Paper Squares, Toothpicks, and Licorice: Making Sense of Functional Relationships

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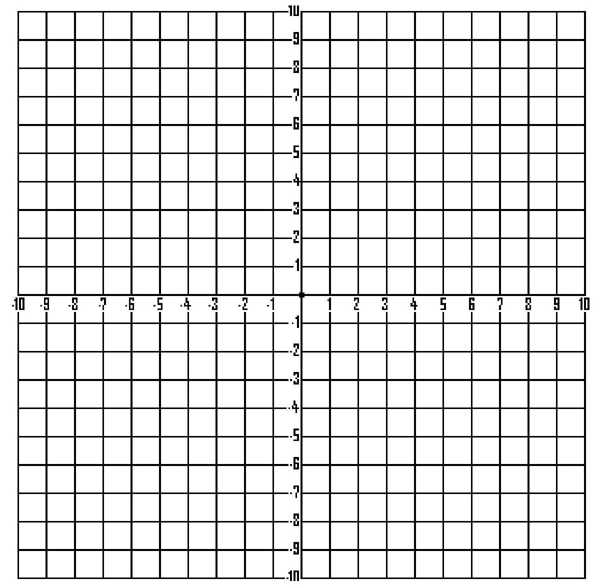
Three Views of a Function

Rule or Equation

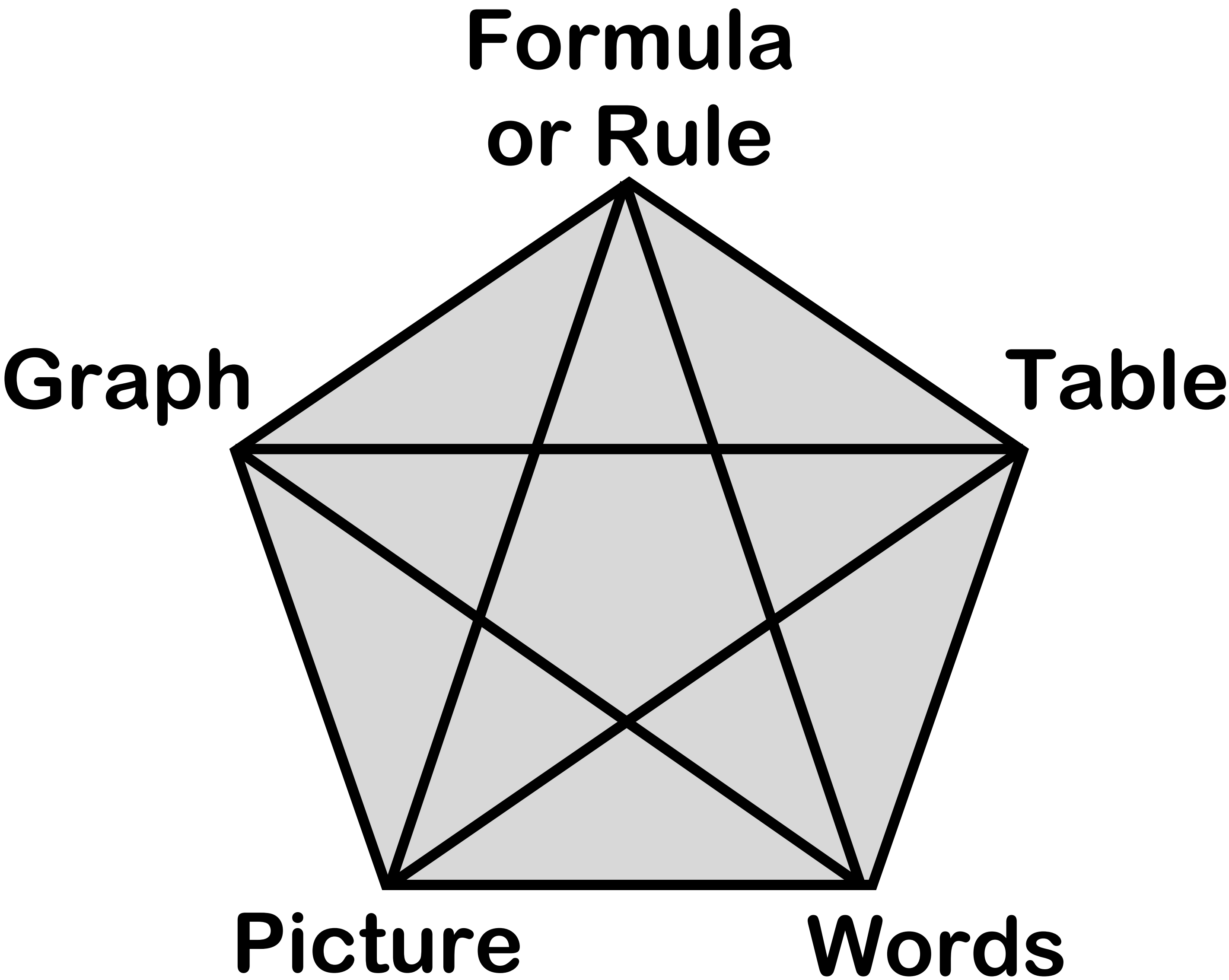
x		y	Coordinates:



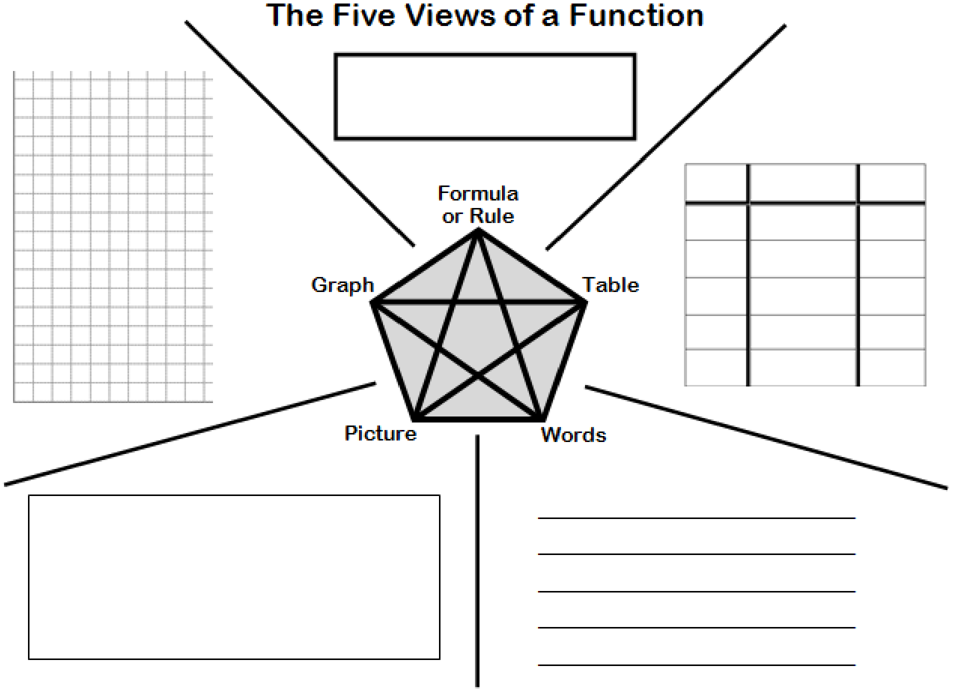
Table

Graph

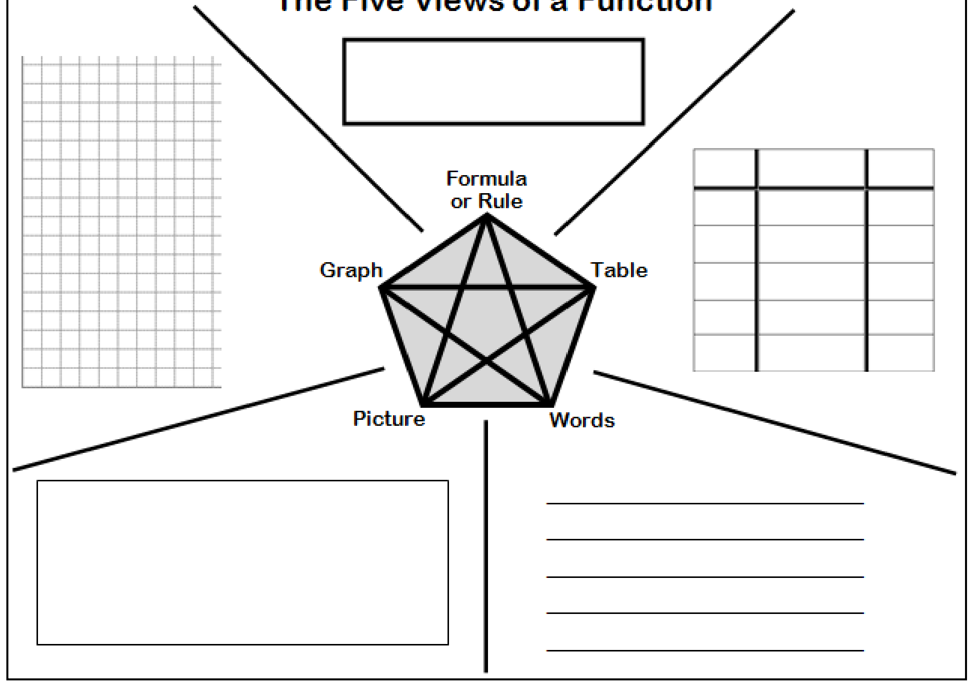
The Five Views of a Function



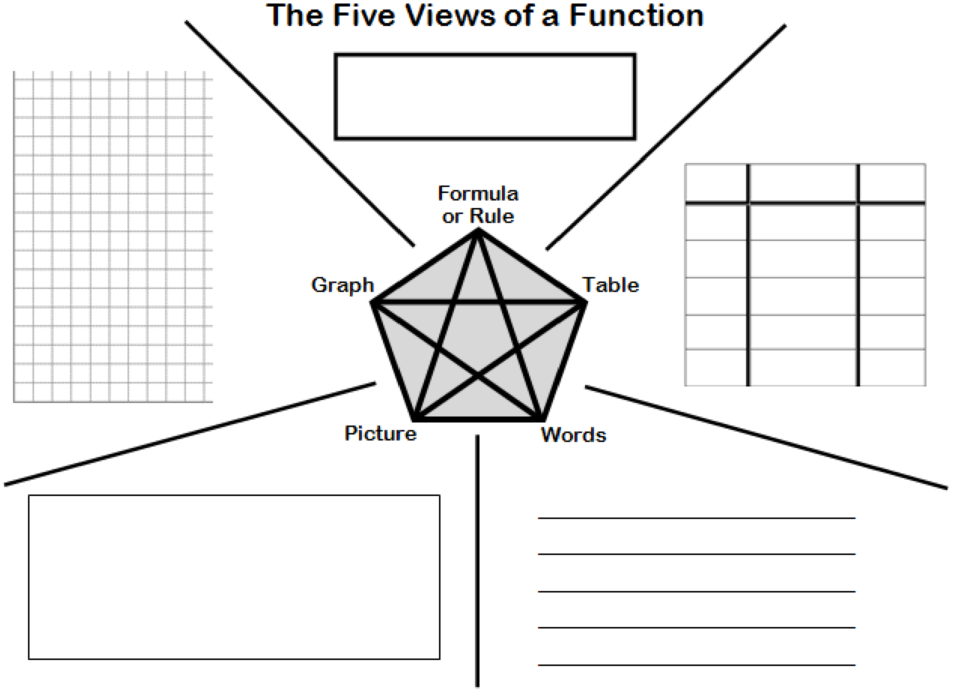
The Five Views of a Function



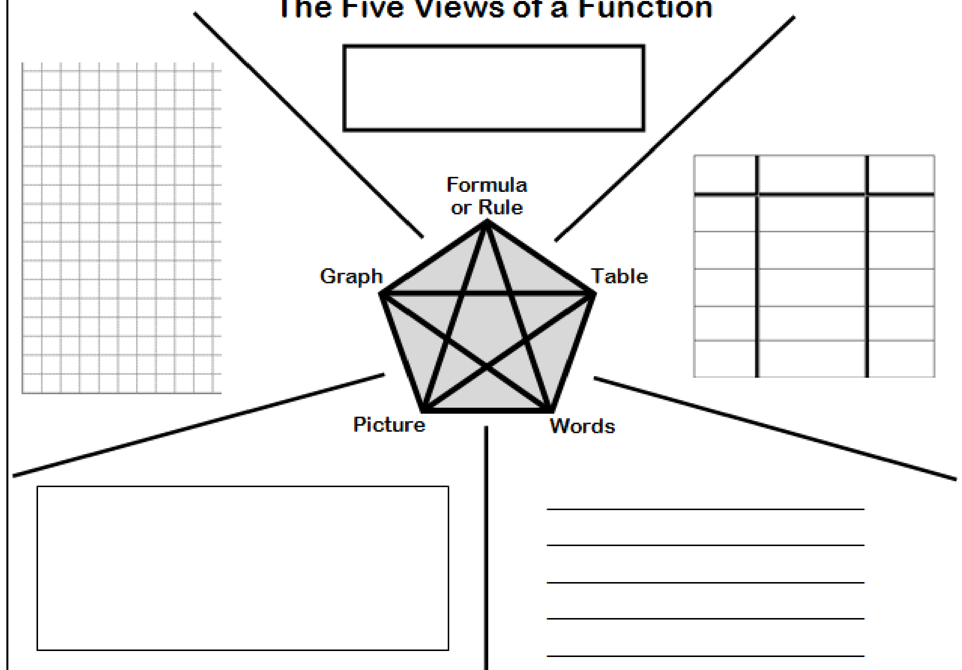
The Five Views of a Function



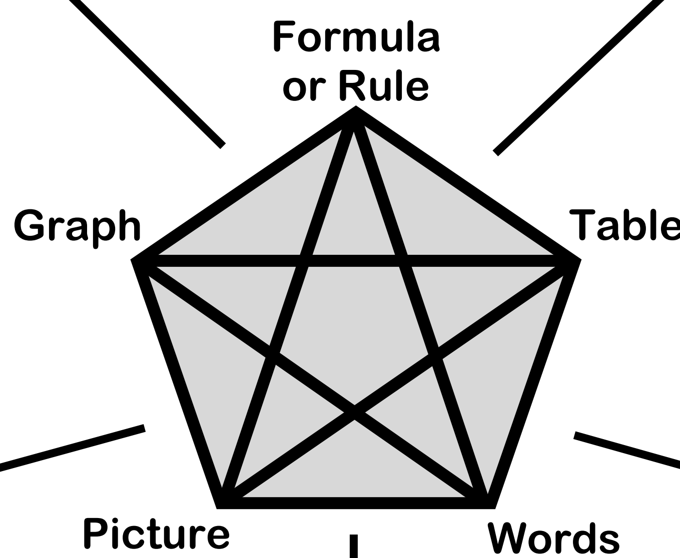
The Five Views of a Function



The Five Views of a Function



