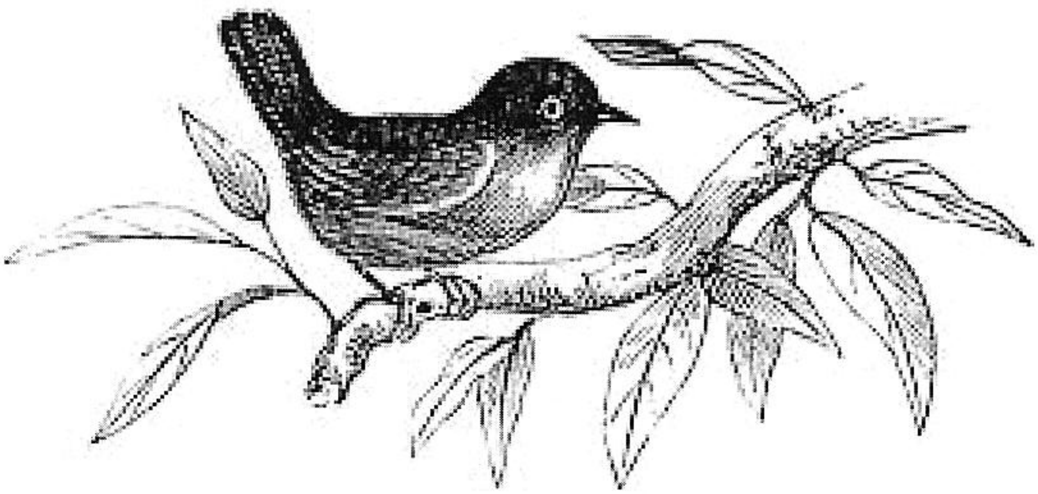
Real Number Explorations and the Pythagorean Theorem



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New Orleans, LA
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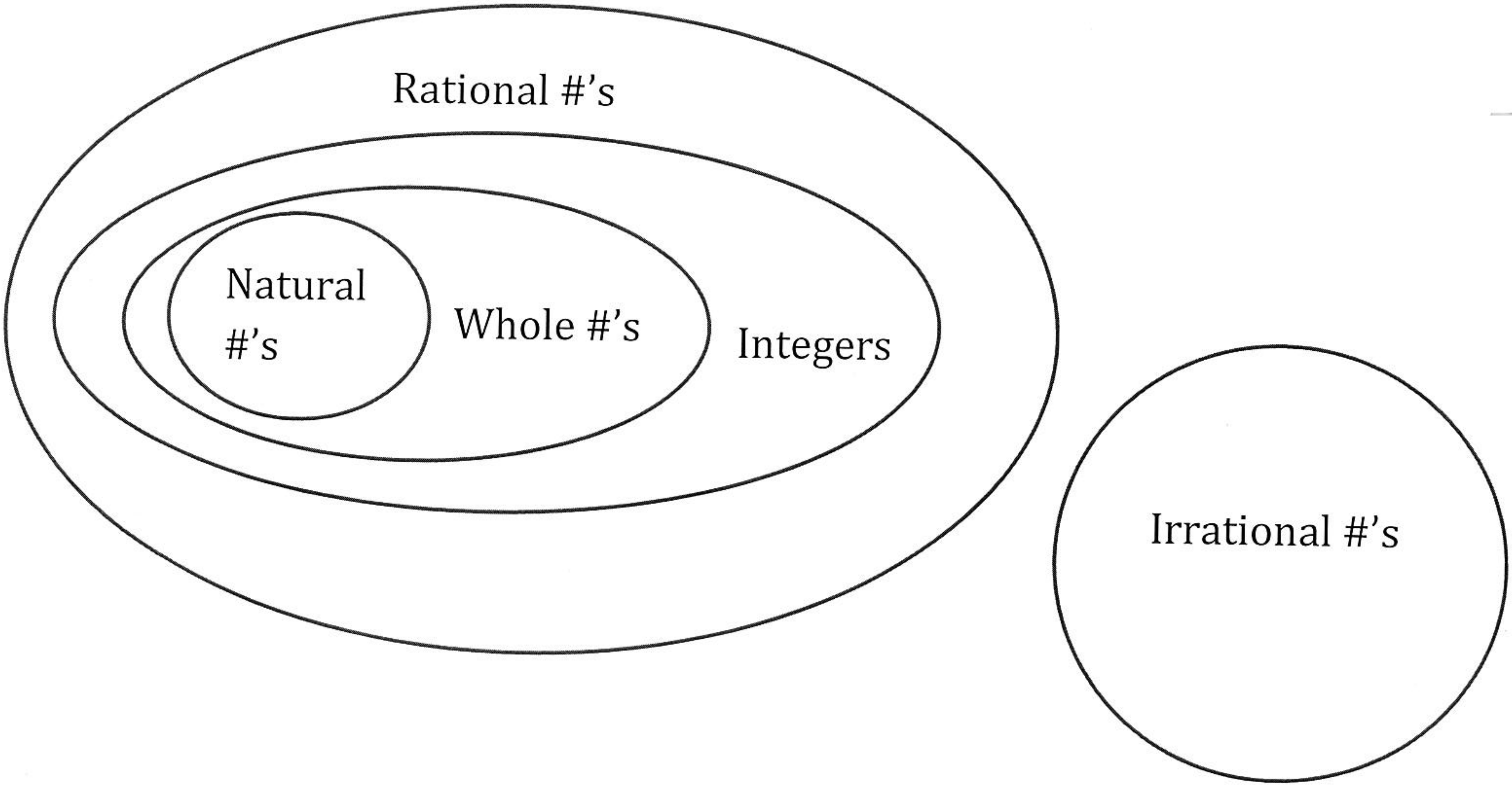
Real numbers: the set of all rational and irrational numbers



Organizing Numbers

Reporting Category Number and Number Sense
Topic Investigating the real number system
Primary SOL 8.2 The student will describe orally and in writing the relationships between the subsets of the real number system.

