Some Algebra-Related PBL Problems

1. A town's building code does not permit building a house that is more than 35 feet tall. An architect working on the design shown at right would like the roof to be sloped so that it rises 10 inches for each foot of horizontal length.
    a. Given the other dimensions in the diagram, describe a way that the builder can carry out this plan?
    b. Two vertical supports (shown dotted in the diagram) are to be placed 6 feet from the center of the building. How long should they be?

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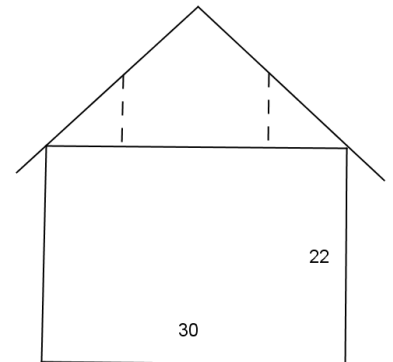
2. In 1986, renovations were made to the stairs in the Status of Liberty in an overall celebration of its bicentennial. Frank Schettino was the draftsman in charge of the calculations for the details of the structural steel for those stairs.   He found that the stairs in the Lady's pedestal needed to have a vertical rise of 7.8 inches and a horizontal run of 9.75 inches. The more complicated spiral stairs the led up the body had a constant vertical rise of 9 inches and average horizontal run of 5.06 inches.
    a. Using graph paper (or GeoGebra), create a graphical representation of these two sets of stairs.
    b. Which flight of stairs do you think is steeper? Why?
    c. Calculate the $rise/run$ ratio for each flight and verify that the greater ratio belongs to the flight you thought was steeper.

3. (Continuation) A flight of stairs goes from point A(2,2) to B(6,4) to C(10,6) as shown in the diagram at the right. We define the $rise/run$ ratio of the line AB made by the points on the stairs as the *slope* of the line. What's a way you would calculate that slope? What do you think the slope of the line AB is in this diagram?

C = (10, 6)

B = (6, 4)

A = (2, 2)
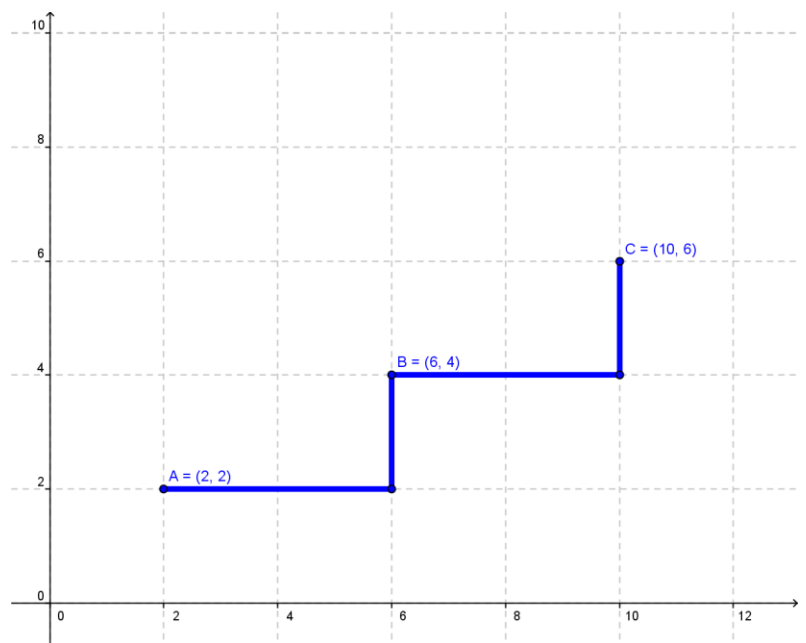
4. Find the slope of the line through
    (a) (1, 0) and (2,-4) **(b)** (3, 1) and (3 + 4t, 1 + 3t);   **(c)** ($m-$ 5, n) and (5 + m, n^2).

5. Draw the segment from (3, 1) to (5, 6), and the segment from (0, 5) to (2, 0). Calculate their slopes. You should notice that the segments are equally steep, and yet they differ in a significant way. Do your slope calculations reflect this difference?

6. Consider the segment from A(1,5) to B(6,2). Find a point C that makes triangle ABC a right triangle with C the right angle.   Find the lengths of AC and BC. How would you find the length of AB?

7. A river runs along the line $x = 3$ and a dog is tied to post at the point D=(10,5). If the dog's leash is 25 units long (the same units as the coordinates), if a fence were going to be placed at the edge of the river along $x = 3$, name the two coordinates along the river where it would be safe for the fence to end so that the dog could not fall in the river.

Where do you place the safety fence?