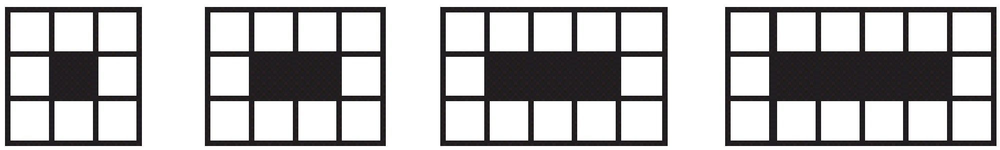
The Five Views of a Function



Garden Size	Number of Tiles
1	
2	
3	
4	
10	
47	
n	

TILES AROUND THE GARDEN

Based on the first four gardens, what will the fifth garden look like? Draw it.



Garden 1 **Garden 2** **Garden 3** **Garden 4** **Garden 5**

Complete the table below to determine the number of tiles in the 10^{th} garden.

Garden Size	Number of Tiles
1	
2	
3	
4	
10	
47	
n	

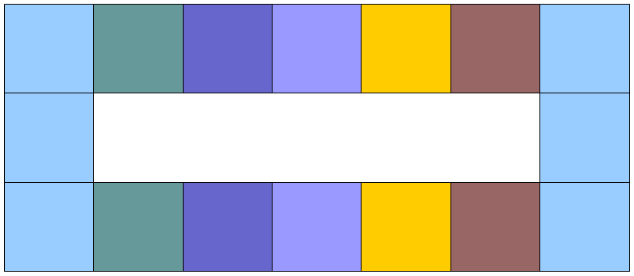
Describe in pictures or words how to find the number of tiles in a garden, given the number of tiles in the garden before it (the recursive rule).

Describe in pictures or words how to find the number of tiles in any size garden.

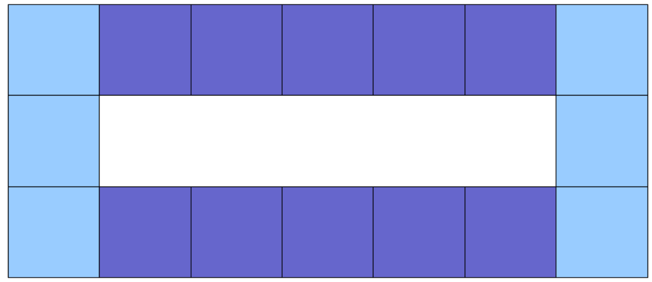
Give your explicit rule. Define the meaning of each variable.

