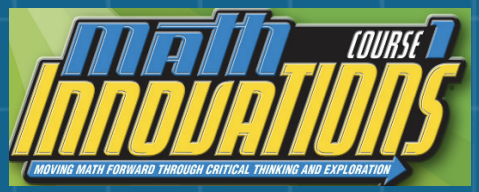
Model with Mathematics



Calvin and Homer raced a 50-yard dash, and Calvin won by 5 yards.

1. The next time, they each ran at the same speed as before, but Homer got a five-yard head start. Who won this time?
2. For the third race, Calvin and Homer ran at the same speeds as before. However, this time Homer ran 50 yards and Calvin started back 5 yards and ran 55 yards. Who won this third race?

Make sense of problems.



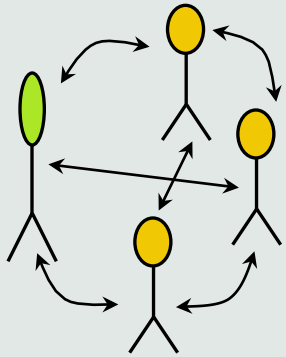
At This Rate, p. 9

Reason abstractly and quantitatively.

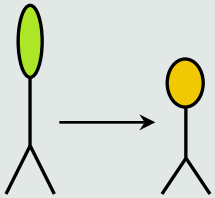
Talk Moves

Time to compare and share

Towards this…



…rather than this.



Students as mathematicians

1. Think, model and write time
2. Partner talk
3. Repeat/rephrase and check
   - Teacher
   - Student
4. Agree/disagree and explain why
5. Add on

Math Innovations

Why is proportional reasoning so important?

- **Affects understanding of fractions and percents**
- **Builds understanding of measurement and derived attributes in mathematics and science (speed, acceleration, density, etc.)**
- **Supports students' understanding of linear functions and graphs (understand that $y = mx$ is a proportional statement)**
- **Is foundational for understanding algebra, geometry, trig, calculus, statistics, etc.**

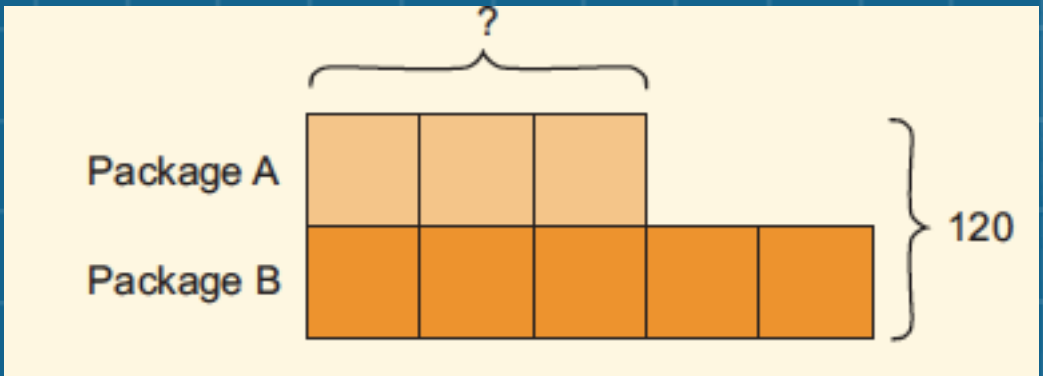
NCTM Research Clip & Research Brief for Ratio & Proportion posted March 11, 2013 www.nctm.org

"Proportional relationships provide a powerful means for students to develop algebraic thinking and function sense."
Cai & Sun, 2002, p. 195

Proportional relationships and Fractions

8. a) The weight of Package A is $\frac{3}{5}$ the weight of Package B. If the two packages together weigh 120 pounds, how much does Package A weigh?

b) Explain how to use a tape diagram to solve the problem.



Use tools appropriately.

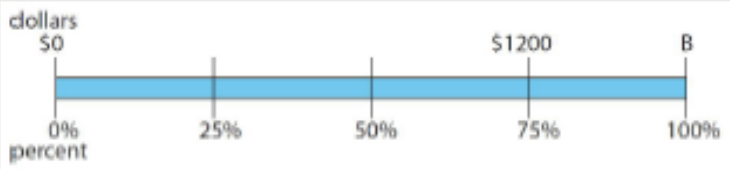
Math Innovations Course 2
Moving math forward through critical thinking and exploration

Puzzling Proportions, SE p. 7

Proportional Relationships and Percents

Solving a percent problem

If 75% of the budget is $1200, what is the full budget?

