institute for learning

# Assessing Understanding and Advancing Rigorous Thinking of the Common Core through Questioning


## Institute for Learning
## Learning Research and Development Center
## University of Pittsburgh


**PARTICIPANT HANDOUT**

National Council of Teachers of Mathematics

Laurie Speranzo and Sandra Campo

New Orleans

April 10, 2014

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## Rationale

Effective teaching requires being able to support students as they work on challenging tasks without taking over the process of thinking for them.

Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*. New York, NY: Teachers College Press.

Asking questions that *assess* student understanding of mathematical ideas, strategies, or representations provides teachers with insights into what students know and can do. The insights gained from these questions prepare teachers to then ask questions that *advance* student understanding of mathematical concepts, strategies, or connections between representations.

By analyzing students' responses, teachers will have the opportunity to develop questions to advance student understanding of mathematical concepts and mathematical practices and provide opportunities for rigorous thinking.

## Session Goals and Activities

Participants will:

- dissect Mathematical Practice Standards 2 and 3;
- learn the importance of anticipating student responses in order to be able to press for conceptual understanding;
- identify questions to advance student understanding; and
- determine how questioning can move students toward rigorous thinking.

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# Mathematical Practice Standard 2:
# *Reason abstractly and quantitatively.*

The Common Core State Standards recommend that students:

- make sense of quantities and their relationships in problem situations;
- bring two complementary abilities to bear on problems involving quantitative relationships:
  - the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents; and
  - the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved;
- create a coherent representation of the problem at hand;
- consider the units involved;
- attend to the meaning of quantities, not just how to compute them; and
- know and flexibly use different properties of operations and objects.

Council of Chief State School Officers (CCSSO), & National Governors Association Center for Best Practices (NGA Center). (2010). Mathematics. *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math/Practice

## Mathematical Practice Standard 3:
## *Construct viable arguments and critique the reasoning of others.*

The Common Core State Standards recommend that students:

- construct viable arguments and critique the reasoning of others;
- use stated assumptions, definitions, and previously established results in constructing arguments;
- make conjectures and build a logical progression of statements to explore the truth of their conjectures;
- recognize and use counterexamples;
- justify conclusions, communicate them to others, and respond to the arguments of others;
- reason inductively about data, making plausible arguments that take into account the context from which the data arose; and
- compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is.

Council of Chief State School Officers (CCSSO), & National Governors Association Center for Best Practices (NGA Center). (2010). Mathematics. *Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math/Practice

# The Structure and Routines of a Lesson

## Set-Up of the Task

## The Explore Phase/ Private Work Time

Generate Solutions

## The Explore Phase/Small Group Problem Solving

1. Generate and Compare Solutions
2. Assess and Advance Student Learning

## Share, Discuss, and Analyze Phase of the Lesson

1. Share and Model
2. Compare Solutions
3. Focus the Discussion on Key Mathematical Ideas
4. Engage in a Quick Write

**MONITOR:** Teacher selects examples for the Share, Discuss, and Analyze Phase based on:

- Different solution paths to the same task
- Different representations
- Errors
- Misconceptions

**SHARE:** Students explain their methods, repeat others' ideas, put ideas into their own words, add on to ideas, and ask for clarification.

*REPEAT THE CYCLE FOR EACH SOLUTION PATH.*

**COMPARE:** Students discuss similarities and differences between solution paths.

**FOCUS:** Discuss the meaning of mathematical ideas in each representation.

**REFLECT:** Engage students in a Quick Write or a discussion of the process.

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## Bobby's Hike Task

Bobby said that he wanted to go for a four-mile hike. Bobby stops every $\frac{1}{3}$ mile for a sip of water from his water bottle. How many times does Bobby stop? Be sure to show how you found your answer with both diagrams and an explanation in words. What equations involving fractions match your diagram?

