Exploring Innovative Techniques for Teaching Arithmetic Using the (CRA) Approach

Joseph Sencibaugh, Ph.D, Webster University, St. Louis, MO
Dan Sinclair, Mastery Educational Services, Fallbrook, CA
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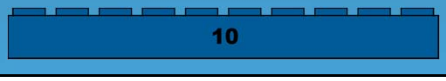


Session Objectives and Overview

- Learn innovative techniques using CRA
- See how to implement techniques
- Strategies for affirming students' knowledge

- What is CRA?
- What Works Clearinghouse Recommendations
- Hands-on Demonstration & Strategies
- Conclusion

Early Predictors of High School Math

A Carnegie-Mellon lead researcher , Robert Siegler, recently published a study:

- Mastery of fractions and early division
- Focus on whole number division and fractions
- Teachers' "lack of a firm conceptual understanding" of the concepts

"Any effort to improve the children's understanding without improving the teacher's understanding is doomed to fail,"
- Robert Siegler

http://www.post-gazette.com/stories/news/education/formula-written-for-math-success-640962

Concrete-Representational-Abstract Approach

- **Concrete** (Doing Stage). In the concrete stage, the teacher begins instruction by modeling each mathematical concept with concrete materials (e.g., red and yellow chips, cubes, base-ten blocks, pattern blocks, fraction bars, and geometric figures).
- **Representational** (Seeing Stage). In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting.
- **Abstract** (Symbolic Stage). At this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, –, x, /) to indicate addition, subtraction, multiplication, or division.

Guidelines for the CRA Technique

- Before abstract experiences, instruction must proceed from concrete (manipulative) experiences to representational experiences.
- The main objective of manipulative aids is to help students understand and develop mental images of mathematical processes.
- The activity must accurately represent the actual process, there should be a correlation between manipulative activities and paper-pencil activities.
- More than one manipulative object should be used in teaching a concept.
- The manipulative experience must involve the moving of objects.

What Works Clearinghouse

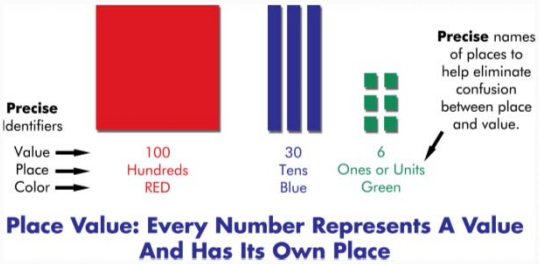
- A Recent Practice Guide Recommended:
  - Intervention materials should include opportunities for students to work with visual representations of mathematical ideas.
  - Interventionists should be proficient in the use of visual representations of mathematical ideas.

Source: http://ies.ed.gov/ncee/wwc/pdf/practice_guides/rti_math_pg_042109.pdf

Innovative Techniques

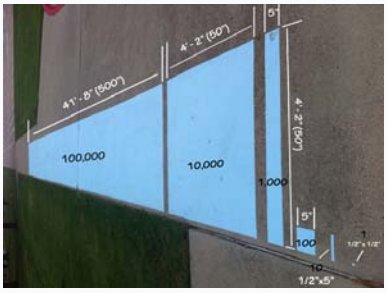
- Moving Beyond Counting Strategies
- Developing CRA Techniques for All Operations
- Engaging Multiple Senses
- Engaging Manipulatives that Demonstrate Each Step
- Developing Teacher Proficiency

Place Value



Place Value: Every Number Represents A Value And Has Its Own Place

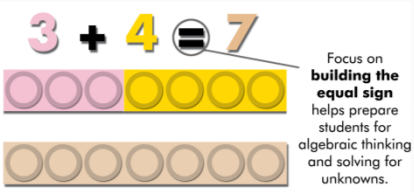
Place Value on the Playground



Addition Strategies

Specific colors for each value helps with subitizing and processing issues.

Specific values help teach addition strategies, rather than counting strategies.



Focus on **building the equal sign** helps prepare students for algebraic thinking and solving for unknowns.

Multiplication Strategies

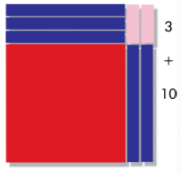
Using Language & Place Value Notation

Clear Modeling

$$\begin{array}{r} 12 \\ \times 13 \\ \hline 36 \\ 120 \\ \hline 156 \end{array}$$

Expand

$$\begin{array}{r} 10 + 2 \\ 10 + 3 \\ \hline 30 + 6 \\ 100 + 20 \\ \hline 100 + 50 + 6 \end{array}$$



Explicit Instruction (**Clear Modeling** **Precise Verbal & Written Expression** **Comprehensive Manipulatives**) = *Mastery*

Division Strategies

Precise Verbal & Written Expression

How many 13's can we count out of 156?

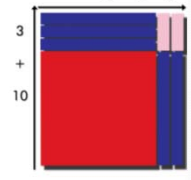
13 x 10 = 130

13 x 2 = 26

This explains why the 6 comes down and eliminates the idea of there being a 15 in 156.

$$\begin{array}{r} 2 \\ 10 \\ \hline 13\overline{)156} \\ -130 \\ \hline 26 \\ -26 \\ \hline 0 \end{array}$$

Clear Modeling



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Algebra Strategies

The FOIL method can be confusing

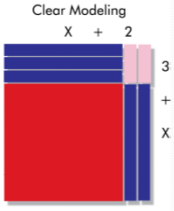
$(X + 2)(X + 3)$

X^2+5X+6

Using Language & Place Value Notation

$(X + 2)$
$(X + 3)$
$3X + 6$
$X^2 + 2X$
$X^2 + 5X + 6$

Clear Modeling



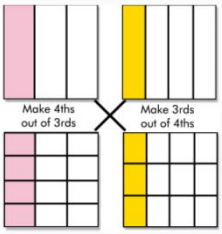
Explicit Instruction (Clear Modeling + Precise Verbal & Written Expression + Comprehensive Manipulatives) = *Mastery*

Fraction Strategies

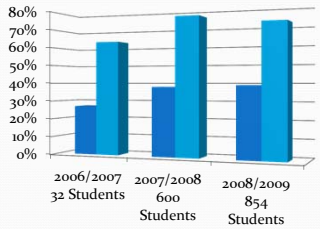
$\frac{1}{3} + \frac{1}{4} =$

Make 4ths out of 3rds × Make 3rds out of 4ths

$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$



Method Results



Source: A school district in New Mexico used with students with math IEP goals.

Results based pre/post testing using curriculum based measurements

Conclusion

- Innovative Techniques:
  - Move Beyond Counting Strategies
  - Use Concrete/Visual Representations
  - Engage Color to Distinguish and Identify
  - Demonstrate Each Step
  - Provide Teachers Tools to Develop Proficiency in Themselves and Their Students

Thank You For Participating!

About the Speakers

- Dan Sinclair
- Mastery Educational Services
- Fallbrook, CA
- Phone: 800-454-6284
- E-mail: Dan@MasteryEd.com
- www.masteryed.com

- Joe Sencibaugh, Ph.D.
- Associate Professor
- Webster University
- St. Louis, MO
- 314-246-7072
- jsencibaugh77@webster.edu

Dan and Joe both share a love of teaching and math. Both are national speakers and have presented at NCTM's national conference previously.

Recommended Recources:

Institute of Educational Sciences ies.ed.gov
Mastery Educational Services www.masteryed.com