1. Make sense of problems and persevere in solving them
6. Attend to precision

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others

4. Model with mathematics

5. Use appropriate tools strategically

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Introduction to the CME Project

CME PROJECT

The CME Project, developed by EDC's Center for Mathematics Education, is a new NSF-funded high school program, organized around the familiar courses of algebra 1, geometry, algebra 2, and precalculus. The CME Project provides teachers and schools with a third alternative to the choice between traditional texts driven by basic skill development and more progressive texts that have unfamiliar organizations. This program gives teachers the option of a problem-based, student-centered program, organized around the mathematical themes with which teachers and parents are familiar. Furthermore, the tremendous success of NSF-funded middle school programs has left a need for a high school program with similar rigor and pedagogy. The CME Project fills this need.

The goal of the CME Project is to help students acquire a deep understanding of mathematics. Therefore, the mathematics here is rigorous. We took great care to create lesson plans that, while challenging, will capture and engage students of all abilities and improve their mathematical achievement.

The Program's Approach

The organization of the CME Project provides students the time and focus they need to develop fundamental mathematical ways of thinking. Its primary goal is to develop in students robust mathematical proficiency.

- The program employs innovative instructional methods, developed over decades of classroom experience and informed by research, that help students master mathematical topics.
- One of the core tenets of the CME Project is to focus on developing students' Habits of Mind, or ways in which students approach and solve mathematical challenges.
- The program builds on lessons learned from high-performing countries: develop an idea thoroughly and then revisit it only to deepen it; organize ideas in a way that is faithful to how they are organized in mathematics; and reduce clutter and extraneous topics.
- It also employs the best American models that call for grappling with ideas and problems as preparation for instruction, moving from concrete problems to abstractions and general theories, and situating mathematics in engaging contexts.
- The CME Project is a comprehensive curriculum that meets the dual goals of mathematical rigor and accessibility for a broad range of students.

About CME

EDC's Center for Mathematics Education, led by mathematician and teacher **Al Cuoco**, brings together an eclectic staff of mathematicians, teachers, cognitive scientists, education researchers, curriculum developers, specialists in educational technology, and teacher educators, internationally known for leadership across the entire range of K–16 mathematics education. We aim to help students and teachers in this country experience the thrill of solving problems and building theories, understand the history of ideas behind the evolution of mathematical disciplines, and appreciate the standards of rigor that are central to mathematical culture.

1.0 Habits of Mind

If you watch college sports on television, you may have seen this message from the National Collegiate Athletic Association (NCAA).

> There are over 380,000 student-athletes, and just about every one of them will go pro in something other than sports.

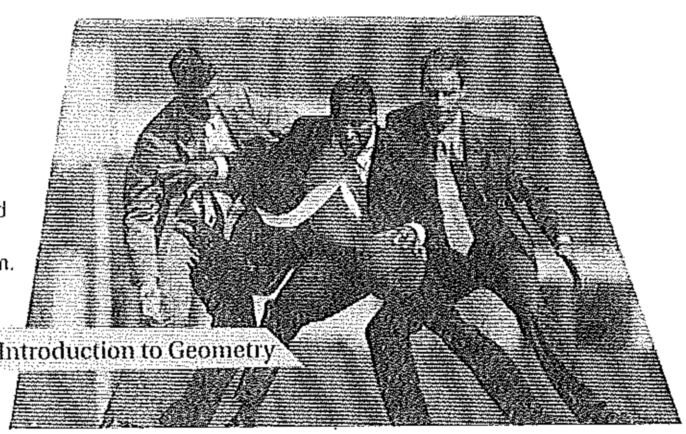
For practitioners of mathematics (mathletes), a similar message could say

> There are over 30,000,000 student-mathletes in this country, and just about every one of them will go pro in something other than mathematics.

So, why play sports? Why study mathematics? Simple. In either venture, the habits you learn have value for the rest of your life. Compare these lists.

Habits of Body	Habits of Mind
Athletics	**Mathematics**
Take care of the body.	Take care of the mind.
Think ahead.	Think ahead.
Build strength.	Build mental agility.
Take a chance.	Take a chance.
Develop conditioning.	Develop persistence.
Respect the rules.	Respect the rules.
Practice mental toughness.	Practice mental discipline.
Visualize perfection.	Visualize relationships.
Plan strategies.	Plan strategies.
Have confidence.	Have confidence.
Model the opponent.	Model the problem.
Study styles.	Look for patterns.
Work as a team.	Work with others.
and many more . . .	**and many more . . .**

To get you started this year, try some of the following activities. As you proceed (and throughout the course), think about how you are thinking. Pay attention to your habits of mind!



Good habits of body will help you on and off the court. Good habits of mind will help you inside and outside the classroom.