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# The CCSS for Mathematical Content: Grade 5

## Number and Operations – Fractions 5.NF

**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

5.NF.B.7a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) x 4 = 1/3.*

5.NF.B.7b Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 x (1/5) = 4.*

5.NF.B.7c Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Council of Chief State School Officers (CCSSO), & National Governors Association Center for Best Practices (NGA Center). (2010*). Mathematics. Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math/Content/5/NF

## The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Council of Chief State School Officers (CCSSO), & National Governors Association Center for Best Practices (NGA Center). (2010). *Mathematics. Common core state standards for mathematics*. Retrieved from http://www.corestandards.org/Math/Practice

# Using Questioning During the Exploration Phase

Imagine that you are walking around the room observing your groups of students as they work on the Bobby's Hike task.

Consider what you would say to the groups who produced responses **A , B, and C,** in order to ***assess*** and ***advance*** their thinking about key mathematical ideas, problem-solving strategies, or use of and connection between representations.

Specifically, for each response, indicate which of the questions you would ask:

- to ***determine what the students know and understand****;* and
- to ***move the students*** towards the target mathematical goals.

| **Student Work** | **Questions to determine what the students know and understand** | **Questions to move the students towards the target mathematical goals** |
|---|---|---|
| **Group A** | | |
| **Group B** | | |
| **Group C** | | |

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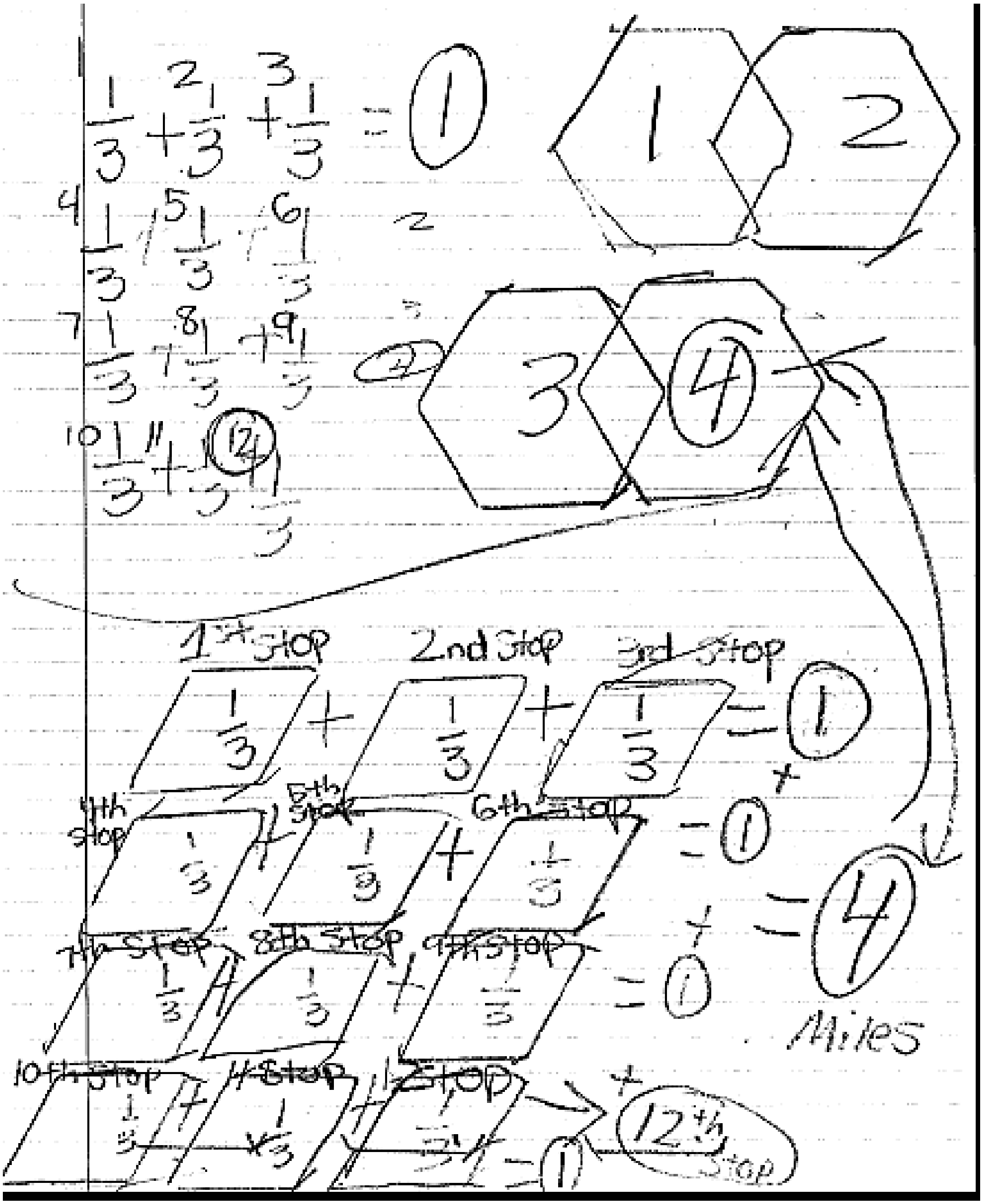
## Group A