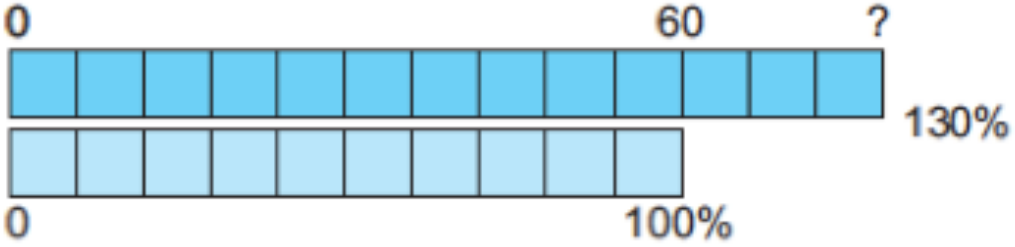


"I said 75% is 3 parts and is $1200
25% is 1 part and is $1200 ÷ 3 = $400
100% is 4 parts and is 4 · $400 = $1600"

Progressions for Ratios and Proportional Relationships, p. 7

Ziv made the diagram below to find 130% of 60. Remember that 130% equals 100% + 30%.

Ziv first drew a strip to show 100% and then drew 30% more. He knew that 130% of 60 was going to be greater than 60 because 100% of 60 = 60.



Puzzling Proportions, SE p. 115

Let's Try It: Percent Sense

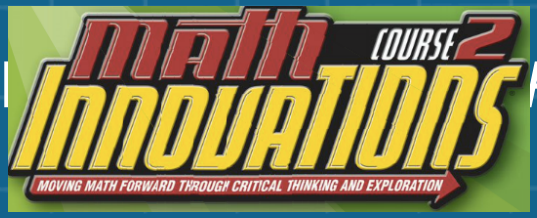
7.8 gallons of gas cost over $25 and only filled 30% of the capacity of the tank.

- **How many gallons will the tank hold when filled to capacity?**

•If 30% of a number, *n*, is 7.8, write a corresponding equation and solve for *n*.

Model with Mathematics.

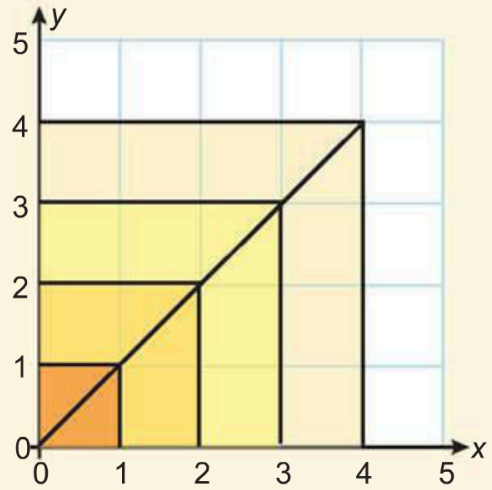
Use tools appropriately.



Puzzling Proportions, SE p. 126

Relating Ratio and Slope

Use the graph to answer Questions 10–14.



Use tools appropriately.

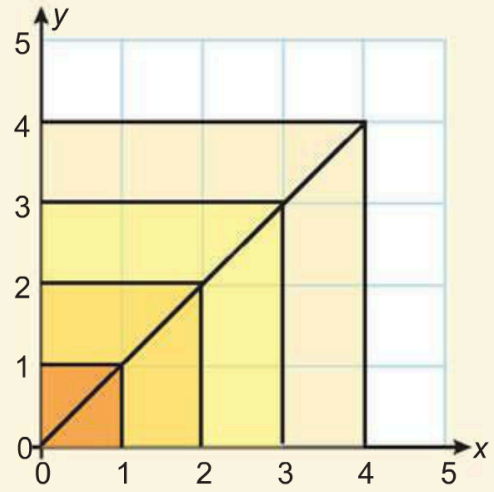
Reason abstractly and quantitatively

11. What $y : x$ ratio describes this set of squares? Record this ratio in simplified form. What is the value of the ratio?

12. Use the diagonal from the origin through the opposite vertex of the squares to create a table that relates the x- and y-values. Determine the equation of this line.

13. How are the slope of the line and the ratio between the y- and x-values related?

Use the graph to answer Questions 10–14.