CME PROJECT

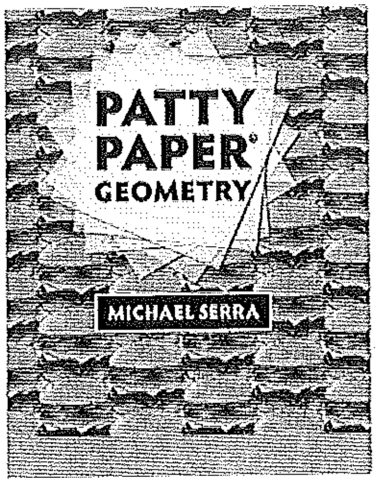
Geometry

PEARSON

Boston, Massachusetts
Chandler, Arizona
Glenview, Illinois
Shoreview, Minnesota
Upper Saddle River, New Jersey

EDC

Education Development Center, Inc.
Center for Mathematics Education
Newton, Massachusetts



Flip to back

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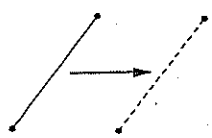
Written by Michael Serra, the best selling author of *Discovering Geometry*, *Patty Paper Geometry* contains 12 chapters of guided and open investigations. Open investigations encourage students to explore their own methods of discovery, and guided investigations provide more direction to students. Use *Patty Paper Geometry* as a supplement to your geometry program or even as a major course of study. Author: Michael Serra, Pages: 262, paperback, Publisher: Playing It Smart, ISBN: 978-1559530723

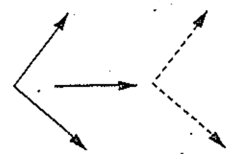
FOLDING THE BASIC GEOMETRIC CONSTRUCTIONS

As mentioned in the historical note, Greek geometers experimented with compass and straightedge constructions over 2,000 years ago. Euclid was the first to organize geometry into a logical structure in the *Elements,* his book on mathematics. The seven basic Euclidean Constructions listed below make it possible to create all of high school plane geometry.

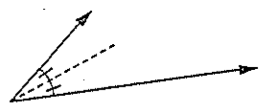
EUCLIDEAN CONSTRUCTIONS



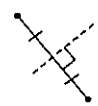
To duplicate a given segment. (C-1)



To duplicate a given angle. (C-3)

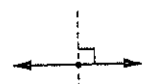


To construct the bisector of an angle.

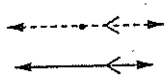


To construct a perpendicular bisector of a given segment.

To construct a perpendicular from a point to a line.



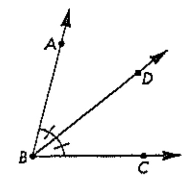
To construct a perpendicular through a point on a line.



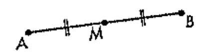
To construct a line through a given point parallel to a given line.

In these investigations you will discover how to use patty papers to perform these Euclidean Constructions. The first two constructions are patty paper construction properties (C-1 and C-3) you've already seen in the introductory lesson. You will discover how to do the remaining five in this lesson. You will need to know the definition of a bisector for these constructions.

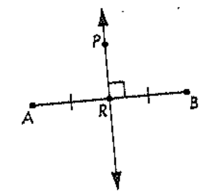
Definition: A **bisector** divides something into two congruent parts. An **angle bisector** is a ray that divides an angle into two congruent angles. A **midpoint** is a point that divides a line segment into two congruent segments. A **perpendicular bisector** is a line that is perpendicular to a line segment and also divides the segment into two congruent segments.



$\overrightarrow{BD}$ is the **angle bisector** of $\angle ABC$.



M is the **midpoint** of $\overline{AB}$.

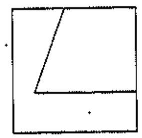


$\overleftrightarrow{PR}$ is the **perpendicular bisector** of $\overline{AB}$.

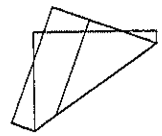
Guided Investigation 2.1

FOLDING AN ANGLE BISECTOR

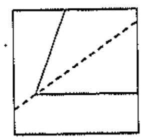
Step 1: Use your straightedge to draw an angle on a patty paper.



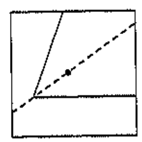
Step 2: Fold one side of the angle on top of the other.



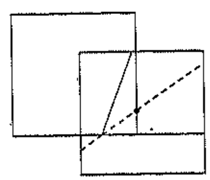
Step 3: Does the crease divide the angle into two congruent parts? Check this by tracing one of the smaller angles and placing the copy on top of the other smaller angle.



Step 4: Draw a point on the angle bisector. What is special about this point? Experiment with your patty paper to find out.



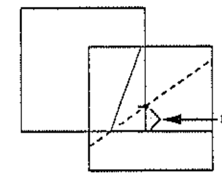
Step 5: Place one edge of a second patty paper on one of the sides of the angle. Slide the edge of the patty paper along the side of the angle until an adjacent perpendicular side of the patty paper passes through the point.



Guided Investigation 2.1 continued

FOLDING AN ANGLE BISECTOR

Step 6: Mark this distance on the patty paper, and then compare this distance with the distance to the other side by repeating the previous step on the other side of the angle.



What is true about the distance from a point on an angle bisector to each of the two sides of the angle? Is the point closer to one side or the other?

Your next patty paper conjecture could be:

PPC 5 If a point is on the bisector of an angle, then it is equally distant from the ____________ ________________________.

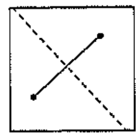
This conjecture gives you a way to find a point that is the same distance from two sides of an angle. In a later investigation you will use this conjecture to find the center of the inscribed circle of a triangle.

Guided Investigation 2.2

FOLDING THE PERPENDICULAR BISECTOR OF A LINE SEGMENT

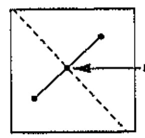
Step 1: Use your straightedge to draw a line segment on a patty paper.

Step 2: Fold the paper so one endpoint lies on top of the other. Open it up.



Does the crease divide the segment into two equal parts?

Step 3: Place a point where the crease intersects the segment. This point is called the midpoint of the segment.