1st Stop
2nd Stop
3rd Stop
4th Stop
5th Stop
6th Stop
7th Stop
8th Stop
9th Stop
10th Stop
11 Stop
12 Stop
12th Stop
Miles

# Group B

Bobby said that he wanted to go for a four-mile hike. Bobby stops every $\frac{1}{3}$ mile for a sip of water from his water bottle. How many times does Bobby stop? Be sure to show how you found your answer with both diagrams and an explanation in words. What equations involving fractions match your diagram?

My teacher said to invert and multiply so

$$4 \div \frac{1}{3} = \frac{4}{1} \times \frac{3}{1} = 12$$

so 12 stops total.

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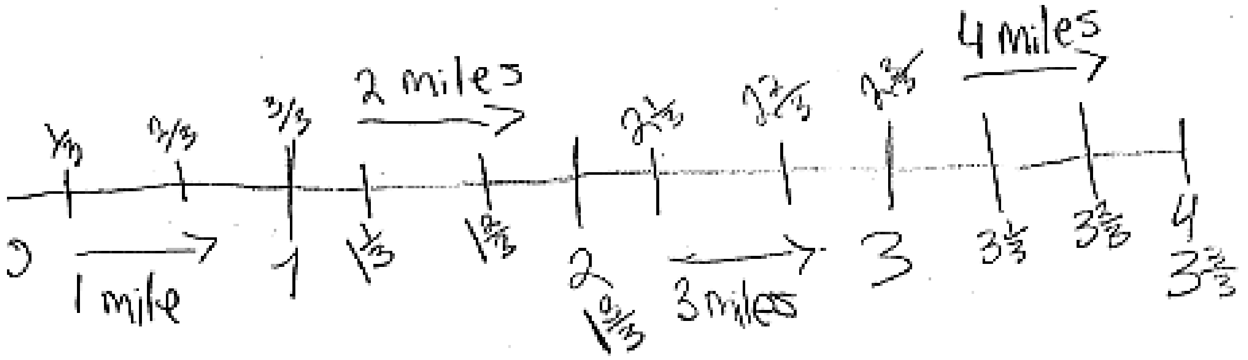
## Group C




What equations involving fractions match your diagram?

$\frac{1}{3} \div 4 = 12$ or $0.33 \div 4.00 = 12^{R4}$

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## Step Back

Looking at the questions that advance student understanding of the mathematical goal, decide which questions give students opportunities to engage in:

- MP2: *Reason abstractly and quantitatively;* and
- MP3: *Construct a viable argument and critique the reasoning of others*.

## What is Reasoning?

People only acquire robust, lasting knowledge if they themselves do the mental work of making sense of it. Good teaching is a matter of arranging for students to do their own knowledge construction, while assuring that the ideas students develop will be in good accord with known facts and established concepts.

Resnick, L.B., Hall, M.W., with the Fellows of the Institute for Learning (2012). *Principles of Learning for effort-based education* (p. 16). Pittsburgh, PA: University of Pittsburgh, Learning Research and Development Center.

# Academic Rigor in a Thinking Curriculum

*Academic Rigor in a Thinking Curriculum* consists of indicators that students are accountable to:

- **a knowledge core;**
- **high-thinking demand; and**
- **active use of knowledge.**

Most importantly, there is an indication that student learning/understanding is advancing from its current state.