Materials
- Real Numbers Cards (attached)
- Scissors
- Real Number System Subset Labels (attached)
- Real Number System Venn Diagram (attached)
- Student whiteboards and markers

Vocabulary

natural numbers, whole numbers, integers (earlier grades)

rational numbers, irrational numbers, real numbers (8.2)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)

1. Distribute scissors and copies of the Real Numbers Cards. Discuss various characteristics of the numbers. Then, direct students to cut the cards apart and sort them any way they like. When they are finished sorting, have them discuss with partners how they did their sorts. Lead a class discussion about the different ways they sorted the numbers, asking them to explain the processes they used.
2. Distribute copies of the Real Number System Subset Labels. Have students cut them apart and arrange them in any order. Have students assign each number card to a subset. Point out how some numbers belong in more than one subset, and discuss the characteristics of each subset.
3. Have students work with partners to sort the number cards into rational and irrational numbers. Then, have them sort the rational numbers into integers, whole numbers, and/or natural numbers. When they have finished sorting, discuss the fact that some numbers can appear in more than one subset, e.g., 4 is a rational number, an integer, a whole number, and a natural or counting number. Explain that the attributes of one subset can be contained in whole or in part in another subset. Explain the process of sorting numbers into the *most specific* subset.
4. Distribute copies of the Real Number System Venn Diagram. Have students write the names of the subsets in the appropriate areas on the diagram and then write the numbers from the number cards in the *most specific* subsets. Finally, have students add two more numbers to each subset, explaining why the only number that can be uniquely in the "Whole Numbers" subset area is zero.

Assessment

- **Questions**
  - To which subset(s) of the real number system does the number −0.75 belong? Why?
  - Is the square root of 15 rational or irrational? How do you know?
- **Journal/Writing Prompts**
  - Identify whether a number can be both whole and irrational, and explain why or why not.
  - Identify which subset of the real number system contains the most rational numbers, and explain why.
  - Explain why rational numbers are "friendly."
  - Identify whether pi (π) is rational or irrational, and explain why.

Extensions and Connections (for all students)

- Display examples of Venn diagrams used in other areas of study to model the purpose of a Venn diagram.
- Have students utilize graphic organizer software to create their own organizers to represent the real number system.
- Prepare a large shopping bag labeled "Real Numbers." Inside the bag, place two smaller equal-size bags, one labeled "Irrational Numbers" and the other labeled "Rational Numbers." Inside the "Rational Numbers" bag, place a smaller bag labeled "Integers." In the "Integers" bag, place a yet smaller bag labeled "Whole Numbers," and in the "Whole Numbers" bag, place the smallest bag labeled "Natural Numbers." Display the "Real Numbers" bag, and pull the smaller bags out one at a time to show the differences in sizes and how they relate to each other. Discuss the types of numbers that would be in each bag. Hand student number cards to put into the *most specific* bags, and then demonstrate putting the set of bags together again.

Strategies for Differentiation

- Use different colors of paper to help students distinguish the different subsets of numbers.
- Use the Venn diagram with only the vocabulary terms, and as a class, create examples to include. Then, have students create their own Venn diagrams and use the real number cards provided to glue into place.
- Place location signs around the room labeled "Natural Numbers, Whole Numbers, Integers, Rational Numbers, Irrational Numbers, and Real Numbers." Give each student a number card, and have him/her match the number to as many locations as possible by going to the location(s) and writing the number on the sign(s). Confirm in class discussion the place or places each number is placed. Repeat this activity throughout the year.

Real Numbers Cards

Copy cards on cardstock, and cut out.

-6	0.5	$0.\overline{4}$	$0.349...$
$\sqrt{25}$	2	0	$\frac{3}{5}$
π	$\sqrt{13}$	$\frac{2}{3}$	$-\frac{10}{2}$
5	2.25	$-\sqrt{25}$	$\frac{9}{3}$

