Why Math? Why Me?

Around 300 BCE near Alexandria, Egypt, a boy was born. We don't know much about his parents or what he looked like or what he ate, but we do know that he opened a school in Alexandria, asked a lot of questions and wrote at least two books, only one of which survives today. Euclid's book,The Elements, was not necessarily written by Euclid, instead he considered his task organizing and systematizing Greek Geometry. Euclid created a system of logic that included a limited number of assumptions and definitions and then everything else derived logically from those assumptions. This book is perhaps the most read book in human history but it's not actually a book. It was written on many scrolls which aren't even the original scrolls but rather copies of the scrolls that have managed to survive history. Pretty good beginning to a story, isn't it?

Big deal! I hear you cry, but do you know that every mathematical conclusion from that time forward has followed on Euclid's criteria of logical deduction? Euclid, this man from ancient Greece, 2300 years ago put together a logical system and we have used it as our basis for discoveries ever since. What might have happened had Euclid not insisted on the rigor of proof before assumption? What would our understanding of the natural world be without the work of those early mathematicians like Euclid?

Our story jumps now to modern day: Perhaps you've heard your parents say it , or a friend, or you've said it yourself "I'm not good at math" or "I don't have a math brain", or "math is not my thing" but nothing could be further from the truth!

Do you know that babies can do math? In a study done with very young babies (under 3 months old), a scientist measured their interest in puppets by watching their eye movement. The baby was shown an empty puppet stage and then a hand putting a puppet on the stage. Then a screen was put up and another puppet was shown going behind the stage. When the screen revealed two puppets the baby noted it and became uninterested relatively quickly. When the screen revealed an incorrect number of puppets like one or zero, the baby was much more interested as revealed by the baby's eye movement. Even when they altered the experiment and changed the color of the puppets or the shape of the objects, the baby was more interested when the wrong number of objects appeared than with another change. Even as a baby, everyone has a math brain!

Do you know that the study of mathematics is entirely created by humans? That we are the ones through experience and reasoning who have made nature and thus mathematics make sense? It may have begun with Euclid, but he was just an ordinary man who asked questions like: how do we know things? How can we prove that to be true? Other humans have thought about the things that we see in nature and our world and wondered? Is my dog doing Calculus? How does the ant know how to get back to her home? or even How early do babies understand numbers? Euclid may be one important person in the history of mathematics, but the truth is that people are responsible for its creation and use and for helping mathematics to explain what nature is doing.

“Math is about patterns. And patterns are what life is all about.” (Devlin, pg.30)

Think about three distinct sets of data, three columns of numbers. Are there any patterns in these numbers? That may be difficult to see if we look only at the numbers, but may be much easier to view if we graph the data. Can you imagine a time in history when we didn’t think about connecting these two ideas? When lists of numbers were all we had to look at… and then a man named Descartes came up with the idea of connecting the algebra and geometry. Thank goodness! That makes patterns, and thus the world, so much easier to understand.

How we see these patterns, how we interpret them and figure them out is how we create new understanding - you as an adolescent in the 21st century have more than 2000 years of mathematics ideas behind you - more than 2000 years of people puzzling over problems and asking questions and trying to make the world we see make sense. Your task is to work through these ideas at the pace wherein they make sense to you - not to anyone else. We can all make these ideas make sense - millions of people throughout history have done this - everyone has used and understood math… because it was created by us throughout the last 2000 years. Your task is to find the places and pieces of history that mean something to you; that intrigue you and then to explore more.

“...The mathematical landscape is filled with these interesting and delightful structures that we have built (or accidentally discovered) for our own amusement. We observe them, notice interesting patterns, and try to craft elegant and compelling narratives to explain their behavior.” (Lockhart pg. 105)

Galileo Galilei said “The great book of nature can be read only by those who know the language in which it was written. And this language is mathematics.” (Devlin pg. 33)

Thanks to Euclid and Descrates and all the other humans who came before us to make mathematics and nature more understandable.

The Real Story of Numbers

You may have heard the Story of Numbers in your Elementary classroom, how our numbers system came about along with writing and language because of human beings inherent curiosity. More than this though numbers came about because humans wanted to create order from chaos. They wanted to understand the world and the creation of numbers was the first attempt at creating order.