Which of the questions we looked at cause students to think rigorously?

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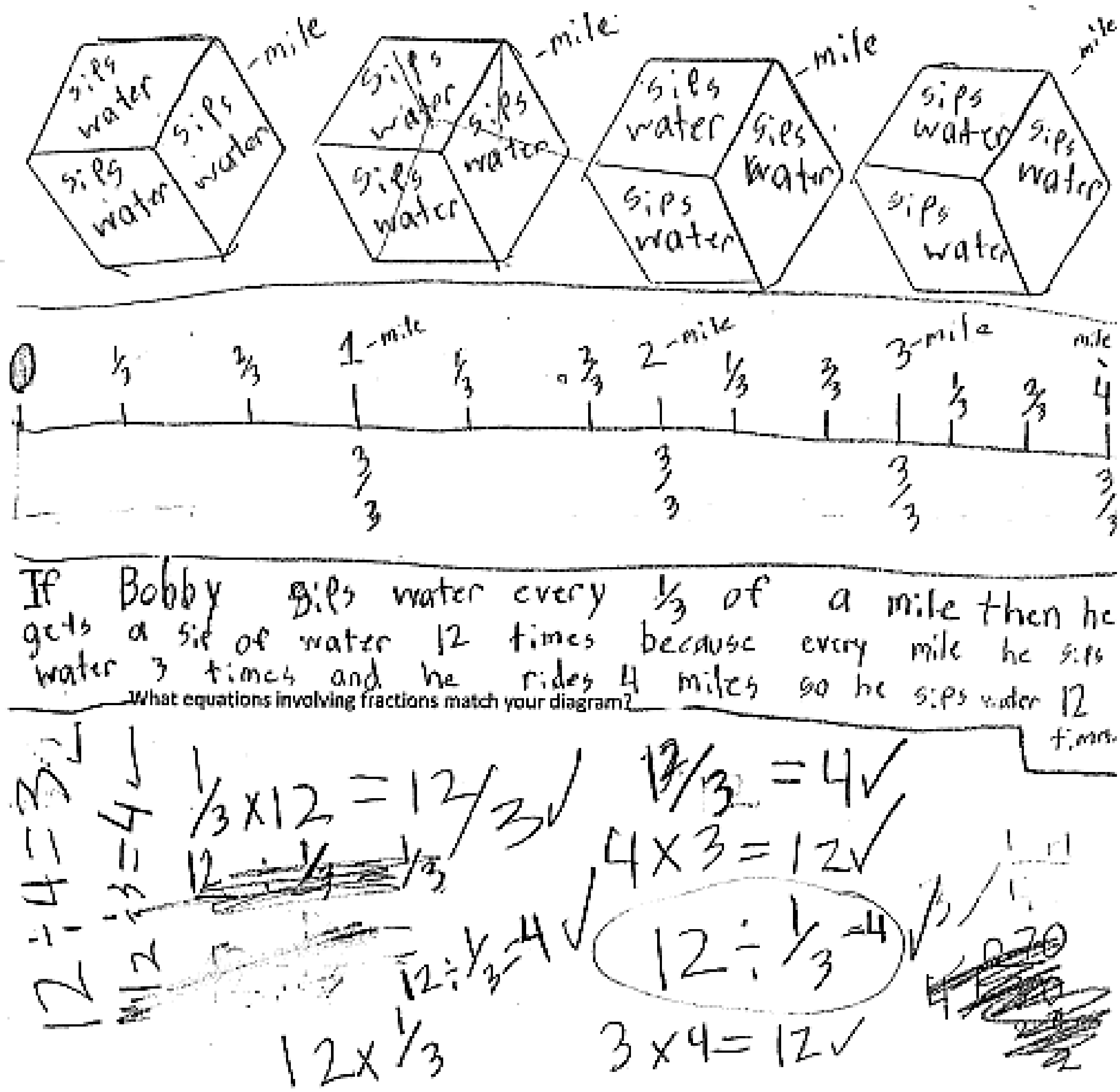
## Give it One More Go . . .

