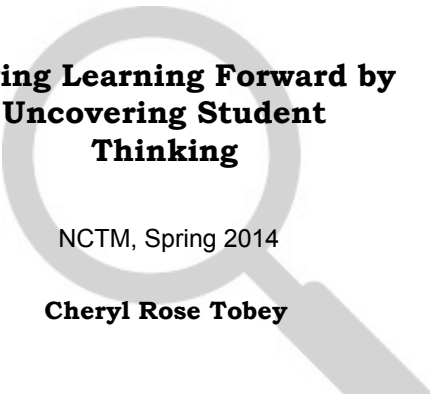


**Moving Learning Forward by  
Uncovering Student  
Thinking**

NCTM, Spring 2014

**Cheryl Rose Tobey**

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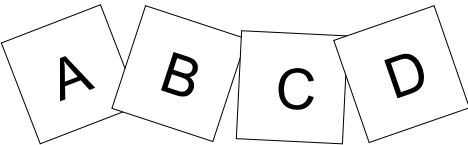
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**Prepare Your Response Cards**

A B C D

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**I work with...**

- A: Grade 3-4 Students
- B: Grade 5-6 Students
- C: Grade 7-8 Students

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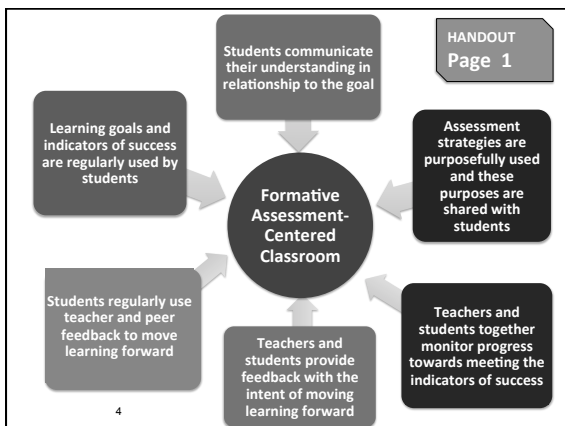
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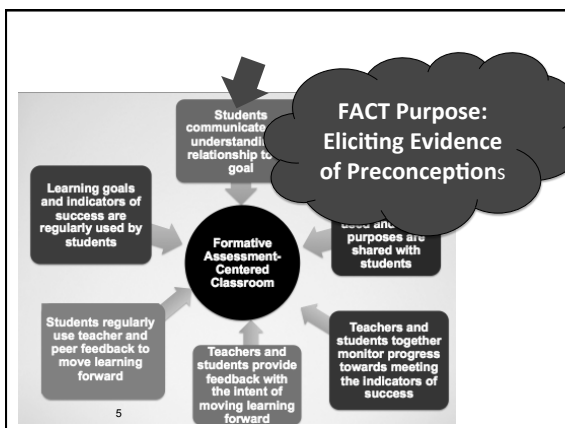
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**Misconceptions in Mathematics**

- alternative conceptions
- student conceptions
- pre-conceptions
- conceptual primitives
- private concepts
- alternative frameworks
- systematic errors
- common errors
- critical barriers to learning
- naïve theories
- overgeneralizations

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Handout p. 6

1.  $\frac{1}{2}$  Circle One: A B C D E F Not Shown

Explain your choice:

Tobey and Arline, Corwin Press

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Student D

**Number Lines**

For each number line, decide which point represents the location of the fraction.

1)  $\frac{1}{2}$  Circle One: A **B** C D E F Not Shown

Explain your choice: Well the number line starts at  $\frac{1}{4}$  and kinda ends at 1 and there is  $\frac{3}{4}$  between those 2 numbers. So if I move 2 spaces ( $\frac{1}{4}$ ) than I'm at half.  $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

2)  $\frac{7}{8}$  Circle One: A B C D E F Not Shown

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**number Lines**

For each number line, decide which point represents the location of the fraction.

1)  $\frac{1}{2}$  Circle One: A B C **D** E F Not Shown

Explain your choice: It is D because  $7 \div 2 = 3.5$

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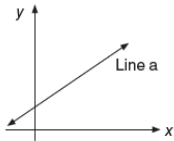
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Determine whether each equation could represent Line a.



Handout p. 8

1.  $y = \frac{2}{3}x + 5$  Circle One: Yes No Can't Determine  
Explain your choice:

2.  $y = -6x + 2$  Circle One: Yes No Can't Determine  
Explain your choice:

Tobey and Arline, Corwin Press

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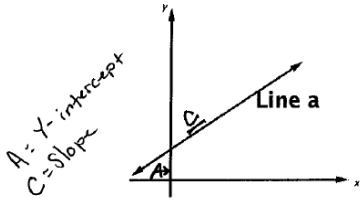
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a)  $y = \frac{2}{3}x + 5$  Circle One: Yes No Can't Determine  
Explain your choice:  
A=5  
C= $\frac{2}{3}$  So it seems perfectly reasonable yes.