R I V E R
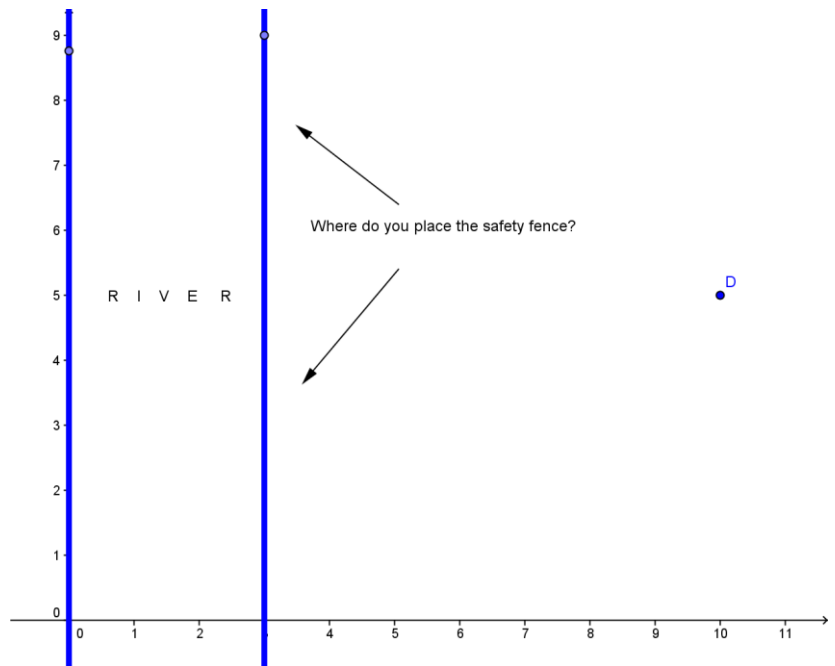
8. A slope can be considered to be a *rate*. Explain this interpretation and give an example.

9. Estimate the slopes of all the segments in the diagram at the right. Identify those whose slopes are negative. Find words to characterize lines that have negative slopes.

10. Explain the difference between a line that has *undefined slope* and a line whose slope is zero. What does it mean for the slope to be undefined?

11. Two different points on the line *y* = 2 are each exactly 13 units from the point (7, 14). Draw a picture of this situation, and then find the coordinates of these points.

12. Find the slope of the line containing the points (4, 7) and   (6, 11). Find coordinates for another point that lies on the same line and be prepared to discuss the method you used to find them.
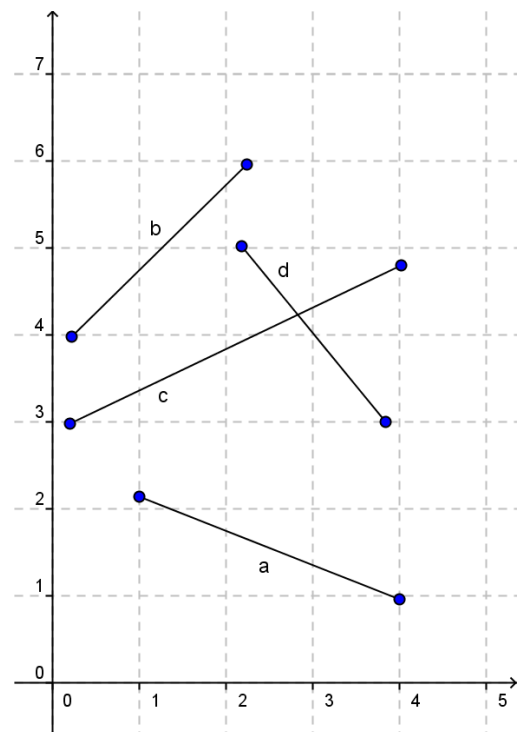
13. Three square buildings are built directly next to each other (as shown) in a city. The vertices A, B, and C are *collinear*. *Collinear* means that those three points are on the same line. Find the dimension n, the width of the third building.
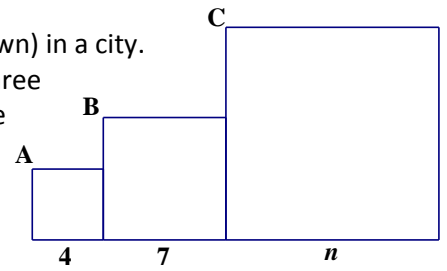
14. (Continuation) Replace the lengths 4 and 7 by *m* and *k*, respectively. Express *k* in terms of *m* and *n*.

15. A five-foot tall student casts a shadow that is 40 feet long while standing 200 feet from a streetlight. How high above the ground is the lamp?

16. (Continuation) How far from the streetlight should the student stand in order to cast a shadow that is exactly as long as the student is tall?

17. Two groups of students from different classes are playing Capture the Flag on the playground at recess. One group's home base is at A (7, 2) and the other group's home base is at B (1, 6). Help them decide where to place the flag so that the game is fair and the both have to travel the same distance to get it.

18. Using GeoGebra, plot the points P (3, 5), Q (0, 0) and R(-5, 3). Measure angle PQR, being careful to select the points in a clockwise manner. Create the segments PQ and QR. Use the Slope tool in the same toolbox as the Angle tool to find the slope of segment PQ. Do the same thing for segment QR. Make a conjecture about how these slopes are related. Verify by calculating the slopes by hand.

19. An airplane 27000 feet above the ground begins descending at the rate of 1500 feet per minute. Assuming the plane continues at the same rate of descent, how long will it be before it is on the ground?

20. Let $P = (x, y)$ and $Q = (1, 5)$. Write an equation that states that the slope of line PQ is 3. Show how this slope equation can be rewritten in the form $y-5 = 3(x-1)$. This linear equation is said to be in *point-slope form*. Explain the terminology. Find coordinates for three different points P that fit this equation.

21. Find a way to show that points $A = (-4, -1)$, $B = (4, 3)$, and $C = (8, 5)$ are collinear.

22. An airplane is flying at 36000 feet directly above Lincoln, Nebraska. A little later the plane is flying at 28000 feet directly above Des Moines, Iowa, which is 160 miles from Lincoln. Assuming a constant rate of descent, predict how far from Des Moines the airplane will be when it lands.