Think about the caveman sitting in his cave talking about the herd of bison out on the plains. Perhaps those early cavemen didn't care how many bison they killed so long as they were not hungry but eventually they got to thinking about how many bison they really needed to get along or how far away the blueberries were, or even more likely they wanted to count the days between full moons to better understand the heavens and the earth.

Counting days between full moons was easy. One, two, three… these were our first numbers and express a primitive or innate part of our experience. These are the numbers given to us by the world and which we took and made our own. Two bison for dinner tonight. Three miles to the blueberries on the hill. 28 days between full moons.

Many civilizations are happy with this much or even less. There is a civilization in Australia called the Warlpiris that has words for one, two, and many. They have no need to count any higher. Or the Vedda tribe in Sri Lanka where if they were going to count a pile of coconuts they would collect a heap of sticks and assign each stick to a coconut. Each time he adds a stick to the pile he says "that is one" and "that is one" but if asked how many coconuts he has all together he would simply point to the pile and say "that many"

Our ancestors didn't stop at "many," however. They kept counting and trading things and discovering things that didn't make sense with the numbers they were working with. For example, they discovered that these natural numbers were perfectly good for addition and multiplication, but left something to be desired when it came to subtraction. 5 + 10 perfectly ok with the natural numbers. 5x10 as well. What about 5-10 though? That doesn't make sense within our world of natural numbers. And at some point it surely occurred that the folks doing the trading wanted to talk about owing money or coconuts or sticks. And thus the second great group of numbers came about – the Integers.

Within the integers we have to talk about the whole numbers. Whole numbers are the counting numbers, the natural numbers, including zero. Why didn't they include zero in the counting numbers to begin with? That's a story for another day, but suffice it to say that the whole numbers include the natural numbers and zero. And that the integers include the natural numbers and their opposites.

But this still doesn't take care of the issue with division. Remember, counting was part of trying to organize the chaos and everything seems pretty tidy at this point with integers and whole numbers and addition, subtraction, and multiplication. But what have we forgotten?

But we've forgotten division and that sends our tidy system into chaos again. Some division problems fit perfectly well in our system, like 12/4 or 36/6, but what about 3/2? Or 12/ 7? Those numbers cannot be found no matter how far up or down the integers we look. In order to work with division we must have something more than integers. The answer comes when we think of division of integers as fractions. With integers we talk about the whole pie. With fractions we talk about the parts of the pie. Individual pieces depending on how many slices we make.

See our system of integers is clear and easy to follow. The integers and whole numbers and natural numbers are discrete. There is one and then two. There is nothing between 1 and 2. There is 5 and then 6. Nothing between 5 and 6 either. It's like stepping stones – like we're preceding from rock to rock across the pond.

With division, suddenly the distance between 1 and 2 has another rock there. What about 5/3 or 7/5 or 12/9? There's a bunch more rocks between 1 and 2. In fact, with the addition of fractions and decimals, there are an infinite number of stones between 1 and 2. These stones that can be expressed as the ratio of two numbers are called rational numbers. The number system is now dense and infinite. It's like we poured sand between the stepping stones and each grain is another number.

So mathematics and human kind skips happily along with its natural numbers, whole numbers, integers, and rational numbers. All kinds of operations are now possible and everything seems good, but the Greeks discovered a problem. See they took a square with sides of 1 and asked a pretty simple question. How long is the diagonal of that square? How would the Greeks have figured that out? They would have turned to Pythagoras and the Pythagorean Theorem. Recall that $a^2 + b^2 = c^2$. In this case, a=1 and b = 1 so $1+ 1 = c^2$ and $c^2 =2$. This means that $c = \sqrt{2}$

What's the problem? The Greeks believed in rational numbers and that everything could be expressed as the ratio of two numbers. $\sqrt{2}$ Doesn't really seem to follow that. The Pythagoreans couldn't figure out what two integers gave you the $\sqrt{2}$. One of the members of the Pythagoreans let slip that they couldn't find two numbers that would create the ratio for the rational number $\sqrt{2}$. The Pythagoreans were so upset that they killed the man… because of math!