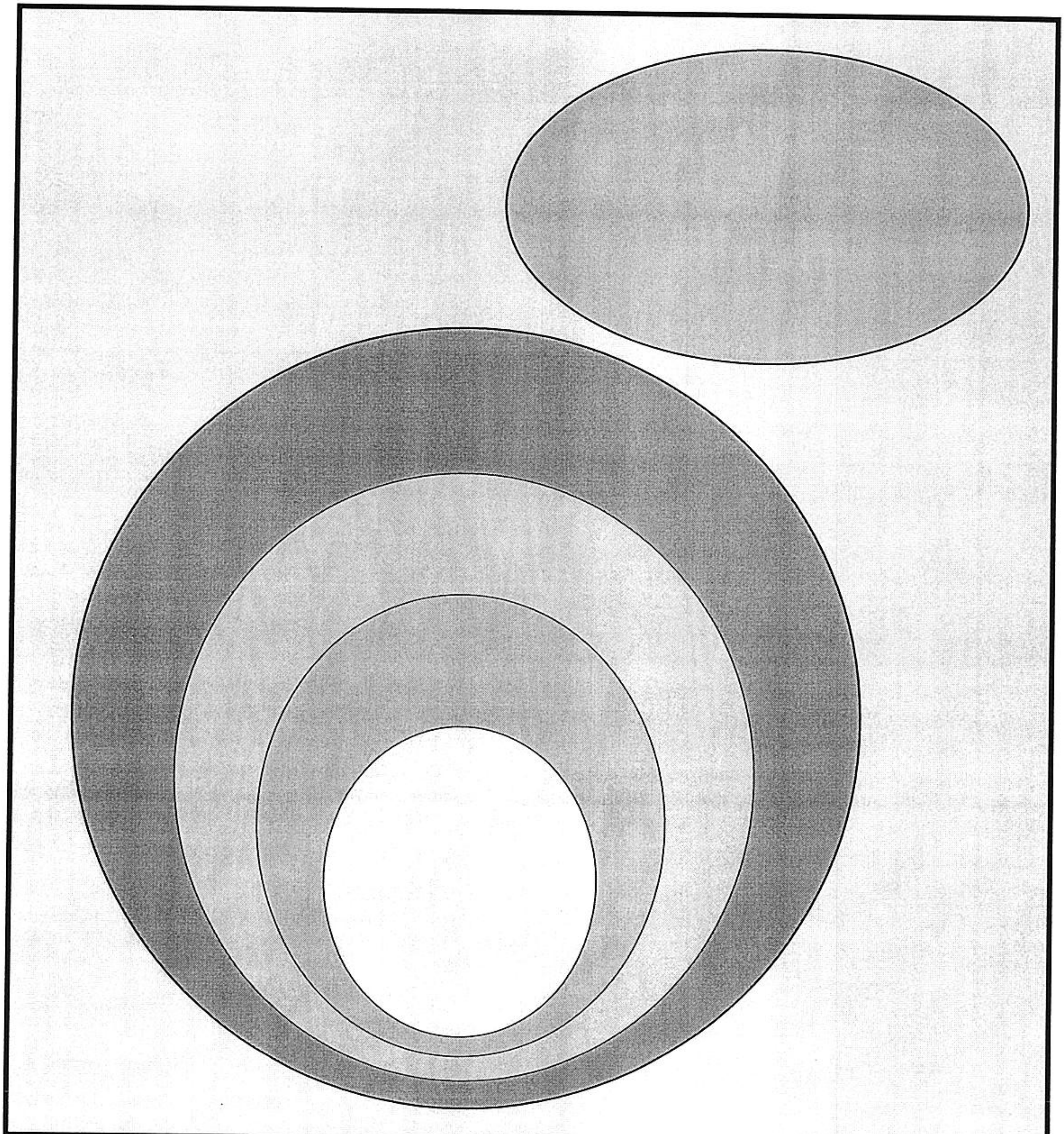
Real Number System Subset Labels

Copy labels on cardstock, and cut out.

Real Numbers
Irrational Numbers
Rational Numbers
Integers
Whole Numbers
Natural Numbers

Real Number System Venn Diagram

Name ______________________________ **Date** ____________________

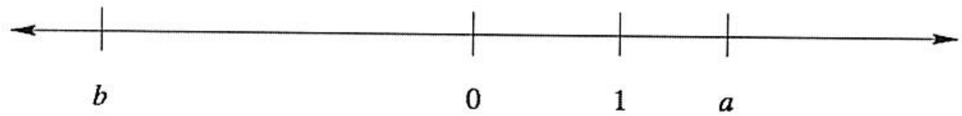


Illustrative Mathematics

7.NS Operations on the number line

Alignment 1: 7.NS.A.1
Not yet tagged

A number line is shown below. The numbers 0 and 1 are marked on the line, as are two other numbers a and b.



Which of the following numbers is negative? Choose all that apply. Explain your reasoning.

a. $a - 1$

b. $a - 2$

c. $-b$

d. $a + b$

e. $a - b$

f. $ab + 1$

Commentary

There is a subtle distinction in the Common Core State Standards between a *fraction* and a *rational number*. Fractions are always positive, and when thinking of the symbol $\frac{a}{b}$ as a fraction, it is possible to interpret it as a equal-sized pieces where b pieces make one whole. The rational numbers are the set of fractions taken together with their opposites: understanding rational numbers requires understanding both fractions and signed numbers. The standard 7.NS.1 signals a significant shift from working exclusively with positive numbers to working with signed numbers. The focus of this task is on the nature of signed numbers rather than the "part-whole" interpretation of fractions.

The purpose of this task is to help solidify students' understanding of signed numbers as points on a number line and to understand the geometric interpretation of adding and subtracting signed numbers. This task (like all tasks featured on the Illustrative Mathematics website) assumes that the number line is drawn to scale.

Solution: Visual inspection

a. a is greater than 1, so $a - 1$ is positive.

b. The distance between a and 1 appears to be less than the distance between 1 and 0, so it looks like a is less than 2. Thus $a - 2$ is negative.

c. b is negative, so $-b$ is positive.

d. The distance between a and 0 appears to be less than the distance between b and 0, so it looks like $|a|$ is less than $|b|$. Since b is negative and a is positive, $a + b$ is negative.

e. $a - b = a + -b$. Since b is negative, $-b$ is positive. a is also positive. Thus, $a - b$ is positive.

f. Since $|a|$ and $|b|$ are both greater than 1, $|ab|$ is also greater than 1 (this builds on the intuition students gained in fifth grade as in 5.NF.5). ab is negative since a is positive and b is negative. Thus, $ab + 1$ is negative.

Division

When you calculate 100 ÷ 6 using a calculator, the result is 16.6666667.

This result can be used to give a **sensible** answer to all the following questions except one.

1. Write down the sensible answers and find the question that cannot be answered using this result.

   a. How much does each person pay when 6 people share the cost of a meal costing $100?

   ____

   b. 100 children each need a pencil. Pencils are sold in packs of 6. How many packs are needed?

   ____

   c. What is the cost per gram of shampoo costing $6 for 100 grams?

   ____

   d. How many CDs costing $6 each can be bought for $100?

   ____

   e. What is the average distance per day, to the nearest mile, traveled by a hiker on the Appalachian Trail, who covers 100 miles in 6 days?

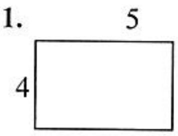
   ____

2. Write another question, together with its sensible answer, that can be answered using 100 ÷ 6.

Distributive Property Using Area

NAME ______________________________

Write the expression that represents the area of each rectangle.

1.


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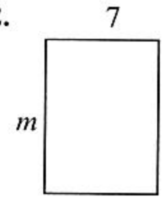

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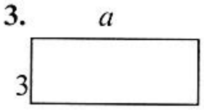

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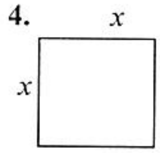
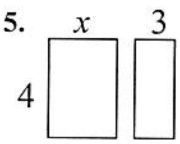


Find the area of each box in the pair.

5.


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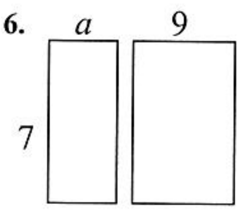

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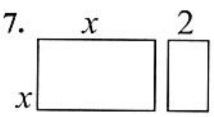


Write the expression that represents the total length of each segment.

8.


____

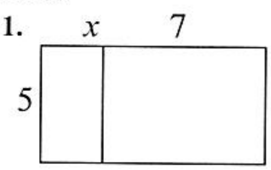
9.


____

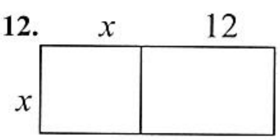
10.


____

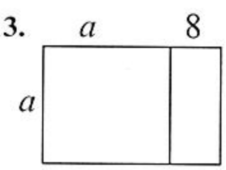
Write the area of each rectangle as the product of ***length* × *width*** and also as a sum of the areas of each box.

11.