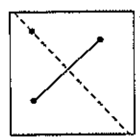


Step 4: Use a corner of another patty paper to check if the angles formed by the crease and the given segment are right angles.

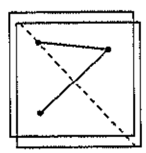
This is a very useful construction. You can use this method to find midpoints, construct perpendiculars, make 90 degree angles, and divide segments into equal parts. If you wish to create just a midpoint, bring the endpoints of the segment together, and pinch!

How can you describe the points on the perpendicular bisector? What is special about each point on the perpendicular bisector of a segment?

Step 5: Place a point on your folded perpendicular bisector.



Step 6: Compare the distances from the point to each endpoint. You can do this by using a second patty paper to measure the distance between the point and one endpoint and then comparing this distance with the distance between the point and the other endpoint.



Guided Investigation 2.2 continued

FOLDING THE PERPENDICULAR BISECTOR OF A LINE SEGMENT

What is true about the distances from the point on the perpendicular bisector to each of the two endpoints of the segment? Is the point closer to one endpoint or the other?

Your next patty paper conjecture could be:

PPC 6 If a point is on the perpendicular bisector of a segment, then it is equally distant from the ______________________.

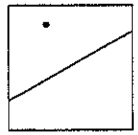
In a later investigation you will use this conjecture to find the center of the circumscribed circle of a triangle.

Guided Investigation 2.3

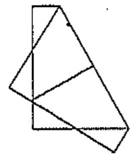
FOLDING A PERPENDICULAR FROM A GIVEN POINT TO A GIVEN LINE

Here is one way to do this construction. You may want to experiment to find other ways.

Step 1: Fold or draw a line on a patty paper. Place a dot on your patty paper to represent the given point.



Step 2: Fold the line on top of itself so that the fold passes through the given point.



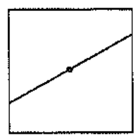
Step 3: Use a corner of another patty paper to check if the angles formed by the crease and the given line are right angles.

This construction allows you to fold an altitude. It is also a way to determine the shortest distance from a point to a line.

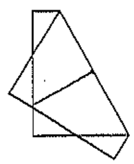
Guided Investigation 2.4

FOLDING A PERPENDICULAR THROUGH A POINT ON A LINE

Step 1: Start with a line and a point on the line.



Step 2: Fold the patty paper so that the crease goes through the point and one side of the line lies on top of the other.



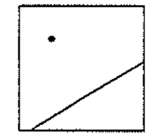
Step 3: Use a corner of another patty paper to check if the angles formed by the crease and the given line are right angles.

This construction allows you to fold a right angle. Many polygons contain right angles. Look around! Right angles are used more than any other angle.

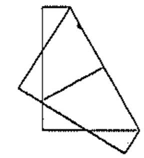
Guided Investigation 2.5

FINDING A LINE PARALLEL TO A GIVEN LINE THROUGH A GIVEN POINT

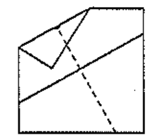
Step 1: Fold your patty paper to create a given line or use your straightedge to draw a line. Draw a point which is not on the given line.



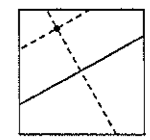
Step 2: Fold the given line onto itself so that the fold passes through the given point. The line formed by this fold is perpendicular to the given line.



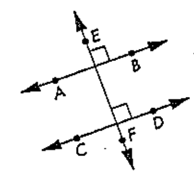
Step 3: Unfold the paper and fold the new line onto itself to form another line perpendicular to the second line. Be sure this last folded line goes through the point.



Step 4: Unfold and you will have two lines (the given line and the last folded line) that are parallel to each other and perpendicular to the second line.



In this investigation you used the fact that if two lines ($\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$) are perpendicular to the same line ($\overleftrightarrow{EF}$) and all three lines lie on the same plane, then the two lines ($\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$) must be parallel. This is an important idea that will be used in future investigations.



This construction will allow you to create parallelograms, rectangles, and other figures with parallel lines. Try folding a parallelogram on your patty paper. You will discover properties of parallelograms in a later lesson.

1.6 Compasses, Angles, and Circles

Geometers distinguish between a drawing and a construction. You make a drawing to aid memory, thought, or communication. A rough sketch serves this purpose quite well. On the other hand, a **construction** is a guaranteed recipe. A construction shows how, in principle, to accurately draw a figure with a specified set of tools.

Drawings are aids to problem solving. Constructions are solutions to problems.

In your study of geometry you will probably use both hand construction tools and computer tools. The computer tools are introduced in the next investigation.

Hand Construction Tools

Compass A compass is any device—even a knotted piece of string—that allows you to move a pencil a fixed distance around a certain point. A compass allows you to copy distances and to construct circles of any size that you can place anywhere.

Straightedge An object with a straight edge—even a piece of paper—helps you draw a segment to look straight. In general, a straightedge is unmarked and you cannot use it to measure distances. You can use a straightedge to draw a line through, or a segment between, two points. You can also use a straightedge to extend a drawing of a line.

Remember...

You use a ruler to draw straight segments and to measure distances. You can also use a ruler as a straightedge, ignoring its markings.

Measuring devices Rulers and protractors are measuring devices. You can use a ruler to measure the length of a segment or the distance between two points. You can use a protractor to measure an angle.

Paper Paper is not just a surface on which to write and draw. You can use the symmetries formed by folding paper to construct geometric figures creatively. You can also use dissection—the process of cutting paper figures and rearranging their parts—as a powerful aid to reasoning.