And it turns out there really wasn't a way to write $\sqrt{2}$ as a ratio of two numbers and so to move forward, humans needed to let go of their adherence to rational numbers and think about a new set of numbers. These numbers are called irrational numbers. What's the most famous irrational number? Pi! But that's a story for another day. Many of the irrational numbers that you will come across are in square roots. Think about all the square roots $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$? $\sqrt{5}$, $\sqrt{6}$, etc. Most of them are irrational.

All of these numbers together – the natural numbers, the whole numbers, the integers, the rational numbers and the irrational numbers are all real numbers. We call these numbers all together the real numbers.

This begs the question, what are non-real numbers? There is a group of numbers that we call imaginary, but that is a story for another day.

So that is my story of numbers, remember that as humans we have come up against problems with the way we do mathematics and we have solved those problems by thinking outside the box. By thinking of more numbers or different theories to make things make sense. Another day I'll tell you a story about pi…

Recipe for Pi

My Aunt asked me the other night "Why pi? What's the big deal?" And I said – are you kidding? Did you know that the Great Pyramid at Giza is related to pi? The ratio of the length of one side to the height is approximately pi/2. Isn't that weird? Isn't that amazing?

Pi shows up in all kinds of places like that. It shows up in this sequence: 1 + -1/3 + 1/5 + -1/7 + 1/9… Isn't that amazing? Isn't that weird? No wonder the ancient Greeks were fascinated by pi.

$\Pi = 4/1 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11\ldots$

Think about it: In ancient Greece they knew more about Geometry than any of you and they proved most of what's in the Geometry textbook that we use. Yet they were afraid of zero and they refused to accept irrational numbers even though they were right before their eyes. The Greeks took philosophy and numbers very seriously.

Do you remember that Greeks loved ratios? Rational numbers are all about ratios – any number that can be written as the ratio of two numbers is a rational number – this happens to include lots of things that aren't ratios – like decimals that end or repeat – these can also be written as ratios.

Irrational numbers don't ever end or repeat and they can't be written as ratios so they didn't fit in the Greeks world. In fact, the Pythagoreans, who were a little on the nutty side of mathematics cults, once killed a man, Hippasus of Metapontum, who let slide the great secret about irrational numbers.

See a square which is such a basic shape that the Greeks could see and touch had an irrational diagonal. (Think about it – use a square that has a side of one. Then according to the Pythagorean Theorem, the diagonal would be sqrt2). The Greeks could not deny the existence of this segment, but they also couldn't get that diagonal to work out to any ratio… and when Hippasus admitted to someone outside the cult, that it wasn't a rational number the Pythagoreans – who had this whole ratio theory of the universe, right – had him killed.

The Greeks created a famous mathmematical problem using the circle and square – two really basic shapes that they could see and touch. They puzzled how you would create a circle and a square that had exactly the same area using only a compass and a straightedge? This idea seemed so simple, and yet no one was able to solve it. For 2000 years mathematicians worked on this problem off and on with no success until finally in 1882 someone proved that it was actually an impossible task. Imagine working on a problem for 2000 years only to find out it couldn't be done!

So the Greeks refused to believe in irrational numbers, but they never gave up hope that pi would be something more. Of course, they never bothered to name this quantity. People referred to it as "the quantity which, when the diameter is multiplied by it, equals the circumference" for nearly 2000 years. That's a mouthful!

Leonard Euler finally and formally decided on the Greek letter pi (the 16^{th} letter of the Greek alphabet) to represent the ratio of circumference to diameter. And we have used it ever since. We have Euler to thank for a lot of mathematical notation we now use – not the least of which is saving us from the quantity which, when the diameter is multiplied by it, equals the circumference.

The Greek mathematician Archimedes first tried to compute the value of pi by drawing a hexagon inscribed in a circle and a hexagon circumscribed about that circle. He figured that pi would be somewhere between the inscribed and circumscribed hexagon. To be more accurate he doubled the sides of the polygon each time trying to narrow the gap between the inscribed and circumscribed polygons. For 2000 years, this technique was repeated again and again eventually attaining polygons with upwards of 96 sides trying to find more and more precise values of pi.

Why we are compelled to find all these decimals of pi? For ANY computing we might want to do we need no more than maybe 10 or so at best. Why, then do we spend so much time and energy finding decimals of pi? Some might argue that it's for the same reasons that we climb Mt. Everest. Because it's there. It's the same feat.