AREA AS PRODUCT	AREA AS SUM
5(X+7)	5X+35

12.


AREA AS PRODUCT	AREA AS SUM

13.


AREA AS PRODUCT	AREA AS SUM

This process of writing these products as a sum uses the **distributive property**.

Use the distributive property to re-write each expression as a sum. You may want to draw a rectangle on a separate page to follow the technique above.

14. $4(x+7)=$______________

15. $7(x-3)=$______________

16. $-2(x+4)=$______________

17. $x(x+9)=$______________

18. $a(a-1)=$______________

19. $3m(m+2)=$______________

20. $-4(a-4)=$______________

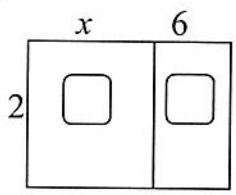
21. $a(a-12)=$______________

Factoring a Common Factor Using Area

NAME ____________________

Fill in the missing information for each: dimensions, area as product, and area as sum

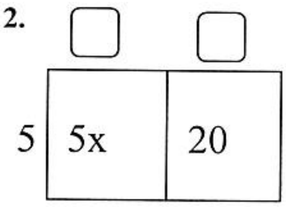
1.



2(x+6)

2x+12

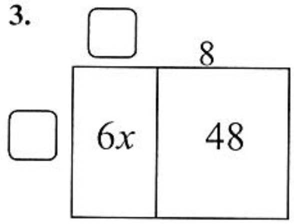
2.



______

______

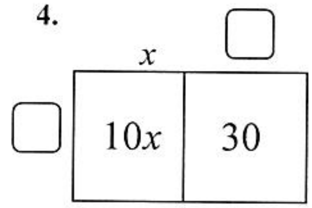
3.



______

______

4.

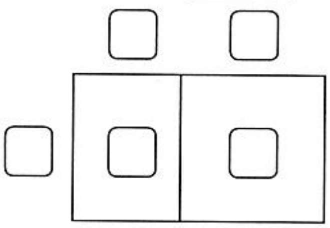


______

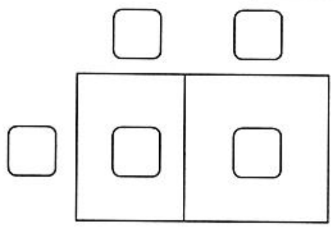
______

Fill in the missing dimensions from the expression given.

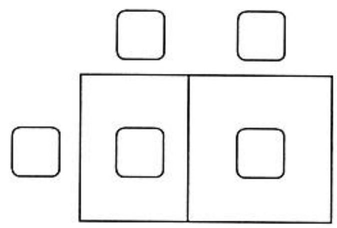
5. $5x + 35 = 5(\_\_\_\_\_)$



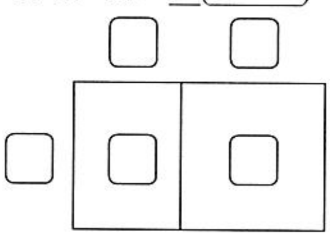
6. $2x + 12 = 2(\_\_\_\_\_)$



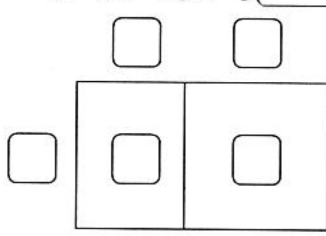
7. $3x - 21 = \_\_(\_\_\_\_\_)$



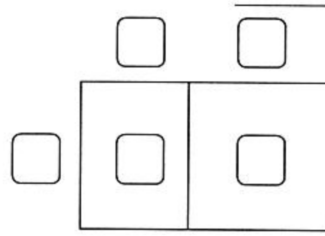
8. $7x - 21 = \_\_(\_\_\_\_\_)$



9. $-3x - 15 = -3(\_\_\_\_\_)$



10. $-5x + 45 =$ ______



This process of writing a sum or difference as the product of factors is called **factoring**.

Factor these:

11. $4x - 16 =$ ____________

12. $-7x - 35 =$ ____________

13. $9x - 81 =$ ____________

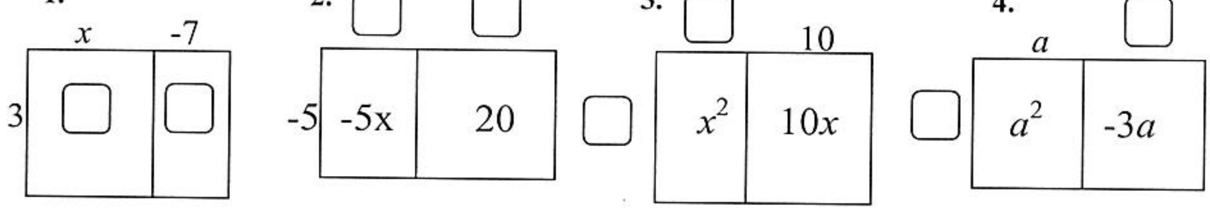
14. $4x + 18 =$ ____________

More Factor Using Area

NAME ______________________

Fill in the missing information for each: dimensions, area as product, and area as sum

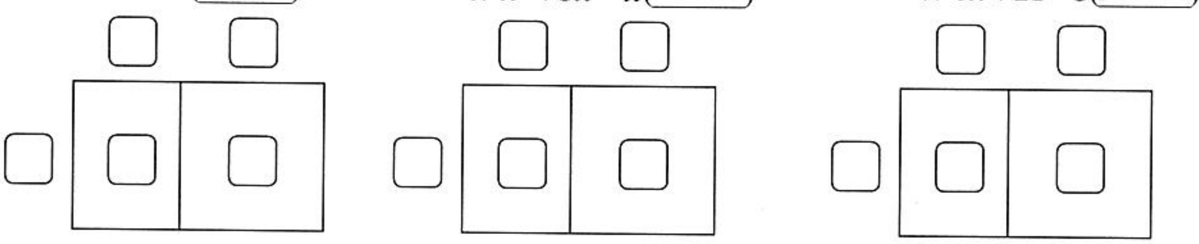
1. 2. 3. 4.