String You can use string and tacks to build devices that you can use to construct circles, ellipses, spirals, and other curves.

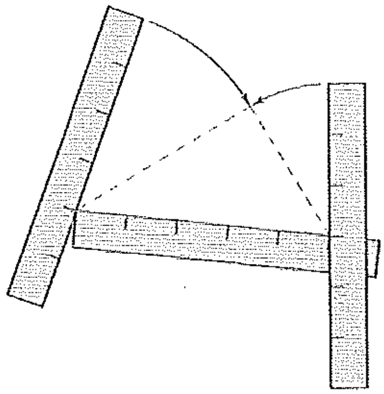
Minds in Action — episode 1

Sasha and Tony are trying to draw a triangle with side lengths 3 in., 4 in., and 5 in.

Sasha I'm going to use three rulers to draw this triangle.

Tony Why *three* rulers?

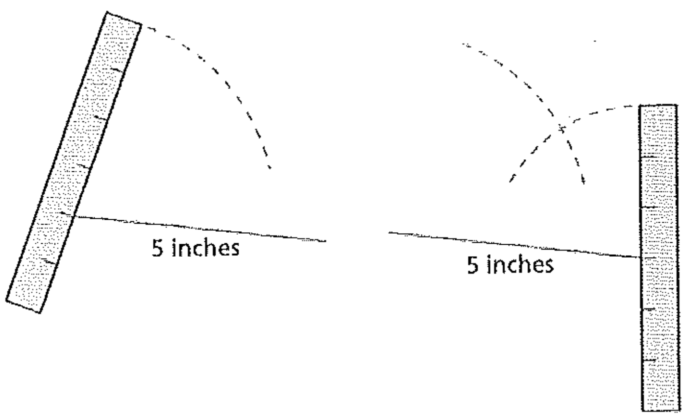
Sasha Watch and learn, Tony. First, I'll draw a segment that is one of the given lengths, say the 5-inch segment. Then I'll use the other two rulers to represent the other two sides of the triangle and swing them toward each other until they meet. I'll connect the point where the two rulers meet to each end of the 5-inch segment. That gives me my triangle.



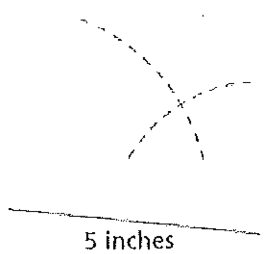
Tony Very nice, but I bet I can do the same thing with only one ruler.

Sasha Let's see!

Tony Okay. First I'll start just like you did and draw the 5-inch segment. Then I'll put the ruler at one end of the 5-inch segment, mark off 4 inches, and swing the ruler around with my pencil to sketch an arc.



Then I'll put the ruler at the other end of the 5-inch segment, mark off the 3-inch side, and swing the ruler around to make another arc. And, voilà, there's my triangle!



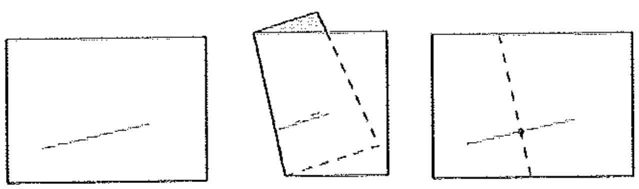
For Discussion

1. Do you see Tony's triangle? Explain how you know his triangle has the correct side lengths.
2. The title of this lesson is *Compasses, Angles, and Circles*, but this lesson has not yet mentioned compasses, angles, and circles. How do Sasha's and Tony's ruler tricks both imitate a compass?
3. How can a compass make a geometric construction easier?

Example

Problem Draw a line segment. Without measuring, construct its midpoint. A **midpoint** is the point on a segment that is halfway between the two endpoints.

Solution The simplest approach is to use symmetry by folding. Fold the segment so that its endpoints lie on top of each other. This matches its two halves exactly. The point that separates the two halves is the midpoint.



In fact, all points on the fold line are equidistant from the two endpoints of the segment. This is easier to see when the paper is folded. Any point on the fold is the same distance from each of the two original endpoints, because the endpoints are now at the same place.

Equidistant means "the same distance."

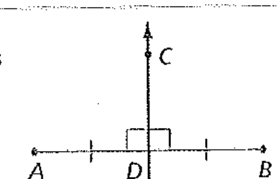
The fold line is also perpendicular to the segment. You can show this by

- matching angles around the bisector to show they are congruent
- showing that the sum of the measures of the adjacent angles is 180°

Therefore, the fold line is the perpendicular bisector of the segment. The **perpendicular bisector** of a segment is a line that is perpendicular to a segment at the segment's midpoint.

Theorem 1.1 Perpendicular Bisector Theorem

Each point on the perpendicular bisector of a segment is equidistant from the two endpoints of the segment.



$CA = CB$ and $DA = DB$

A theorem is a statement that has been proven. Although you will not prove the Perpendicular Bisector Theorem or its converse until Chapter 2, you can use both as theorems now.

6.6 Some Special Triangles

Every triangle has three side lengths and three angle measures. In Chapter 2, you saw that you do not have to know all six measurements to prove that two triangles are congruent. In fact, you only need to know three of the measurements—all three side lengths (SSS), two sides and the included angle (SAS), or two angles and one side length (ASA and AAS). There must be some way to figure out what the other three measurements are from the ones you know.

Minds in Action — episode 25

Tony and Sasha are talking about situations in which they could use three triangle facts to determine the others.