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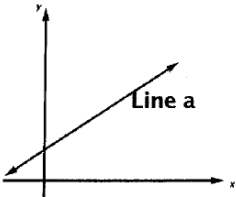
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a)  $y = \frac{2}{3}x + 5$  Circle One: Yes No Can't Determine  
Explain your choice:  
The slope of the line goes up too fast.

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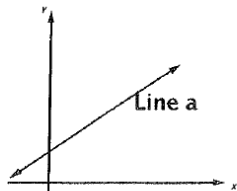
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a)  $y = \frac{2}{3}x + 5$  Circle One: Yes No Can't Determine

Explain your choice:  
*The axes are not labeled*

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**What are Formative Assessment Probes?**

- Diagnostic questions based on research on student learning & misconceptions
- Format allows for quick analysis
  1. Multiple choice Yes/No Example/Non-example
  2. Show or explain choice
- Provides you with evidence of students' understanding/difficulties to inform instruction

Source: Uncovering Student Thinking in Mathematics Series, Rose Tobey, Arline, Fagan, Minton

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
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Circle the figures that are parallelograms. Handout 1/2 sheet 


A. 

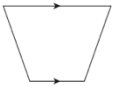
B. 

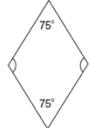
C. 

D. 

E. 

F. 

G. 

H. 

Rose, Minton, Arline, Corwin Press

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### Completing the Probe

Circle the figures that are parallelograms.

A.  B. 

E.  F. 

1. INDIVIDUALLY answer the probe.  
2. Write a very brief explanation about why you listed these shapes

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

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### Adding Confidence Level

Circle the figures that are parallelograms.

A.  B. 

E.  F. 

1. Fold your paper  
2. Add confidence rating of 0 to 5

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### FOLD and PASS

Circle the figures that are parallelograms.

A.  B. 

E.  F. 

1. Fold your paper  
2. When I say go: continually take and hand off papers from others  
3. When I say stop: make sure you have one piece of folded paper

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**FACT Example: Fold and Pass**

**Response Card CHECK-IN**  
How long has it been since you have taught or taken high school geometry?

- ▶ **A: 0-5 years**
- ▶ **B: 6-10 years**
- ▶ **C: 11-15 years**
- ▶ **D: 16+ Years**

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**FACT Example: Fold and Pass**

**Response Card CHECK-IN**  
Hold up a response card if the paper you are holding includes one or more of the following:

- ▶ **Shape C, E, F OR G**

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**FACT Example: Fold and Pass**

**Response Card CHECK-IN**  
Hold up a response card if the paper you are holding includes one or more of the following:

- ▶ **A: Shape C**
- ▶ **B: Shape E**
- ▶ **C: Shape F**
- ▶ **D: Shape G**

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**FACT Example: Fold and Pass**

**Response Card CHECK-IN**

Hold up a response card if the paper you are holding includes one or more of the following:

- ▶ **A: Shape A**
- ▶ **B: Shape D**

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**FACT Example: Fold and Pass**

**Response Card CHECK-IN**

Hold up a response card if the paper you are holding includes

- ▶ **A: ONLY Shape B and Shape H**

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**Example: Is it a Parallelogram?**

The correct responses are B and H. Parallelograms are a special type of quadrilateral. A quadrilateral is a parallelogram if both pairs of opposite sides are parallel, both pairs of opposite sides are congruent, both pairs of opposite angles are congruent, consecutive angles are supplementary, one pair of opposite sides is congruent and parallel, or diagonals bisect each other. Figure B has both pairs of opposite sides congruent. Figure H has both pairs of opposite angles congruent. (See Student 1 in *Student Responses* section.) Some students choose B and H with incorrect reasoning. They see parallelograms as quadrilaterals with two congruent obtuse angles and two congruent acute angles. (See Student 2 in *Student Responses* section.)




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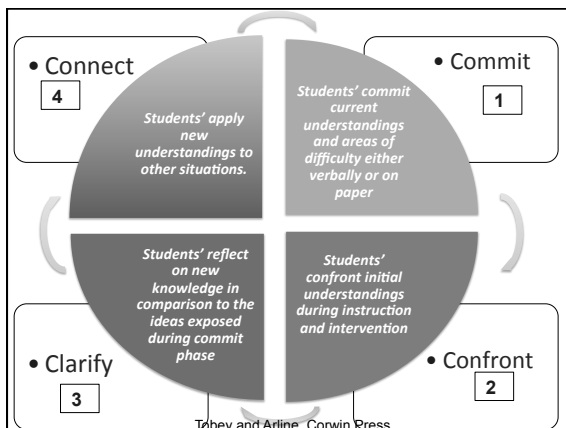
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### Other Strategies Used

- Response Cards
- Confidence Ratings
- Fold and Pass: (Commit and Toss)

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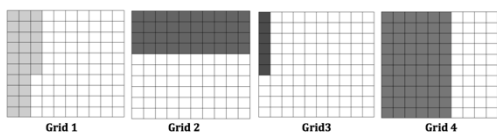
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### Sticky Bar Example: Which Grid Shows 0.6?