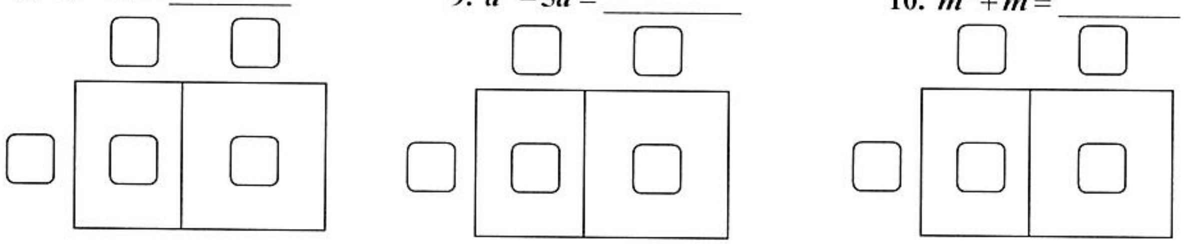
1. 3(x-7) ; 3x-21
2. ______ ; ______
3. ______ ; ______
4. ______ ; ______

Fill in the missing dimensions from the expression given.

5. $x^2+3x=x(\_\_\_\_\_\_)$
6. $x^2+5x=x(\_\_\_\_\_\_)$
7. $6x+21=3(\_\_\_\_\_\_)$



8. $4x-10=$ ________
9. $a^2-5a=$ ________
10. $m^2+m=$ ________



> When factoring expression, you have to consider that the common factor may be a variable instead of (or addition to) a number.

Factor these:

11. $t^2-6t=$ ____________
12. $10x-35=$ ____________
13. $6x+21=$ ____________
14. $9a^2-15a=$ ____________

Greatest Common Factor

NAME ___________________________

Complete the factorization for each:

1.

a. $3x^2 + 15x = 3(\_\_\_\_\_\_)$ b. $3x^2 + 15x = x(\_\_\_\_\_\_)$ c. $3x^2 + 15x = 3x(\_\_\_\_\_\_)$

2. Which of the above factorizations for $3x^2 + 15x$ do you think is the 'best'? Why?

____

3.

a. $4a^2 - 12a = 4(\_\_\_\_\_\_)$ b. $4a^2 - 12a = a(\_\_\_\_\_\_)$ c. $4a^2 - 12a = 4a(\_\_\_\_\_\_)$

4. Which of the above factorizations for $4a^2 - 12a$ do you think is the best? Why?

____

> In each case the third factorization is the *best* because you've factored out the **greatest common factor (GCF)**.

Factor each expression below. Be sure to find and use the *greatest* common factor.

5. $5x^2 + 15x =$ __________

6. $3x^2 + 12x =$ __________

7. $6x - 4 =$ __________

8. $7x^2 - 9x =$ __________

9. $5x^2 + 5x =$ __________

10. $9a^2 - 12a =$ __________

MULTI OPERATION BLACKOUT

LEVEL: Grade 6 - 9

SKILLS: Order of operations, exponents, square roots, problem solving

PLAYERS: 2 vs. 2

EQUIPMENT: Two ten-sided (0-9) dice and one twelve-sided (1-12) die, two hundred boards, one per team (see reproducibles), bingo chips or other markers

GETTING STARTED: The goal of the game is to be the first team to cover up every number on their hundreds chart. Team One rolls the three dice. Players figure out all of the combinations they can make with the three numbers using all operations.

EXAMPLE: Roll 6, 9, 2

NOTE: Both teams work with these numbers for the three minutes.

$6 + 9 + 2 = 17$
$(9 - 6) + 2 = 5$
$(9 \times 6) + 2 = 56$
$(6 \times 2) - 9 = 3$
$6^2 + 9 = 45$
$9^2 - 6 = 75$
$\sqrt{9} \times 2 \div 6 = 1$
$\sqrt{9} + 2 + 6 = 11$

Player may cover up numbers 17, 5, 56, 3, 1, 11, 45, 75, etc.

etc.

For every answer calculated, teams cover the corresponding number on their hundred board. Teams are allowed a maximum of three minutes per roll. Teams alternate turns until one team successfully covers up all of their numbers.

NOTE: Players could keep a written record of their rolls and multi operation sequences.

VARIATION I: Only one team works covering up the numbers while the other team observes.

At the end of a team's time limit the opposing team can "capture" any number combinations that their opponents have missed. In the above example, Team One had missed 87 ($9^2 + 6 = 87$). Team Two can cover 87 on their gameboard as a capture).

VARIATION II: Use two twenty-sided (1-20) dice of one colour and one ten-sided (0-9) die of a different colour. Choose one set of dice as positive and the other as negative.

Illustrative Mathematics

8.NS Converting Decimal Representations of Rational Numbers to Fraction Representations

Alignment 1: 8.NS.A.1
Not yet tagged

Represent each of the following rational numbers in fraction form.

a. $0.33\overline{3}$

b. $0.3\overline{17}$

c. $2.1\overline{6}$

Commentary

Standard 8.NS.1 requires students to "convert a decimal expansion which repeats eventually into a rational number." Despite this choice of wording, the numbers in this task are rational numbers regardless of choice of representation. For example, $0.33\overline{3}$ and $\frac{1}{3}$ are two different ways of representing the same number.

So what is a rational number? Sometimes people define a rational number to be a ratio of integers, but to be consistent with the CCSSM, we would need to say a rational number is any number that is the *value* of a ratio of two integers. Sometimes people define a rational number based on how it can be represented; here is a typical definition: *A rational number is any number that can be represented as $\frac{a}{b}$ where a and b are integers and $b \neq 0$.* It is interesting to compare this with the definition of a rational number given in the Glossary of the CCSSM (as well as the more nuanced meaning developed in the standards themselves starting in grade 3 and beyond).

A more constructive definition for a rational number that does not depend on the way we represent it is:

> *A number is rational if it is a quotient $a \div b$ of two integers a and b where $b \neq 0$.*

or, equivalently,

> *A rational number is a number that satisfies an equation of the form $a = bx$, where a and b are integers and $b \neq 0$.*

So $0.33\overline{3}$ is a rational number because it is the result we get when we divide 1 by 3, or equivalently, because it is a solution to $1 = 3x$. However, numbers like π and $\sqrt{2}$ are not rational because neither of them satisfies an equation of the form $a = bx$ where a and b are integers. This is actually tricky to show and is an exercise left to high school or college.