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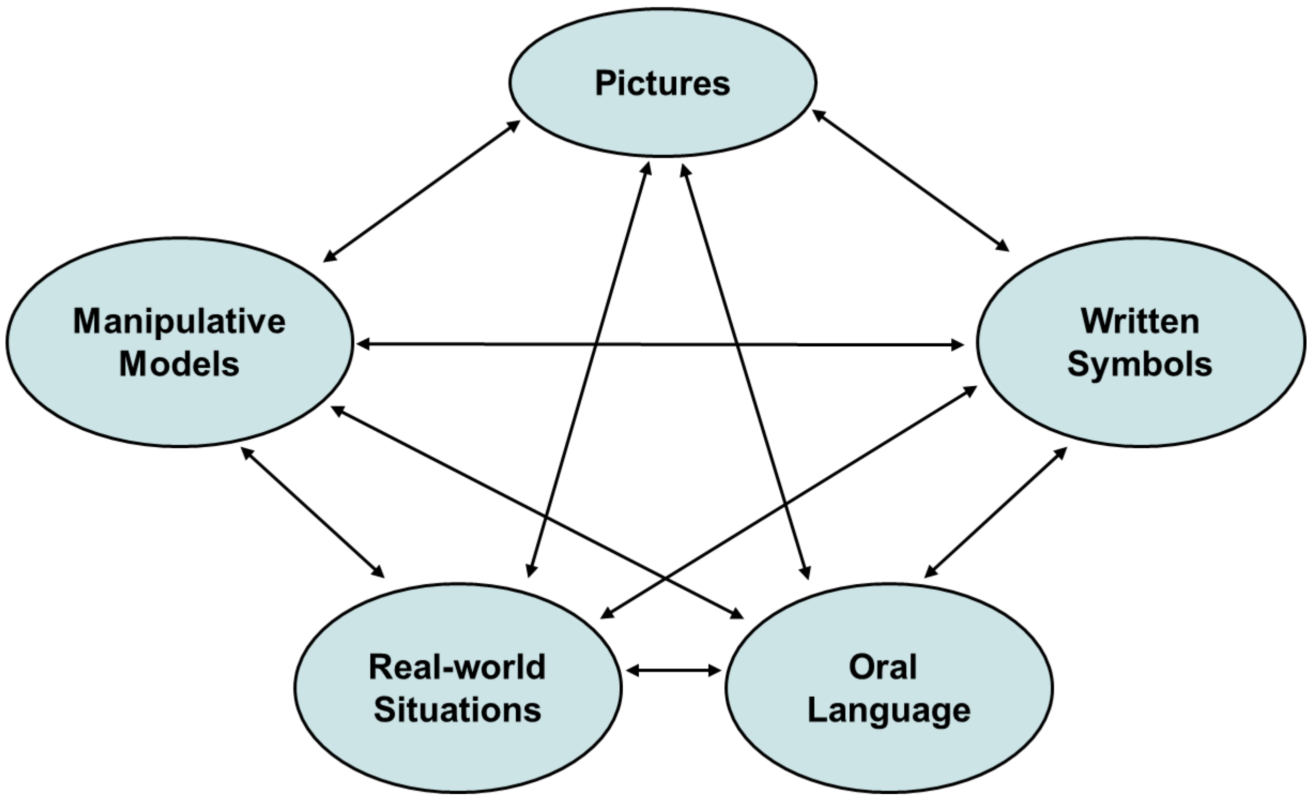
# Five Representations of Mathematical Ideas




Adapted from Lesh, R., Post, T., & Behr, M. (1987). Representation and translations among representations in mathematics learning and problem solving. In C. Janvier, (Ed.), *Problems of representation in the teaching and learning of mathematics* (Ch. 4, pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum Associates.

## Determining Student Understanding

What will you need to see and hear to know that students understand the concepts of a lesson?

As we watch the video, we will be stopping twice. Each time we stop, you will be asked to write the next teacher question in order to press for rigorous thinking.