There is a human compulsion to break records – the current world record is 67,890 digits of pi by Chao Lu of China!

The Chudnovsky brothers have spent a lifetime calculating pi. They built their own supercomputer which snakes between rooms and throughout their apartment in NYC and have spent 20 years repeating various calculations, testing theories, and calculating pi to more than 8 billion digits. David Chudnovsky says, "Exploring pi is like exploring the universe." He may be right.

"When we think of pi, let's not always think of circles. It is related to all the odd whole numbers. It also is connected to all the whole numbers that are not divisible by the square of a prime. And it is part of an important formula in statistics. These are just a few of the many places where it appears, as if by magic. It is through such astonishing connections that mathematics reveals its unique and beguiling charm." (Joy of Pi p.97 from Sherman K. Stein, Strength in Numbers)

Memorize a story w first word three letters, second word one, etc.

Pi is the average ratio of the actual length and the direct distance between the source and mouth in a meandering river!

<u>The Story of Diophantus of Alexandria:</u>

Remember how we've talked about the story of numbers? How people in ancient times chose to mark different numbers and how our own numbers came to be? Remember how the Hindus gave us zero and many people all together over the years gave us the numbers that we use today. Today I'm going to tell you another story.

A long time ago, nearly 2000 years, there lived a man named Diophantus. There is not much known about this man, except for this puzzle. It says:

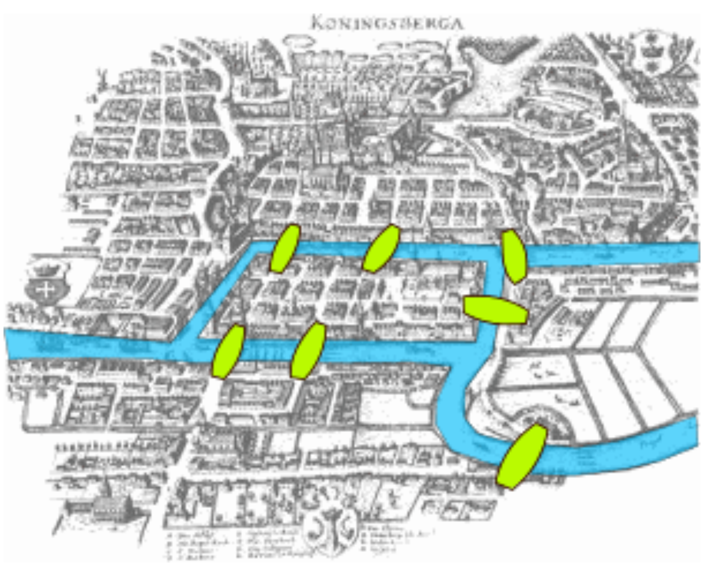
This tomb holds Diophantus. Ah, what a marvel! And the tomb tells scientifically the measure of his life. God vouchsafed that he should be a boy for the sixth part of his life; when a twelfth was added, his cheeks acquired a beard; He kindled for him the light of marriage after a seventh, and in the fifth year after his marriage He granted him a son. Alas! late-begotten and miserable child, when he had reached the measure of half his father's life, the chill grave took him. After consoling his grief by this science of numbers for four years, he reached the end of his life.

Diophantus is sometimes called the "father of Algebra" because he wrote the very first Algebra book composed of 13 smaller books or chapters. A book that is nearly 2000 years old doesn't seem like it would be very important or even necessarily interesting, but that's not the end of the story.

Over the years some of Diophantus' Algebra books got lost, but some of them were looked at by other mathematicians. Hypatia, the first female mathematician lived just after Diophantus in Alexandria, Egypt and she worked on the problems he created and tried to find solutions to them. Hypatia was murdered by Christian fanatics in 415 A.D.

Later mathematicians also studied Diophantus. There was a mathematican named Euler who spent much of his life in Konigsberg, Prussia. He was a great mathematican and created a famous problem of his own called the Konigsberg bridge problem.

Fine ladies and gentleman used to enjoy strolling the bridges and islands in Konigsberg. The problem was: is there a route that would



allow the fine ladies and gentlemen to cross each bridge exactly once and arrive back at their starting place?