Solution: Solution

The solution for all the parts of this take advantage of the repeating structure of the decimal expansions. Namely, by multiplying by a suitable power of 10 (namely, 10^r where r is the length of the repeating segment in the decimal expansion) and subtracting the original number, we can get a multiple of x with a finite decimal expansion.

a. Let $x = 0.33\overline{3}$ Then

$$10x = 3.3\overline{3} = 3 + 0.33\overline{3} = 3 + x$$

Subtracting x from both sides gives $9x = 3$, so

$$0.33\overline{3} = x = \frac{3}{9} = \frac{1}{3}.$$

b. Let $x = 0.31717\ldots$

Then

$$\begin{aligned} 100x &= 31.717171\ldots \\ x &= 0.317171\ldots \end{aligned}$$

Now subtracting the two equations gives $99x = 31.4$, so

$$0.3\overline{17} = x = \frac{31.4}{99} = \frac{314}{990}.$$

c. Let $x = 2.166\ldots$ Then

$$\begin{aligned} 10x &= 21.6666\ldots \\ x &= 2.1666\ldots \end{aligned}$$

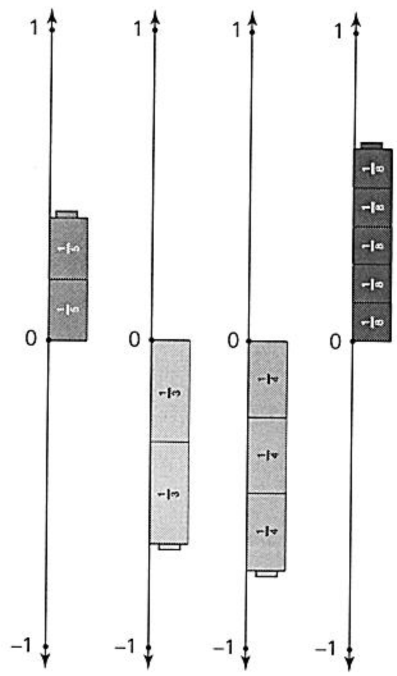
Now subtracting the two equations gives $9x = 19.5$, so

$$2.1\overline{6} = x = \frac{19.5}{9} = \frac{195}{90}.$$

Name ____________________

Use Fraction Towers to model each rational number on a number line. Write each number. Then write the numbers in order from least to greatest.

1.



Numbers:

____

Ordered from least to greatest:

____

Using Fraction Towers, model each rational number. Sketch the models on number lines. Write the numbers in order from least to greatest.

2. $\frac{3}{8}, -\frac{1}{4}, \frac{7}{12}, -\frac{2}{5}$

Ordered from least to greatest: ____________________

Use < or > to compare the numbers.

3. $\frac{7}{8}$ ◯ $\frac{3}{4}$

4. $\frac{7}{10}$ ◯ $\frac{9}{12}$

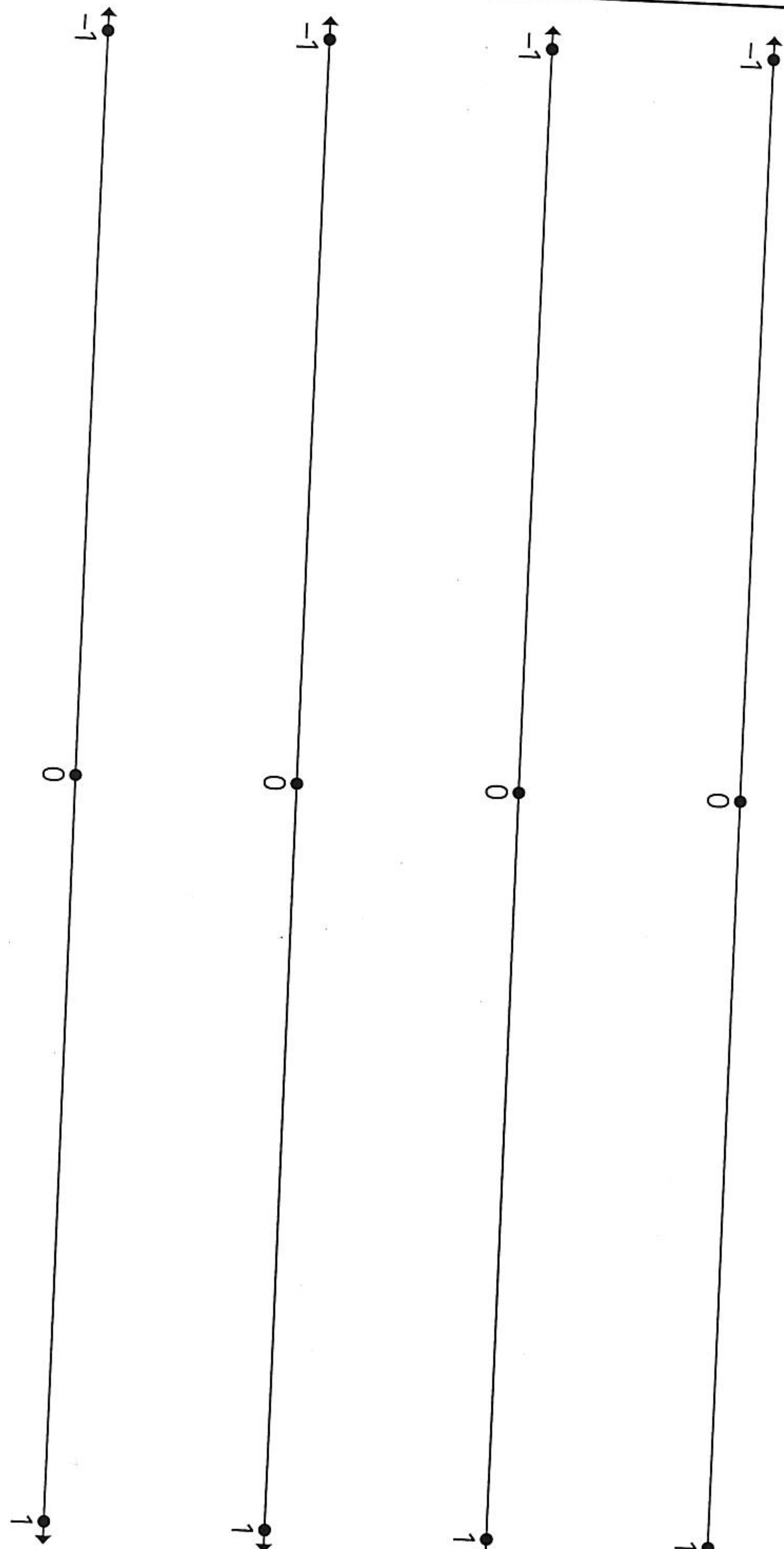
5. $\frac{1}{3}$ ◯ $\frac{1}{4}$

6. $\frac{2}{5}$ ◯ $\frac{1}{2}$

7. $\frac{1}{6}$ ◯ $\frac{1}{4}$

8. $\frac{3}{12}$ ◯ $\frac{2}{6}$

Name ______________________________



BLM 4

Double Fraction Tower® Number Lines

SIMPLY RADICAL

(submitted by Cheri Eck)

