

Learn ← → Reflect Strand
Number and Operations: Be Radical and Get Real!

**Stories, Models, Strategies,
and Algorithms**

Putting Fractions in Real Contexts

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2013 – 2014 Learn ← → Reflect Strand

- What is number sense and how can you promote the development of number sense in your students?
- How are fluency and understanding related in the context of number and operations?
- How can instructional decisions facilitate the development of strategies that are meaningful and transferable for operations on all numbers?
- How are equity and diversity promoted by developing conceptual understandings of number?
- How can the Standards for Mathematical Practice support the development of number sense and computational fluency?
- How are you thinking differently about your learning and teaching of number and operations as a result of participating in the Learn ← → Reflect sessions?

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Stories, Models, Strategies and Algorithms:
Putting Fractions in Real Contexts

GOALS:

- To use **story problems** and **visual fraction models** to develop strategies for making sense of **fraction division** (both partitive and measurement) based on the meanings of numerators and denominators.
- To **connect strategies** for solving **fraction division** problems that use multiplication, addition, subtraction, and counting equal parts.
- To use visual fraction models and real-world problem situations to **explain and demonstrate why** and how “invert and multiply” works for **fraction division**.

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Common Core State Standards - Mathematics:

5.NF.B.7.

- Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

5.NF.B.7.c.

- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

6.NS.A.1.

- Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

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STORIES and STRATEGIES

Two basic types of **equal groups division** stories:

- Knowing how many groups and so you need to **find**
 - **how much goes into each group or**
 - **how much goes into one whole group**

$5 \div \frac{1}{2}$

- Knowing the size of each group so you need to **find**
 - **how many groups can be made or**
 - **how much of a group**

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STORY: Partitive Division
“Knowing how many groups” strategy:

- **Split into equal groups and**
- **FIND how much goes into each group** (find a single unit)

There is $\frac{1}{3}$ cup of lemonade.
I use it to fill 2 glasses.
Both glasses have the same amount of lemonade.
How much lemonade is in each glass?

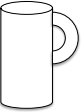
$\frac{1}{3} \div 2$

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STORY: Partitive Division
 “Knowing part of one group” strategy:

- **FIND how much goes into one whole group**
 (complete one whole group)

There are two cups of juice.
 They fill the pitcher $\frac{1}{3}$ full.
 How many cups of juice fill the pitcher?




$$2 \div \frac{1}{3}$$

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STORY: Measurement Division
 “Knowing how much to put in each group” strategy:

- **Split given amount into groups of given size**
- **FIND how many groups can be made**
 (how many times it fits into a given amount)

There are 2 cups of lemonade in the pitcher.
 Each glass holds $\frac{1}{3}$ cup of lemonade.
 How many glasses of lemonade can be filled?




$$2 \div \frac{1}{3}$$

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STORY: Measurement Division
 “Knowing the part of a part” strategy:

- **Show the part of a part of the whole**
- **FIND how much of a whole**

There is $\frac{1}{3}$ cup of lemonade in the pitcher.
 Each pitcher holds 2 cups of lemonade.
 How full is the pitcher?
 (The lemonade fills what fraction of the pitcher?)

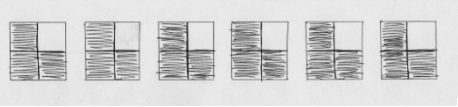


$$\frac{1}{3} \div 2$$

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STORIES, MODELS and STRATEGIES

A 6-ounce package of peanuts says that one serving is $\frac{3}{4}$ of an ounce. How many servings are in the package?

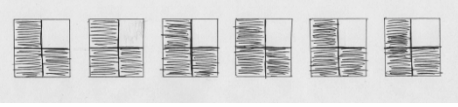


$$6 \div \frac{3}{4} = \frac{24}{4} \div \frac{3}{4} = \frac{24 \div 3}{4 \div 4} = \frac{24 \div 3}{1} = 24 \div 3 = 8$$

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STORIES, MODELS and STRATEGIES

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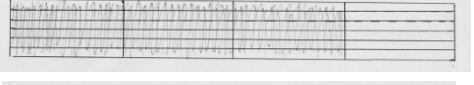


$$6 \div \frac{3}{4} = 6 \times \frac{4}{3} = \frac{6 \times 4}{3} = \frac{24}{3} = 24 \div 3 = 8$$

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STORIES, MODELS and STRATEGIES

Three-fourths of the lasagna is left in the refrigerator. There are 6 of us who share it for supper. How much of the lasagna does each of us get to eat?



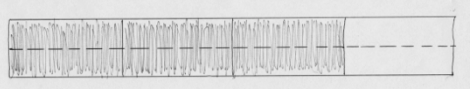
$$\frac{3}{4} \div 6 = \frac{3}{4} \div \frac{24}{4} = \frac{3 \div 24}{4 \div 4} = \frac{3 \div 24}{1} = \frac{3}{24} = \frac{1}{8}$$

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STORIES, MODELS and STRATEGIES

Three-fourths of the lasagna is left in the refrigerator. There are 6 of us who share it for supper. How much of the lasagna does each of us get to eat?




$$\frac{3}{4} \div 6 = \frac{1}{6} \times \frac{3}{4} = \frac{1}{\cancel{3} \times 2} \times \frac{\cancel{3}}{4} = \frac{\cancel{3}}{\cancel{3}} \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$


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STORIES, MODELS and STRATEGIES


$$4 \div \frac{3}{5}$$


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STORIES, MODELS and STRATEGIES


$$\frac{3}{5} \div 4$$


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STORIES, MODELS and STRATEGIES


$$\frac{3}{4} \div \frac{3}{5}$$


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STORIES, MODELS and STRATEGIES

$$\frac{3}{5} \div \frac{3}{4}$$


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STORIES, MODELS and STRATEGIES

I have $\frac{3}{4}$ of an hour to do 3 chores.
How much time can I spend on each chore?


I have $\frac{2}{3}$ of an hour to do 4 chores.
How much time can I spend on each chore?


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STORIES, MODELS and STRATEGIES 


I walked $\frac{3}{4}$ of a mile in $\frac{1}{4}$ of an hour.
How far could I walk in 1 hour?


I rode my bike for $\frac{2}{3}$ of an hour and went 5 miles. How far could I ride in 1 hour?

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STORIES, MODELS, and ALGORITHMS 


- Common denominator algorithm
- Divide Across method
- Invert and Multiply algorithm (a.k.a. Freeze Change Flip)


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STORIES, MODELS, STRATEGIES and ALGORITHMS: 
Putting Fractions in Real Contexts

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THANK YOU!

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