## Context for the Lesson

| | |
|---|---|
| Visiting Teacher: | Marrie Lasater |
| Principal: | Roseanne Barton |
| School: | John Pittard Elementary School |
| School District: | Murfreesboro, Tennessee |
| Grade: | 5 |
| Date: | February 4, 2013 |

The visiting teacher is a TN Common Core Coach. She is demonstrating a lesson for the classroom teacher, who is interested in gaining a better understanding of ways of encouraging classroom talk.

The students have studied fractions in the past, but not division of fractions. *Everyday Mathematics* is the text used in the classroom.

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# Transcript #1

| | |
|---|---|
| *Student:* | Do you do is zero through ‘cause zero – |
| *Student:* | Yeah and then.- |
| *Student:* | Have one, two, three and each one - one, two, three, one, two, three. |
| *Student:* | What you did is you added all of the – okay, you added all of these equal one whole, then you added one whole equals two whole.  All of that added three whole.  All of that equals four. |
| *Student:* | She didn’t do that.  She did one zero – |
| *Student:* | See, you started it in the middle.  Well, the zero is a one third. |
| *Student:* | True.  That is one |
| *Student:* | No ‘cause it starts at zero like the timeline. |
| *Student:* | I bet if we get a ruler, it would show it. |
| *Student:* | Okay ‘cause the first beginning is like – the first beginning is usually one third and then you go to your – and then you keep counting and it goes to your one whole. So you keep counting and it goes to your two whole.  Three equal parts. |
| *Student:* | Into three equal parts and it didn’t start at zero. |
| *Student:* | Yeah, it didn’t. |
| *Student:* | So that’s how I got to it. |
| *Student:* | No, it started with zero and it ended with one. |
| *Teacher:* | Yeah, it did start with zero.  I think you’re right.  I think I did have a zero on that little – |
| *Student:* | Yeah, one was whole and then zero was  there the one.  But I was thinking – I was thinking that – |
| *Teacher:* | So we haven’t convinced her that we don’t have nine.  Is that what I'm hearing? |
| *Student:* | Yeah ‘cause there’s three in each one and there’s four miles, so – and four times three equal twelve, so that’s how I got twelves. |

**<u>Next teacher question to press for rigorous thinking:</u>**

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# Transcript #2

| | |
|---|---|
| *Teacher:* | Okay. I'm going to start listing the equations that we have so far because I want us to be able to come back and look at them and thinking about what do the numbers mean in the equation. For example, can anybody talk to me about what does the four represent, what does the one third represent and what does the twelve represent? Turn and talk to your neighbor. What do those represent? |
| *[Crosstalk]* | |
| *Teacher:* | What do you think? Who can talk to me about what those numbers mean? Yes, sir? Would you say that again for me? |
| *Student:* | In a different way? |
| *Teacher:* | Well, you can. |
| *Student:* | Well, since he did that equation, four is like the miles that he hiked and the one third is how many times that he stopped to get a sip of water. Then the twelve is how many times that he stopped and that's four miles. |
| *Teacher:* | Okay. Anybody else have any thoughts? Do you all have something you'd like to share? Yes? Okay. |
| *Student:* | Twelve divided by one third equals four. |
| *Student:* | *[Inaudible]*. It would equal 36 'cause I'm pretty sure there's three times into one, six, two, nine, three, twelve – |
| *Teacher:* | Okay. |
| *Student:* | Yeah, it's be 6. |
| *Teacher:* | Okay, I'm seeing he's talking in his group. What is your equation that you're saying? Say it again. |
| *Student:* | Twelve divided by one third. |
| *Teacher:* | Twelve divided by one third. Talk in your groups. What is that meaning of twelve divided by one third. |
| *Student:* | Twelve divided by one third does not equal four, but twelve times one third equals four. |
| *Teacher:* | Hmm. |
| *Student:* | Because one third will go into twelve four – one divided – one third would go into twelve – wait, now I'm getting confused. |

**<u>Next teacher question to press for rigorous thinking:</u>**

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## Preparing to Ask Assessing and Advancing Questions

How does a teacher prepare to ask good, pressing questions?

## Supporting Student Thinking and Learning

In planning a lesson, how does the process of considering how students are likely to respond to a task and developing questions in advance help us press for:

- argumentation;
- reasoning; and
- rigorous thinking?

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