LEVEL: Grade 8 - 11

SKILLS: Simplifying radicals, factoring

PLAYERS: 2 of equal skill level

EQUIPMENT: Two twenty-sided (1-20) dice, calculators, paper, pencil

GETTING STARTED: Each player rolls a twenty-sided die. At the same time players multiply the two numbers and find the square root in simplified mixed form. The first player to verbalize the correct answer out loud earns the point.

EXAMPLE: Numbers rolled: 15 and 5

$\sqrt{75} = \sqrt{25 \times 3} \quad \sqrt{25} = 5\sqrt{3}$

Numbers rolled: 18 and 12

18 x 12 = 216

$\sqrt{216} = \sqrt{9 \times 2 \times 4 \times 3} = \sqrt{9 \times 4}\,\sqrt{2 \times 3} =$

$\sqrt{36}\,\sqrt{6} = 6\sqrt{6}$

Play continues for a set period of time. The player with the most points is the winner.

EXPONENT WAR

LEVEL: Grade 7 - 10

SKILLS: Multiplication (exponents), with positive and negative integers

PLAYERS: 2

EQUIPMENT: Cards Ace - 5 (Ace = 1) or Ace - 9 (Ace = 1) for advanced players

GETTING STARTED: Players divide the cards evenly between themselves. Players turn over two cards each. Black cards are positive and red cards are negative. The first card turned up is the base card and the second card is the exponent. Example: Player One turns up a 3 and then a 4. The total is 3 x 3 x 3 x 3 = 81. The player with the highest total wins all four cards. Play continues until one player earns all of the cards. In the event of a tie (i.e. both players have the same totals), each player deals three cards face down. Two more cards are turned face up in the same manner as above and the higher total wins all of the cards.

EXAMPLE:

Player One		**Player Two**
red 2, black 4		black 4, black 2
$(-2)^4 = (-2) \times (-2) \times (-2) \times (-2)$		$4^2 = 4 \times 4$
= 16		= 16
–	{three cards	–
–	face down}	–
–		–
red 3, black 5		red 1, black 4
$= (-3) \times (-3) \times (-3) \times (-3) \times (-3)$		$= (-1) \times (-1) \times (-1) \times (-1)$
= -243		= 1

Player Two would collect all of the cards.

√RADICAL RULES: A power is a number with a base and an exponent (e.g. 3^4 - 3 is the base and 4 is the exponent). When you use an exponent, the base is a repeated factor and the exponent determines the number of times the base is used as a factor.

EXAMPLE: $3^4 = 3 \times 3 \times 3 \times 3$

Reasoning about the Pythagorean Theorem

Goals

- Become familiar with the Pythagorean theorem
- Use an area model to discover the Pythagorean theorem

Materials and Equipment

- Two or more sheets of grid paper (available on the CD-ROM that accompanies this book) and one ruler for each student
- A copy of the blackline master "Reasoning about the Pythagorean Relationship" for each student

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Activity

The students draw two right triangles, one with sides measuring three, four, and five units, and the other with sides measuring five, twelve, and thirteen units. On each triangle, they draw squares, using the sides of the triangle as sides of the squares. For each triangle, they find the area of the squares and enter their results in a table, along with the measures of the sides of each triangle.

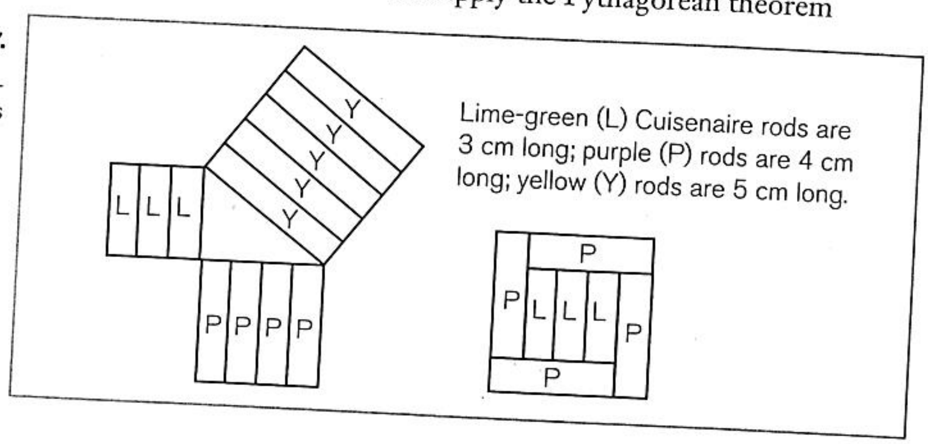
Discussion

This exploration should be continued with two or three more triangles. You might model another example for the students so you can address precision in measurement (many teachers prefer to use metric units). In cases where nonintegral measures occur, rounding may be necessary to make the Pythagorean relationship clear. For example, a triangle with legs measuring 2 centimeters and 3 centimeters has a hypotenuse of approximately 3.6 centimeters. The students may continue to draw the triangles on graph paper, but they will need to use a ruler to measure the sides and compute the areas of the squares. Some students may focus more on completing measurements than on the relationship among the squares of the sides. The students may need to be prompted specifically to look for patterns in the data that indicate relationships among the parts of the figures.

A seventh-grade teacher reported using Cuisenaire rods to demonstrate the Pythagorean relationship, as shown in figure 1.7.