Another mathematician who studied Diophantus' problem was Pierre Fermat. Fermat lived from 1601 – 1665. He was an amateur mathematician who is known for his work with early calculus. In addition though, as he read Diophantus' work he wrote after one particular problem that he had an elegant proof for this problem that was too large to fit in the margin of the book. And then he died leaving Fermat's Last Theorem to puzzle mathematicians for the next 300 years! This proof has been listed in the Guinness Book of World Records as the most difficult math problem ever.

Do you want to know the problem that Fermat thought he could prove, but didn't have room to show the proof for? Do you remember the Pythagorean Theorem? It states $a^2 + b^2 = c^2$. Fermat puzzles: does $a^n + b^n = c^n$ work? (Where n can be any number)

Which brings us to a young boy named Andrew Wiles. As a ten year old, Andrew was introduced to this problem and it intrigued him all his life. It actually led him into mathematics as a profession and he continued to dabble with the problem for many years. At one point, Andrew abandoned all other research, cut himself off from the world, and for seven years worked on proving Fermat's Last Theorem. He revealed his proof to the world in 1993 – it was more than 200 pages long!

So you see, Diophantus of Alexandria, has continued to be relevant and interesting to people for nearly 2000 years!

Diophantus of Alexandria: Arithmetica Book 1

1. To divide a given number into two having a given difference. Given number 100, given difference 40.

2. To divide a given number into two having a given ratio. Given number 60, given ratio 3:1

3. To divide a given number into two numbers such that one is a given ratio of the other *plus* a given difference. Given number 80, ratio 3:1, difference 4.

4. To find two numbers in a given ratio and such that their difference is also given. Given ratio 5:1, given difference 20.

5. To divide a given number into two numbers such that given fractions (not the same) of each number when added together produce a given number. *Necessary condition*. The latter given number must be such that it lies between the numbers arising when the given fractions respectively are taken of the first given number. First given number 100, given fractions 1/3 and 1/5, given sum of fractions 30.

6. To divide a given number into two numbers such that a given fraction of the first exceeds a given fraction of the other by a given number. *Necessary condition*. The latter number must be less than that which arises when that fraction of the first number is taken which exceeds the other fraction. Given number 100, given fractions ¼ and 1/6 respectively, given excess 20.

7. From the same (required) number to subtract two given numbers so as to make the remainders have to one another a given ratio. Given numbers 100, 20, given ratio 3:1.

8. To two given numbers to add the same (required) number so as to make the resulting numbers have to one another a given ratio. *Necessary condition*. The given ratio must be less than the ratio which the greater of the given numbers has to the lesser. Given numbers 100, 20, given ratio 3:1

9. From two given numbers to subtract the same (required) number so as to make the remainders have to one another a given ratio. Necessary condition. The given ratio must be greater than the ratio which the greater of the given numbers has to the lesser. Given numbers 20, 100, given ratio 6:1.

10. Given two numbers, to add to the lesser and to subtract the greater the same (required) number so as to make the sum in the first case have to the difference in the second case a given ratio. Given numbers 20, 100, given ratio 4:1.

11. Given two numbers, to add the first to, and subtract the second from, the same (required) number, so as to make the resulting numbers have to one another a given ratio. Given numbers 20, 100, given ratio 3:1.

12. To divide a given number twice into two numbers such that the first of the first pair may have to the first of the second pair a given ratio, and also the second of the second pair to the second of the first pair another given ratio. Given number 100, ratio of greater of first parts to lesser of second 2:1, and ratio of greater of second parts to lesser of first parts 3:1.

13. To divide a given number thrice into two numbers such that one of the first pair has to one of the second pair a given ratio, the second of the second pair to one of the third pair another given ratio, and the second of the third pair to the second of the first pair another given ratio. Given number 100, ratio of greater of first parts to lesser of second 3:1, of greater of second to lesser of third 2:1, and of greater of third to lesser of first 4:1.

14. To find two numbers such that their product has to their sum a given ratio. [One is arbitrarily assumed.] Necessary condition. The assumed value of one of the two must be greater than the number representing the ratio. Ratio 3:!, x one of the numbers, 12 the other >3.