Use the rule to determine which size garden will use 80 tiles.

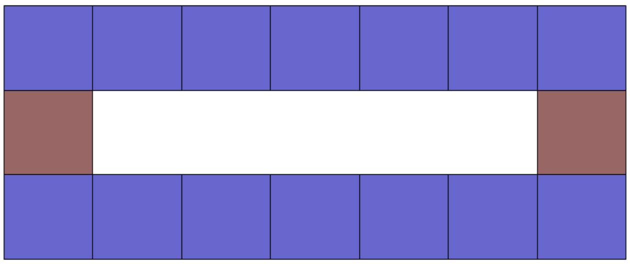
Spatial Patterns for Tiles Around the Garden



3 + n groups of 2 + 3

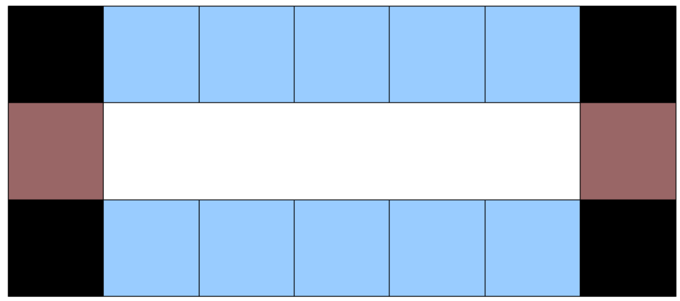


3 + 2 groups of n + 3

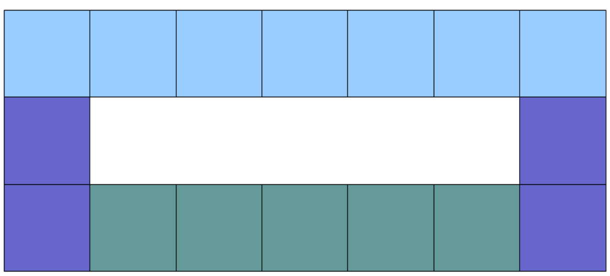


2 on the sides +
2 groups of (n+2)