Tony If you just know three sides of a triangle, how could you ever figure out what the angles are without constructing it?

Sasha Well, I don't know how to do it for *every* triangle. I bet I can give you three facts that you could use to solve for the other three.

Here's one I *know* you know. What if $\triangle ABC$ has three sides that all measure 6.17 cm? What are its angle measures?

Tony Okay. That one I get. So I agree that there are some special cases that are possible to do. How about a side-angle-side set?

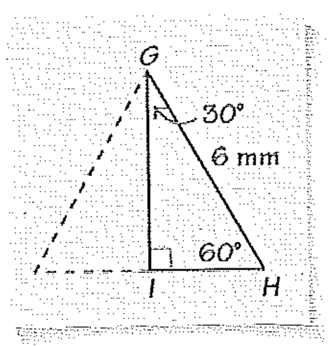
Sasha How about $\triangle DEF$ with $DE = 4$ in., $EF = 4$ in., and $m\angle E = 90°$?

Tony With two sides of a right triangle, I can always figure out the length of the third side, but how can I figure out the other two angle measures? Oh, it's an isosceles right triangle.

Now I've got an angle-side-angle set for you, Sasha: $\triangle GHI$ where $m\angle G = 30°$, $m\angle H = 60°$, and $GH = 6$ mm.

Sasha That one's tricky, but at least it must be a right triangle. I'll draw a sketch.

Hey! This is just half of an equilateral triangle!



For You to Do

Answer these questions about the several triangles that were mentioned in the Minds in Action.

1. What are the measures of the angles of $\triangle ABC$? Explain.
2. What are the missing measures of $\triangle DEF$? Explain.
3. What are the missing measures of $\triangle GHI$? Explain.
4. Create a new problem in which you specify only three measurements for a triangle. Make sure the other three measurements can be determined from the given information. Exchange problems with a classmate.

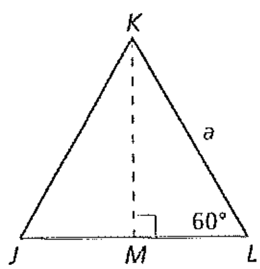
In the Minds in Action, Tony and Sasha discussed an isosceles right triangle and a triangle that is half of an equilateral triangle. You already know many properties of these two triangles. Now it is time to learn how the angle measures and the three side lengths in each are related.

A 30-60-90 triangle is a right triangle that is half of an equilateral triangle.

Example

Problem An altitude of an equilateral triangle cuts it into two right triangles. If the side length of the original triangle is a, find all the side lengths and angle measures for one such right triangle.

Solution Here is a diagram of the situation.



The measure of $\angle KLM = 60°$, because $\angle KLM$ is one of the angles of the original equilateral triangle. You also know that $m\angle LMK = 90°$, because $\overline{KM}$ is an altitude of the equilateral triangle. $m\angle MKL = 30°$, because the sum of the three angle measures is $180°$.

$KL = a$, because $\overline{KL}$ is one of the sides of the original equilateral triangle. $LM = \frac{a}{2}$, because the altitude of an isosceles triangle bisects the base. You can find KM by using the Pythagorean Theorem.

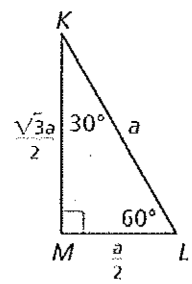
$$KM^2 + LM^2 = KL^2$$

$$KM^2 + \left(\frac{a}{2}\right)^2 = a^2$$

$$KM^2 = \frac{3a^2}{4}$$

$$KM = \frac{\sqrt{3}a}{2}$$

Here is the triangle with all of its angle measures and side lengths marked. Learn to look for and recognize triangles with these angles, or with side lengths that are proportional to those in this triangle.



You can also call this triangle a 30-60-90 triangle. The ratio of side lengths in a 30-60-90 triangle is $1 : \sqrt{3} : 2$.

Habits of Mind

Prove. If you did not remember that the altitude of an isosceles triangle must bisect its base, you could quickly prove it using AAS congruence of the two right triangles.

For You to Do

A diagonal of a square cuts it into two congruent right triangles.

5. If the side length of the square is a, find all the side lengths and angle measures for one such right triangle.

A right triangle that is half of a square is an isosceles right triangle. You can also call it a 45-45-90 triangle. Learn to look for and recognize triangles with these angles, or with side lengths that are proportional to those of an isosceles right triangle. The ratio of side lengths in a 45-45-90 triangle is $1 : 1 : \sqrt{2}$.

Investigation 3A

Cut and Rearrange

In *Cut and Rearrange,* you will cut a given shape into parts. Then you will rearrange the parts to form a second shape. Cutting and rearranging will not convert a given shape into just any other imaginable shape. For example, you would not expect the resulting shape to have greater or lesser area. Chih-Han Sah was a mathematician who specialized in dissections of this sort. He called two figures that can be cut into each other scissors-congruent.

By the end of this investigation, you will be able to answer questions like these.

1. What is an algorithm?
2. Why is it important to justify each step in an algorithm?
3. What does the Midline Theorem say about the relationship between a midline and the sides of a triangle?

You will learn how to

- devise and follow algorithms to dissect and rearrange one figure into an equal-area figure
- justify each cut in a dissection
- write general algorithms for dissections
- test algorithms with standard and extreme cases

You will develop these habits and skills:

- Visualize ways to make equal-area figures through dissection.
- Write clear and precise algorithms.
- Reason by continuity to identify extreme cases.