15. To find two numbers such that each after receiving from the other a given number may bear to the remainder a given ratio. Let the first receive 30 from the second, the ratio being then 2:1, and the second 50 from the first, the ratio being then 3:1; take x + 30 for the second.

16. To find three numbers such that the sums of pairs are given numbers. *Necessary condition.* Half the sum of the three given numbers must be greater than any one of them singly.

17. To find four numbers such that the sums of all sets of three are given numbers. *Necessary condition*. One-third of the sum of the four must be greater than any one singly.

18. To find three numbers such that the sum of any pair exceeds the third by a given number. Given excesses 20, 30, 40. 2x the sum of all three.

The Story of Zero
Taken from Zero: The Story of a Dangerous Idea

Have you ever thought about what a crazy number zero is?

Add a number to itself and it changes. 1+1=2; 2+2 =4; but 0+0 = 0. Zero refuses to get bigger. It also refuses to make any other number bigger: 2+0 = 2; as though you never bothered to add 0 to begin with. Or think about multiplication – multiplication is like stretching or shrinking a rubber band – multiply by 2 and everything gets bigger – stretches that way – multiply by ½ and everything gets smaller – multiply by zero and the whole thing collapses on itself. Zero is a crazy number.

Or what about division? Undoing multiplication is division. So 2x3 = 6 and (2x3)/3 = 2; but this doesn't work for zero 2x 0 = 0 but (2x0)/0 doesn't get us 2. By this logic, one might assume that (2x0)/0 = 2 but that's the same as saying that 0/0 = 2 which just makes no sense at all!

It is no wonder ancient mathematicians didn't trust this number. It puts all rules of mathematics to the test.

What can I tell you about nothing? Plenty.

Imagine being back in Ancient Greece. You know a lot about things that you can see and touch like triangles and circles and the beauty that exists in nature. Interestingly, you believe that much of the beauty in nature has to do with ratios. The ratios of width to height of rectangles create the most beautiful rectangles and the Greeks use them in their architecture, in their statues, in everything you do. Take a look at these rectangles and pick out the one that you think is most beautiful. Which one is it? Greeks believed that rectangles with this width to height ratio were the most beautiful rectangles in the world. In many ways, we still do too since we use tehm all the time as well...
So these rectangles with the perfect ratio are things you can see and touch: the beauty of certain rectangles over others, the ratio of various parts of stars and squares and triangular numbers. Zero had no place in this universe. How could you see zero? What did a square or rectangle look like that had a width of zero?

According to Pythagoras, the earth is the center of the universe and the sun, moon, and planets rotate in spheres – spheres with special ratios – around the earth. The outermost planets (Jupiter and Saturn) moved the fastest there at the outside of the spheres. The spheres, as they move, make music and the outermost ones make the highest pitched music. The innermost ones make lower notes and taken altogether the planets make a harmony of spheres so that the heavens are a beautiful mathematical orchestra. But zero was not a part of this.

For everything in the universe to be governed by ratios including these heavenly spheres, there could be no zero. What is the ratio of 0 to 2? The other number is swallowed up by the 0. What's the ratio of 2 to 0? A number divided by 0? It defies logic. It destroys logic. For everything in the universe to be governed by ratios, everything that made sense in the universe had to be related to a nice neat proportion. This meant that infinity could not exist for the same reasons as zero. What's the ratio of anything with infinity?

What's more, since infinity didn't fit into the Greeks' understanding, the universe couldn't be infinite. Their theory of spheres with the earth at the center needed some end. The last sphere held all the stars that could be seen and beyond that the Greeks thought that God was the master conductor of this orchestra of the stars. In many respects, denying the existence of zero and infinity, gave the Greeks proof of God's existence. This is, in part, the reason why this theory of concentric spheres lasted so long: it proved the existence of God.

In order to understand some of this next part, you must understand the different number systems that were in place back in the day. In Roman times they used Roman numerals – note that there is no zero in Roman numbers but also note how cumbersome multiplying or dividing with Roman numbers is. What's VIII + IX? Blech. It doesn't even make sense, but notice that there's no zero anywhere in the Roman system.