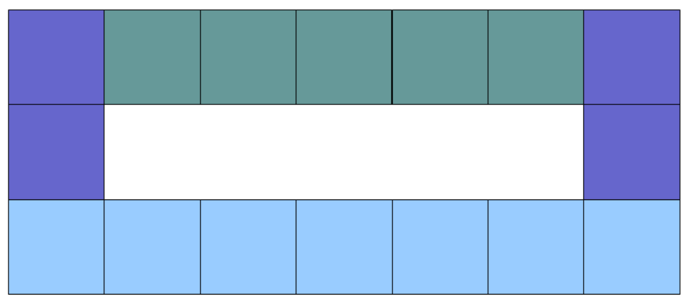
2+2(n+2)
=2+2n+4
=2n+6



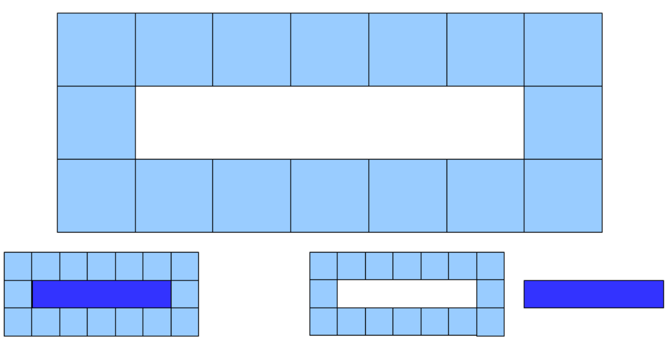
2 on the sides + 2 x n + 4 corners



(n + 2) on top + 2 x 2 sides + n on the bottom



(n + 2) on bottom + 2 x 2 sides + n on the top



whole area minus middle strip 3 x (n+2) – n

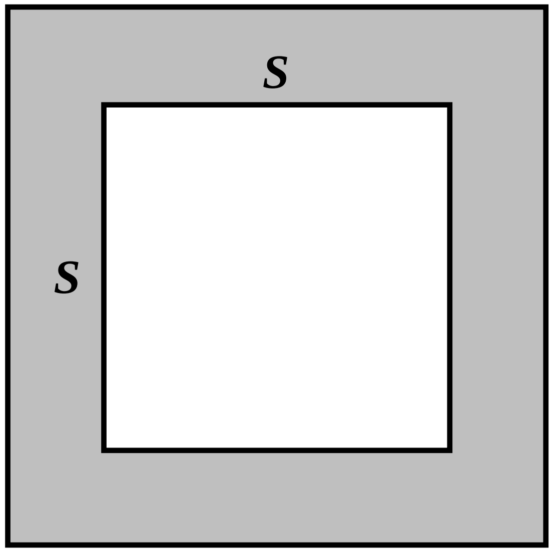
All these algebraic expressions are equal because they all represent the same quantity.

Area of the border
$= 3 + n \times 2 + 3$
$= 3 + 2 \times n + 3$
$= 2 + 4 + 2 \times n$
$= (n + 2) + 2 \times 2 + n$
$= 3 \times (n + 2) - n$

$$= 2 \times n + 6$$

TILING THE POOL

You are building a square pool that is ***S*** feet by ***S*** feet in size. You plan to border the pool with 1 foot tiles. How many tiles will you need for your pool?



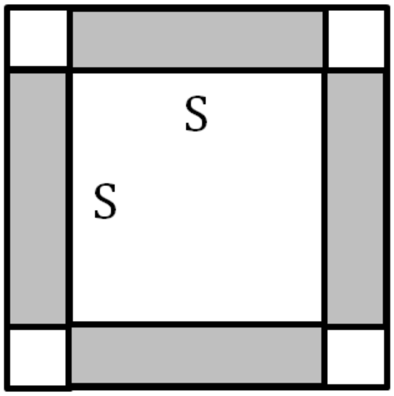
Use a the pattern below to find an expression that shows the number of tiles you'll need for any size pool.

Pool Length (ft)	Number of Tiles
1	
2	
3	
4	
5	
10	
100	
S	

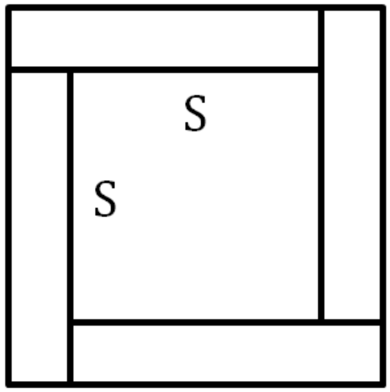
What expression represents the number of tiles you'd need to tile any size pool (S)?

Possible Student Responses:

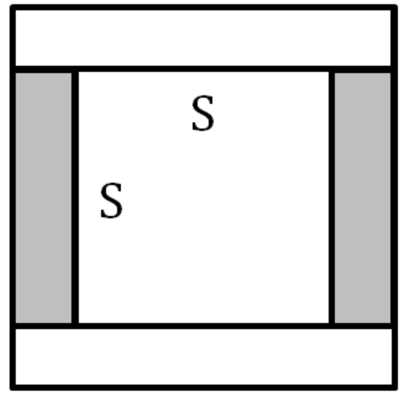
$4s + 4$



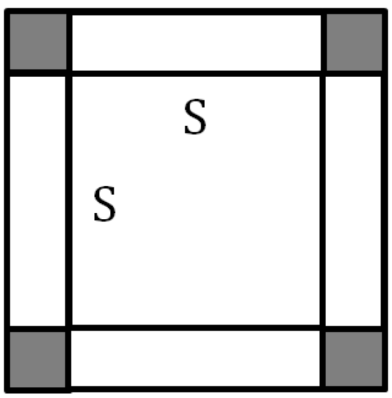
$4(s+1)$



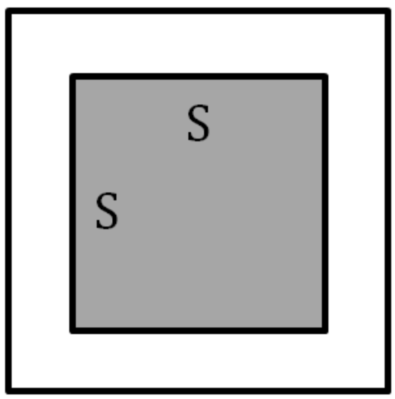
$2(s+2) + 2s$



$4(s+2) - 4$



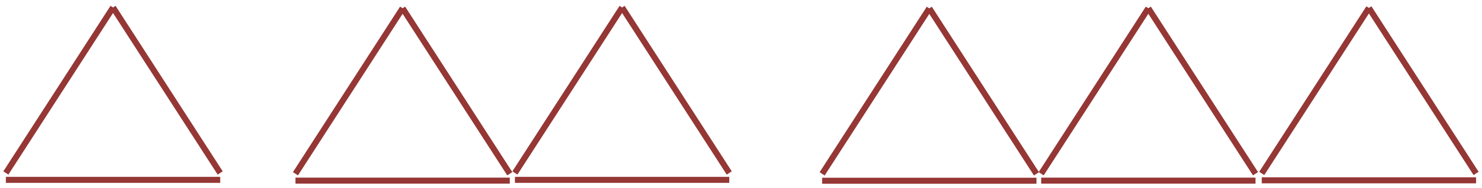
$(s+2)^2 - s^2$



Remember that students might not see these in terms of the variables, but will probably see them in terms of the sample numbers that you gave them (1, 2, 3, 4, 5, 10, 100). It is your job to help them to generalize this to variables, but more importantly, to see that all of the expressions they can come up with are equivalent to $4s+4$.

Toothpick Triangles

How many toothpicks would you need to make the n^{th} shape in this series?



Construct a table and write a rule for the pattern.

How many toothpicks would you need to make the n^{th} shape in this series?

Construct a table and write a rule for the pattern.

How are these series alike? How are they different?

Toothpick Squares

Aisha used toothpicks to construct squares. This is the pattern she created.

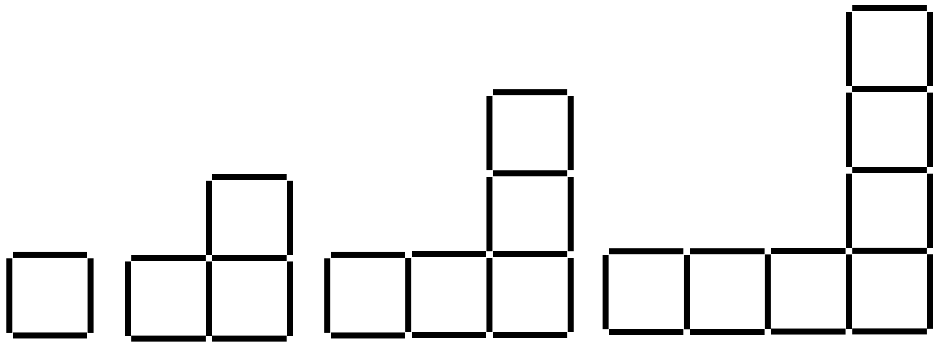


Figure 1 ***Figure 2*** ***Figure 3*** ***Figure 4*** ***Figure 5***

- Draw Figure 5.

- Complete the table below to determine the number of squares, the number of toothpicks used, and the perimeter for each figure.

Figure	**1**	**2**	**3**	**4**	**5**
# squares	1	3	5	7	
perimeter	4	8	12		
# toothpicks	4	10			

- Write an expression that can be used to determine the perimeter of the nth figure in this pattern. Let *x* represent the figure number.

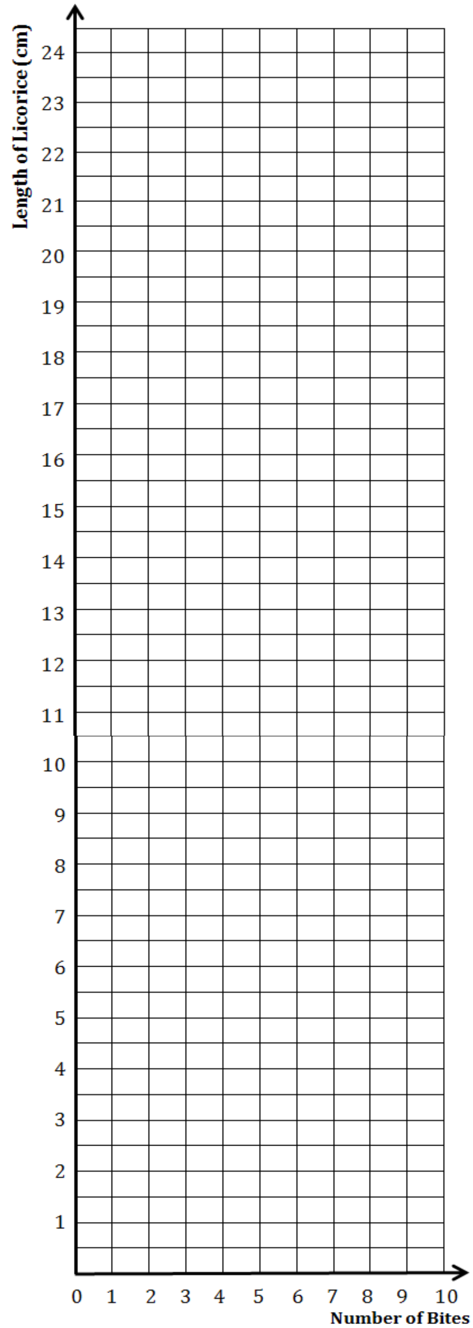
- Write an expression that can be used to determine number of toothpicks in the *n*th figure in this pattern. Let *x* represent the figure number.