The total area remains the same no matter how you arrange the pieces.

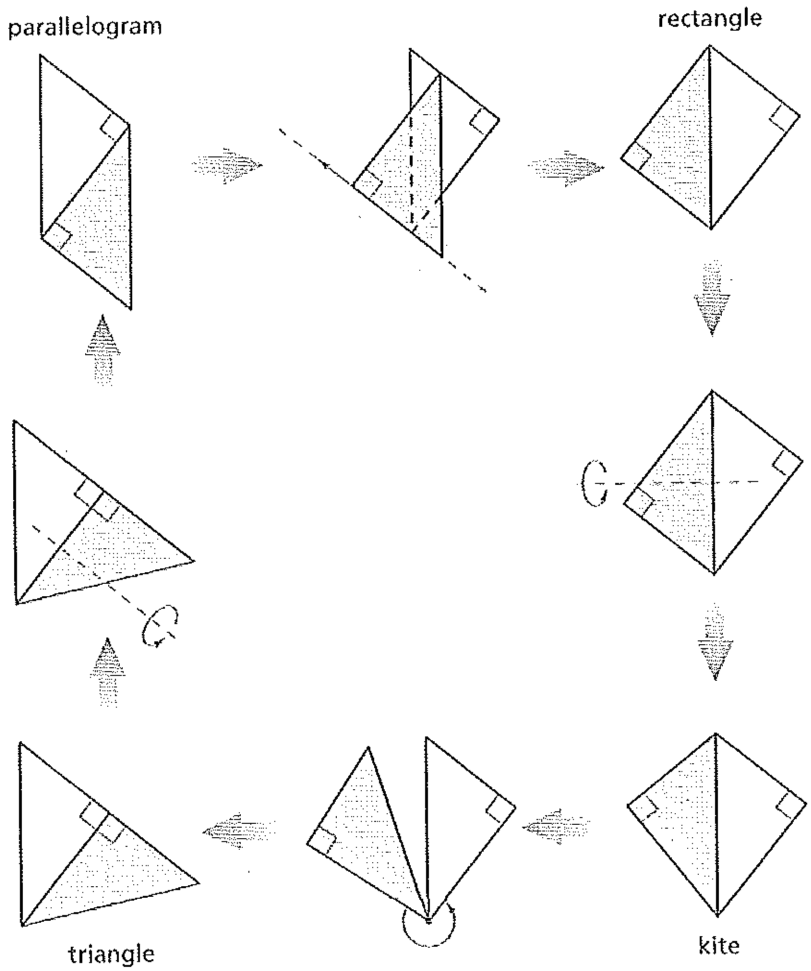
3.1 Getting Started

Activating Prior Knowledge
Exploring New Ideas

Geometric language helps you clearly describe how to move pieces to change one shape to another. Assume that you have a shape made of two congruent right triangles. You can use three types of moves to change the shape into another.

In the figure at the upper left, the two right triangles form a parallelogram. Moving clockwise, you can

- slide the gold triangle parallel to a base of the parallelogram to form a rectangle
- reflect the gold triangle across the dashed line to form a kite
- rotate the gold triangle about a vertex to form a triangle
- reflect the gold triangle across the dashed line to form the original parallelogram



For You to Explore

For Problems 1–5, start with the shape described. (Your teacher may give you copies of shapes to work with.) Cut this shape into pieces that you can rearrange to form the second shape described in the problem. The two shapes are **scissors-congruent.** Write a complete description of the cuts you made and how you moved the pieces.

1. Start with a parallelogram. Cut and rearrange to form a rectangle.
2. Start with a right triangle. Cut and rearrange to form a rectangle.
3. Start with a (not right) scalene triangle. Cut and form a parallelogram.
4. Start with a (not right) scalene triangle. Cut and form a rectangle.
5. Start with a trapezoid. Cut the trapezoid and form a rectangle.

Each problem can be solved in more than one way. Work with classmates to find a few solutions. Then pick one that you like best. Save your written work. You will need it later.

Remember...

A scalene triangle has no two sides the same length.

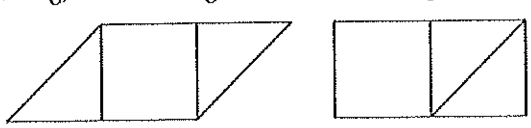
Exercises *Practicing Habits of Mind*

On Your Own

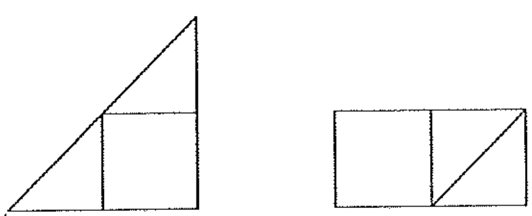
6. Study each pair of figures below.

   Describe how to turn the first shape into the second shape by reflecting, rotating, or translating one or more of the pieces.

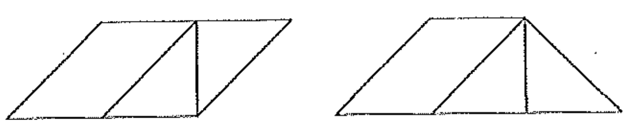
   a.



   b.



   c.



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3.2 Do the Cuts Really Work?

Have you ever expected a dissection to work, but then discovered that the pieces did not quite fit? Or perhaps the pieces looked like they fit, but you found it difficult to be sure?