Which incorrect choice are students most likely to choose? Why?

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**Video Focus Question: Which Grid is 0.6?**

*What do you notice about the teachers' strategy for gathering information on the students' understanding?*

- Small rural school in Maine, on State Priority list for mathematics
- 94% free and reduced lunch
- Reminder: please watch without judging teacher or students

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**Probe Examples**

Handout

- Scan and briefly review 2-3 of the probes
- For each Probe:
- What conceptual and procedural knowledge is targeted by the items?
  - What conceptual misconceptions and/or procedural difficulties is targeted by the items?

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**Example Item: Representing Fractions I**  
[em2.edc.org](http://em2.edc.org)

Is the shaded part  $\frac{1}{4}$  ?

- Yes  
 No



Explain your thinking.

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### Representing Fractions I

<p>Is the shaded part <math>\frac{1}{4}</math> ?</p> <p><input checked="" type="radio"/> Yes <input type="radio"/> No</p>	<div style="display: flex; align-items: center;">  <div style="margin-left: 10px;"> <p>Explain your choice using words and/or pictures.</p> <p>1 space is shaded and 3 arnt and 3+1=4</p> </div> </div>
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Handout p. 5

### Example Item: Representing Fractions II

em2.edc.org

<p>Is the shaded part <math>\frac{1}{4}</math> ?</p> <p><input type="radio"/> Yes <input type="radio"/> No</p>	<div style="display: flex; align-items: center;">  <div style="margin-left: 10px;"> <p>Explain your thinking.</p> </div> </div>
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### Representing Fractions II

<p>Is the shaded part <math>\frac{1}{4}</math> ?</p> <p><input type="radio"/> Yes <input checked="" type="radio"/> No</p>	<div style="display: flex; align-items: center;">  <div style="margin-left: 10px;"> <p>Explain your thinking.</p> <p>1 shaded &amp; All But doesnt work becuz they dont look the same</p> </div> </div>
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### Representing Fraction II

Is the shaded part $\frac{2}{5}$ ? <input type="radio"/> Yes <input type="radio"/> No		Explain your thinking.
Is the shaded part $\frac{1}{4}$ ? <input type="radio"/> Yes <input type="radio"/> No		Explain your thinking.

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### Sample Student Response

Is the shaded part $\frac{2}{5}$ ? <input type="radio"/> Yes <input checked="" type="radio"/> No		Explain your thinking. <i>The 3 shapes are way different.</i>
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### Sample Student Response

Is the shaded part $\frac{1}{4}$ ? Yes <input checked="" type="radio"/> No		Explain your thinking. <i>Too many parts / all are different</i>
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## Rational Number Misconceptions

A)  $\frac{11}{12} + \frac{6}{7}$  I know  $12 + 7 = 19$ , (Almost) 20.

a) $\frac{1}{2}$	d) 2
b) 1	<b>e) 20</b>
c) $1\frac{1}{2}$	f) 40

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The screenshot shows the homepage of the Eliciting Mathematics Misconceptions (EM2) website. It features a navigation menu with links for Home, About, Research, Assessments, Testimonials, Online Administration, and News. The main content area includes a 'Welcome!' message and three key sections: 'Our Long Term Goal' (developing an online diagnostic assessment system), 'Our Audience' (upper elementary school and middle-grade mathematics teachers), and 'Our Focus' (identifying and categorizing student misconceptions).

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The screenshot shows the 'Assessments' page of the EM2 website. It details the target areas of focus for probes, organized into three columns: 'Understanding Fractions', 'Understanding Decimals', and 'Operations of Fractions & Decimals'. The 'Understanding Fractions' column includes 'Representing Fractions I', 'Representing Fractions II', and 'Comparing Two Fractions'. The 'Understanding Decimals' column includes 'Locating Decimals on a Number Line', 'Representations of Decimals', and 'Comparing Decimals and Fractions'. The 'Operations of Fractions & Decimals' column includes 'Estimating Addition of Fractions', 'Adding and Subtracting Fractions', 'Multiplying and Dividing Fractions', and 'Multiplying and Dividing Fractions'.

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**Thanks for Joining Today's Session!**

[ctobey@edc.org](mailto:ctobey@edc.org)

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