The Babylonians didn't have a zero either, but they did have a placeholder. See Babylonians had what we now call a sexagesimal system, which means that instead of our base 10, they had a base of 60. Think about what this means: They were using base 60 which means that the next place over were the 3600 – trust me when I tell you that the Babylonians needed a place holder and used two slanted lines for just such a purpose. This helped them differentiate various numbers like 1, 61, and 3601.

So the Babylonians were using zero, but it had no meaning in and of itself. In about the fourth century B.C. , Alexander the Great marched with his Persian troops from Babylon to India and he took the Babylonian concept of zero as a place holder with him. This is how Indians first learned of zero. When Alexander died in 323 B.C. they split up his great empire and much of it was taken by the Romans, but the Romans did not reach to India and thus India continued to develop and work with number systems independently of the west. Although India was influenced by some of the Greek works via Alexander, they never embraced this fear of infinity or the idea of the concentric spheres.

In fact, India, and Hinduism in particular, have always embraced infinity and the idea of duality: yin and yang, creation and destruction, good and evil. Shiva, their god was creator and destroyer as well as representing nothingness. India was not afraid of zero.

Sometime around the fifth century A.D., the Indians switched from a Greek style of numbering to a Babylonian-style system. An important difference between the Babylonian style system and the Indian system is that Indian numbers were base 10 instead of base 60 – our own numbers evolved from the symbols that the Indians used (as an aside, we often call our number system Arabic, but it would be more rightfully called Indian).

The Indian system of numbering allowed them to use fancy tricks to multiply, divide, add, and subtract. Instead of the clunky methods that the Romans had to use with those cumbersome Roman Numerals, Indians created lattices for multiplying that did a pretty neat trick of keeping track of numbers and place values. Though the Indian number system was useful for everyday tasks like addition and multiplication, the true impact of Indian numbers was considerably deeper. Numbers no longer were inextricably linked to geometry. Indians didn't think about 2 squared being lengths of sides of squares – instead they were simply numbers for numbers sake. You can't remove a 3 acre farm from a 2 acre farm, but it's easy enough to do 2 – 3 = -1. In the same way, India was far more willing to talk about a square with 0 units on a side and thus they were willing to accept zero with all of its faults and oddities.

By the time that Indian mathematics made it's way back to Europe, there was no escaping the power that zero had within the system, and what's more, even European mathematicians were beginning to doubt the veracity of the Earth at the center of the universe and the concept of zero and infinity which were so neatly tied together.

This acceptance of zero was the foundation for all of modern mathematics and many of the resulting discoveries and understanding of mathematics and the world occurred only because we were finally willing to embrace the idea of zero.

Name ______________________

The story of Zero

The axiom of Archimedes is destroyed by the idea of zero. Remember, that the Greeks only believed in natural numbers. What are the natural numbers?

The axiom of Archimedes states:

If x and y are numbers *there exists* a number n such that nx > y.

What does the phrase *there exists* imply?

Pick values for x and y. Then find an n value that make this statement true.
If x = ______ and y = _____ and n = _______ then nx > y.
If x = ______ and y = ____ and n = _______ then nx > y.

Now find values for x and y so that there is no n-value that makes this true. What x value has NO n value that will make it bigger than y?

If x = ______ and y = _______ and n = ______ then nx is not > y.

Clearly, this axiom is not true as stated. It can be made true if we make **one change** to the original statement. What do you think that change needs to be?

Roman Numerals: use the following 7 symbols and represent numbers by essentially adding individual symbols (or sometimes subtracting)

I = 1	V = 5	X = 10
L = 50	C = 100	D = 500
M = 1000		

So 4 = IV (one before 5) and 21 = XX1 (10+10+1)

Try to perform the following operations on the Roman numerals. Can you do them without converting them to our Arabic number system? Are some easier than others?

1. XVI + VI

2. XL + XC – XX

3. XIII * XXI

4. CXX * X

5. CLX / XL

6. MMX – XXXIX

Think critically: How would our world be different if we still used the Roman Numerals? Give at least two differences and explain your reasoning. Complete sentences are important! You may type this response.

The following resources in whole or part were used to create the above stories.

Euclid's Window by Leonard Mlodinow
Mathematicians are People Too (and the sequel)
A Tour of the Calculus by David Belinski
The Math Instinct by Keith Devlin
Zero: The Biography of a Dangerous Idea by Charles Seife