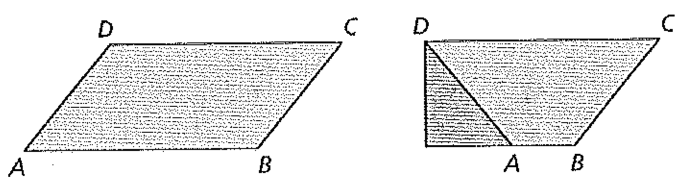
A dissection involves cutting something into pieces. In this book, a dissection also involves rearranging those pieces.

To understand why a dissection works, you must know properties of the shapes you are cutting. Here are some properties of parallelograms that you learned in Chapter 2.

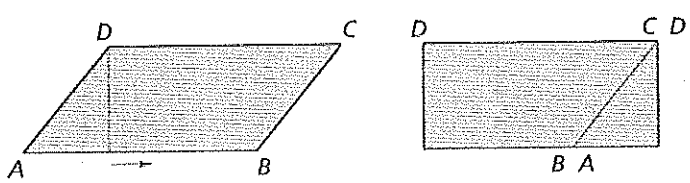
- Parallelograms have exactly four sides.
- Opposite sides are parallel.
- Opposite sides are congruent.
- Opposite angles are congruent.
- Consecutive angles are supplementary.
- The diagonals bisect each other.

Here is Tony's method for dissecting a parallelogram into a rectangle.

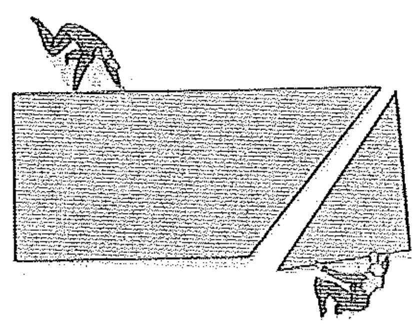
Cut out the parallelogram. Make a fold through vertex D so that A lines up on the bottom, on $\overline{AB}$.



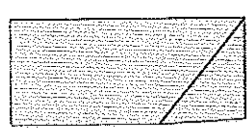
Then unfold and cut along the crease. Slide the triangular piece along $\overline{AB}$ so that $\overline{AD}$ matches up with $\overline{BC}$ and you have a rectangle.



Tony cut two pieces: a triangle and a trapezoid. He then rearranged those pieces. But what guaranteed that the rearrangement had four sides? Here are two ways that his dissection might fail.



The newly glued edges might not match.



The new bottom edge might be crooked.

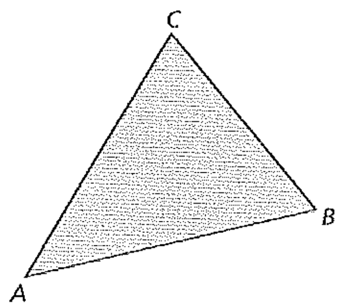
For Discussion

In Tony's method on the previous page, you slide the triangle to the opposite side of the trapezoid. Explain how the properties of parallelograms guarantee each of the following.

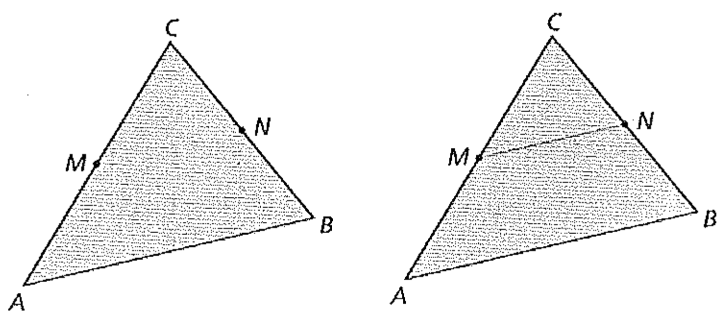
1. The two pieces fit together exactly.
2. The new bottom edge is straight.

Minds in Action episode 5

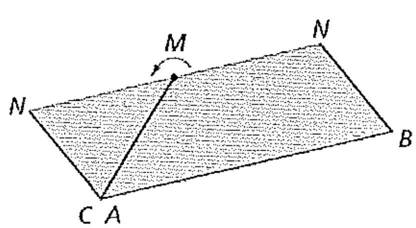
Sasha and Derman are trying to dissect $\triangle ABC$ into a parallelogram.



Sasha A midline is parallel to one side of the triangle, so let's make one. Find the midpoint of $\overline{AC}$. Name it M. Find the midpoint of $\overline{BC}$. Name it N. Cut along $\overline{MN}$.

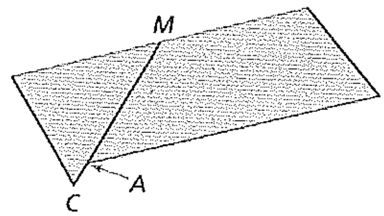


Derman I see where you're going with this. You can rotate $\triangle MCN$—the triangle you just cut off—around M, until $\overline{MC}$ matches up with $\overline{MA}$.



Sasha Now we have a parallelogram, right?

Derman I guess we do, but what if it just *looks* like a parallelogram? What if our scissors techniques aren't accurate and the picture should really look like this?



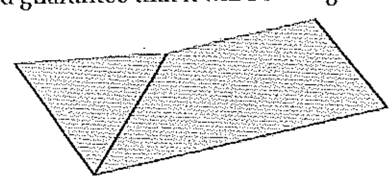
Sasha Good try Derman. But the steps we took *guarantee* that we can stick those pieces together.