Look for and make use of structure

17. For each equation, list the slope and give the dimensions of three rectangles whose diagonals would lie on the line.

a) $y = 3x$

b) $y = \frac{2}{3}x$

c) $y = \frac{4}{5}x$

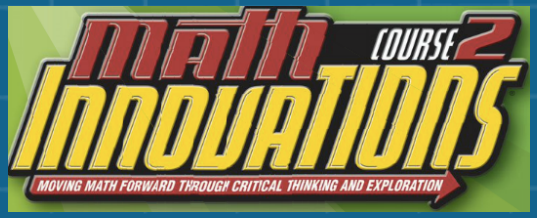
d) $y = 2x$

Liza has 50 songs on her MP3 Player. She wants more songs and decides to use part of her allowance to download two songs per week. Ned just got an MP3 Player for his birthday. He also got money for his birthday and plans to download five songs per week.

a. Write an equation to represent the relationship between week number and the number of songs Liza has. Use x for the independent variable and y for the dependent variable.

Make Sense of Problems

Model with Mathematics



Line It Up, SE p. 132

Proportional Reasoning, Slope, and Direct Variation

b. Write an equation to represent the relationship between week number and the number of songs Ned has. Use x for the independent variable and y for the dependent variable.

c. Is either of the equations you wrote in Parts a or b a direct variation? Why or why not?

d. Graph your two equations.

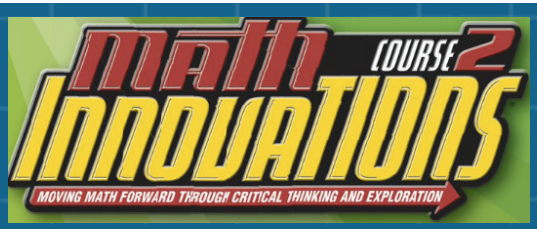
Model with Mathematics

Reason abstractly

e. Find the slope and y-intercept of each line. What does each represent in this situation?

Use appropriate tools

f. When will Ned have as many songs as Liza? How do you know?



Line It Up, SE p. 133

CCSS-Based Assessment for *Math Innovations*

George made a graph that represents the total number of dollars he earned for every mile he walked during the annual school walkathon. For each mile he walked he earned $5.00.

Circle all the statements below that must be true.

A) The graph is a straight line.

B) The point (4, 25) is on the line.

C) If George earned $60, he walked 12 miles.

D) The graph includes the origin.

- Does the graph show a proportional relationship? Explain your answer.

Adapted from SBAC released items from Showcase Math Materials for Grade 7.

5. Part I. George made a graph that represents the total number of dollars he earned for every mile he walked during the annual school walkathon. For each mile he walked he earned $5.00.

Circle all the statements below that must be true.

(A) The graph is a straight line.

B) The point (4, 25) is on the line.

(C) If George earned $60, he walked 12 miles.

(D) The graph includes the origin.

$\frac{20}{4}$ $\frac{10}{2}$: $\frac{5}{1}$

Part II. Does the graph show a proportional relationship? Yes Explain your answer.

There are three ways that this shows a proportional relationship. One way is that the graph is graphed as a straight line (increasing by equal intervals) and going through the origin. Then there is the equation that could be written as $5w = e (5 dollars for every week(w) equals the earnings(e)), $5 represents the constant that is proportional. The last way is that all the points will all have the same value of the ratio $(\frac{1}{5})$

9	10	11	12
45	50	55	60
$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Part II. Does the graph show a proportional relationship? yes Explain your answer.

The graph shows a proportional relationship because it shows a straight line that goes through the origin. This means that the two variables (distance walked and money earned) are related multiplicatively in a $y = kx$ equation. The slope of the line is k, the constant of direct variation. When one variable increases, the other does as well. Also, the proportions of the two variables will always simplify down to the same proportion. For example $\frac{\$60}{12\text{ mi}} = 5$, and $\frac{\$5}{1\text{ mi}} = 5$ $(5=5)$. If this data was to be put on a table, it would have a constant recursive rule, previous + 5 = new. The straight line graph, constant slope, $y = kx$ equation, and the recursive rule from the table all show that these two variables are in a proportional relationship.

6. Which of these expressions is equivalent to $4(4a + 2)$? Circle all that apply.

A) $16a + 2$ $= 16a + 2$

(B) $4(2 + 4a)$ $= 8 + 16a = 16a + 8$

C) $4(4a) + 2$ $= 16a + 2$

(D) $4a + 2 + 4a + 2 + 2 + 4a + 2 + 4a = 16a + 8$

(E) $16a + 8$

For more information

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- *Common Core State Standards*, www.corestandards.org/
- NCTM Research Clip and Research Brief for Ratio and Proportion: www.nctm.org
- Progressions Documents for the CCSS-M: ime.math.arizona.edu/progressions/
- *Math Innovations: Moving Math Forward through Inquiry and Exploration* by Gavin, Sheffield and Chapin, http://www.kendallhunt.com (For a free 30-day Flourish trial visit www.kendallhunt.com/freeflourishtrial)