- If this pattern continues, which figure will be made up of 70 toothpicks? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

Licorice Bites

How can we relate the length of a licorice whip to the number of bites we have taken?

Number of Bites	Length (cm)



Plot the points on the graph to the right,
Draw a line through the y-axis and the points.

Here are some questions you might use in a discussion about this problem:

1. What does your graph tell you about what happened to the licorice as you ate it?
2. How can you determine the size of your average bite?
3. Is your data exactly linear? How do you know?
4. Write the equation of your line.
5. What is the slope of your line?
6. What does the slope mean in the context of this problem?
7. What is the y-intercept of the line?
8. What does it mean in the context of this problem?
9. You might have students make a prediction about the number of bites it will take to finish the whip after they've only finished half of it. How could you use your equation to determine how many bites are require to get a licorice whip that is 4 cm long?
10. How might your graph look different if you took bigger bites? Smaller bites? Compare your graph to the graph of someone next to you. How do they look different? What does this mean?

Other questions:

Cool Ideas for
Teaching Linear Relationships
Using Real World Examples

Proportional Relationships (Direct Variations):

- Relationship between thickness of a single book and the height of a stack of books.
- Conversion between centimeters and inches
- Heart rate vs. Elapsed time
- Height of an object vs. its shadow length
- Exchange Rates between US Dollars and other currencies
- Length of a string of paper clips vs. Number of paper clips
- Diameter of a given circle vs. the circle's circumference
- Side lengths of equilateral triangles vs. number of toothpicks needed to make triangles
- ❤ Number of triangles vs. number of toothpicks needed to make triangles

Non-Proportional Linear Relationships:

- Number of people who can sit at a square table vs. number of tables lined up
- Conversions between Celsius and Fahrenheit temperatures
  - *For a system of equations, compare the actual formula, F = 1.8C + 32 to an estimate of the conversion, F = 2C + 30, to see where it's most accurate.*
- Perimeter vs. the number of pieces in a pattern block series
- Surface area vs. the number of one inch cubes stacked
  - *Or, use Cuisenaire rods and compare the surface area (in cm^2 with the volume (in cm^3)*
- ❤ Size of a rectangular garden vs. Number of tiles around the garden
- ❤ Side length of a square pool vs. Number of tiles around the pool

Relationships that fit lines, but aren't exactly linear:

- Number of times a nut is screwed into a carriage bolt vs. distance between nut and bolt head
- Number of cups stacked vs. height of stack
- ❤ Number of bites in licorice vs. Length of licorice left
- Number of sips taken of a drink vs. Height of drink left
- Number of knots tied in a section of rope vs. Length of rope left
  - *For a system of equations, use ropes of different thicknesses and lengths, and graph them on the same grid.*
- Number of times a ball is bounced vs. Time elapsed
- Bounce height of a ball vs. Height at which ball is dropped
- Height of a student vs. Jump height against a measuring tape on the wall

❤ *= Activities we did in our session today.*

Sources:

- *AIMS Looking at Lines*
- *Fulton, Brad & Lombard, Bill. The Pattern & Function Connection Key Curriculum Press, © 2001*
- *NCTM. Navigating Through Algebra in Grades 6-8.*

Online Resources that might be helpful:

- *www.nctm.org (and http://illuminations.nctm.org)*

Thanks for attending my session today!

You can email me at jcampbel@wcboe.org for more resources and a Microsoft Word version of this handout.