Teachers recount that although many students may be able to manipulate the formula $c^2 = a^2 + b^2$ and apply the Pythagorean theorem

Fig. **1.7.**
The Pythagorean relationship demonstrated with Cuisenaire rods



appropriately to situations, they do not have a deep understanding of the relationship. Therefore, multiple approaches should be used to allow students to explore this important relationship in depth. The students will encounter this theorem, which has a rich history, again and again. Those who are able to develop a sound rationale for why the formula works are demonstrating reasoning that will continue to be developed through more-formal studies of geometry. Students should be encouraged to engage in oral and written communication about their reasoning. The following questions can stimulate some fruitful thinking:

- What conclusion can you make on the basis of your work with right triangles?
- How do the various tasks you have undertaken support the Pythagorean theorem?

What does this theorem say about the possibility of a right triangle with sides measuring 3, 5, and 7 units? Support your conclusion with an illustration and an argument.

Exploring Relationships with Polyhedra

The CD-ROM that accompanies this book contains nets of polyhedra, including the five regular polyhedra, which can be used with this and other geometric explorations.

The next task continues students' investigations of the properties and characteristics of shapes by focusing on three-dimensional shapes. It requires students to form conjectures about relationships on the basis of data and connects algebra with geometry.

Have the students use nets to construct several polyhedra, including a cube, a pentagonal pyramid, and a square pyramid. Ask the students to count the vertices, faces, and edges of each polyhedron. A table such as the one in figure 1.8 will help the students keep track of their observations.

Name of Polyhedron	Number of Faces (F)	Number of Vertices (V)	Number of Edges (E)
Tetrahedron	4	4	6
Hexahedron (cube)	6	8	12
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30
Pentagonal prism	7	10	15
Triangular prism	5	6	9
Pentagonal pyramid	6	6	10
Square pyramid	5	5	8

Fig. **1.8.**

An example of a chart in which students have recorded their observations about the properties of polyhedra

Ask the students to study their data carefully in order to detect any patterns in the numbers of faces, vertices, and edges. They should begin to uncover Euler's formula, which can be expressed in varying forms. The formula Vertices (V) + Faces (F) = Edges (E) + 2 is perhaps the most common. Sometimes, the relationship is written as $V + F - E = 2$. The formula holds for both regular and irregular convex polyhedra. Allow time for the students to explore the relationship among the faces, vertices, and edges and express this relationship in words and in more-formal expressions such as those above. Your students might enjoy using

Allowing students to develop conceptual meanings for definitions of various polyhedra assists in developing a broader and more flexible understanding of important geometric terms. See Jane Keiser (2000), "The Role of Definition," included on the CD-ROM.

Counting Parts of Solids on NCTM's Illuminations Web site to explore these relationships (address: www.illuminations.nctm.org; click on i-Math Investigations, and scroll down to the section for grades 3–5).

You could also demonstrate this relationship using a potato or a piece of Styrofoam. Randomly cut flat slices of potato until a flat-faced polyhedron emerges. (You could also cut potatoes ahead of time at home and then have the students use them in small groups.) Count the faces, edges, and vertices. Have the students keep a record of the information they collect. Point out that the totals for an irregular polyhedron fit Euler's formula. The students could also build various irregular polyhedra themselves and verify that the formula holds. Another extension of the task is to have the students truncate (cut off) a corner of one of their polyhedra and discover that the formula still holds.

Conclusion

The activities, tasks, and examples in this chapter were selected to illustrate ways to provide students with important experiences that extend their ability to analyze the characteristics and properties of shapes and to use geometric knowledge to make conjectures and reason effectively. The activities address important goals from *Principles and Standards for School Mathematics* (NCTM 2000): understanding relationships among two- and three-dimensional shapes, understanding relationships among similar figures, and developing inductive and deductive arguments. The instructional goal of these activities and tasks should be to encourage students to use sound reasoning based on the characteristics and properties of shapes as they make conjectures and construct arguments.

Reasoning about the Pythagorean Relationship

Name ___________________________

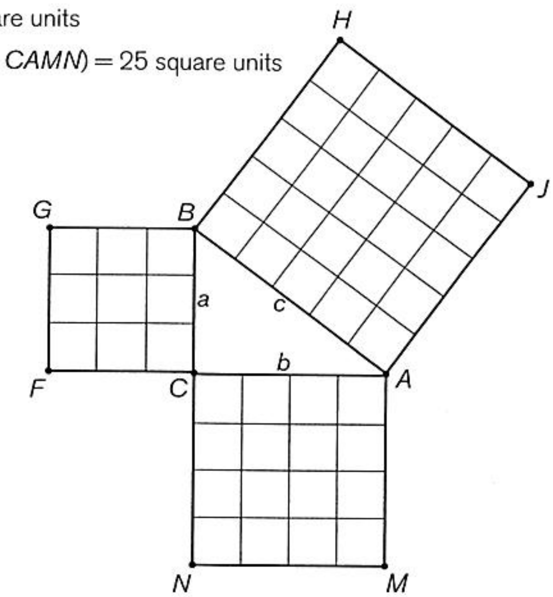
1. Using the distance between adjacent points of intersection on grid paper as units, draw a right triangle with sides measuring 3, 4, and 5 units. Let the legs be 3 units and 4 units and lie on grid lines. Place a second sheet of grid paper along the hypotenuse to verify that it has a length of 5 units.

2. Draw squares, using the sides of the triangle as sides of the squares, as illustrated at the right. Find the area of each square, and enter the information in the table below.

3. Similarly, draw a right triangle with sides measuring 5, 12, and 13 units. Draw squares on the sides of the new triangle. Find the area of each square, and enter the information in the table.

Area *BCFG* = 9 square units
Area *CAMN* = 16 square units
Area *ABHJ* = 25 square units
(Area *BCFG*) + (Area *CAMN*) = 25 square units

$a = 3$ units
$b = 4$ units
$c = 5$ units



Triangle Number	Length of Side *a* (Units)	Length of Side *b* (Units)	Length of Side *c* (Units)	Area of Square with Length *a* (Square Units)	Area of Square with Length *b* (Square Units)	Area of Square with Length *c* (Square Units)
1	3	4	5	9	16	25
2	5	12	13			
3						
4						

4. Draw other triangles, as instructed by your teacher, and record the appropriate data in the table. Look for patterns in the data. Describe any relationships among the parts of the figure.

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