See, since M is the midpoint of $\overline{AC}$ in the original triangle, MC must equal MA. So, sides $\overline{MC}$ and $\overline{MA}$ fit together exactly.

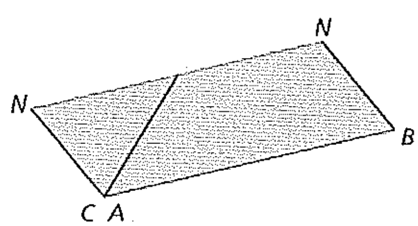
Derman Aaaahhh. It's a proof!

For Discussion

3. If the final figure is really a parallelogram, then the top edge must be straight. It must not look like the picture below. How does Sasha and Derman's method guarantee that it will be straight?



4. If the final figure is really a parallelogram, then it must be true that opposite sides are congruent. How does Sasha and Derman's method guarantee that $\overline{CN} \cong \overline{BN}$? That $\overline{AB} \cong \overline{NN}$?



Later in this chapter you will see a more formal discussion about area. For now, assume that a polygon has area and that the area does not change when you dissect the polygon.

Exercises Practicing Habits of Mind

Check Your Understanding

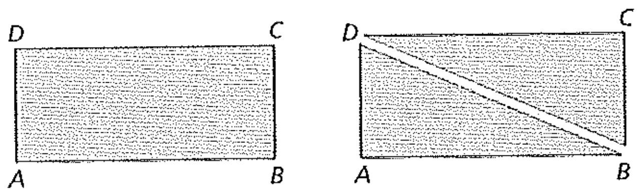
Habits of Mind

Make a list. Listing all properties—even obvious or redundant ones—can help you notice things you might otherwise overlook.

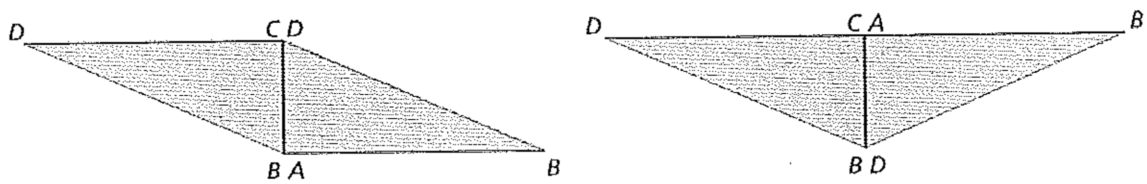
1. **Write About It** List all the properties of rectangles that you can think of. Here is a start.
   - Rectangles are parallelograms.
   - Rectangles have exactly four sides.
   - All angles measure 90°.

Here is Diego's method for turning a rectangle into an isosceles triangle. Use this for Exercises 2 and 3.

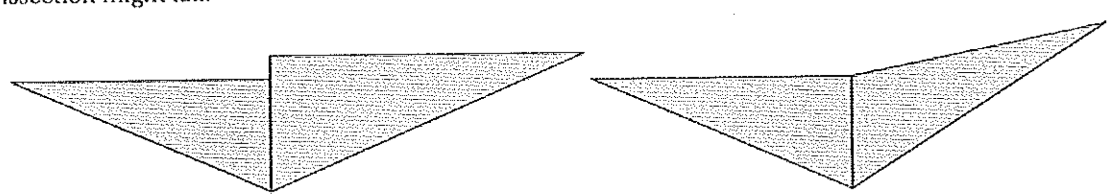
Start with a rectangle. Cut along a diagonal.



Slide $\triangle ABD$ to the right, along the bottom, so $\overline{AD}$ lines up with $\overline{BC}$. Then flip $\triangle ABD$ so you have a triangle instead of a parallelogram.



Diego's diagonal cut makes two triangles. Below are two ways that the dissection might fail.



The newly glued edges might not match.

The new top edge might be crooked.

3. Choose one of your algorithms for dissecting parallelograms, scalene triangles, and trapezoids into rectangles. Justify each step to show why your algorithm reliably produces the desired result.

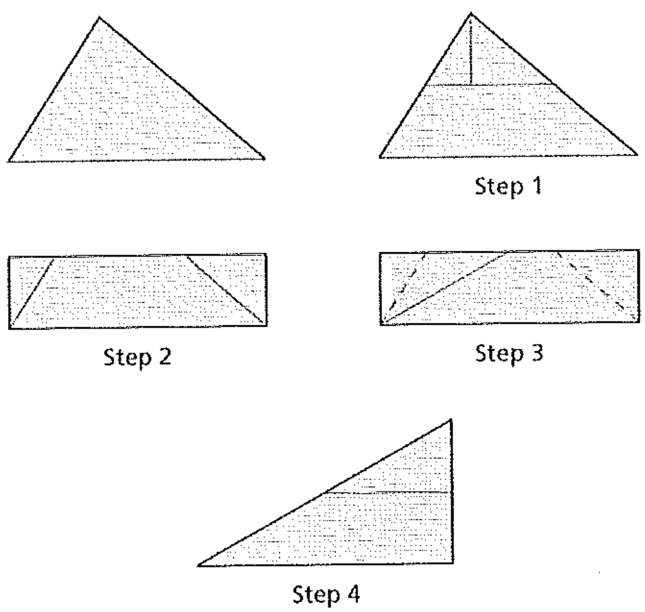
   a. Why do pieces match?

   b. Why are line segments straight?

   c. Why are angles right angles?

   d. Once more, pay close attention to whether your justification works in general, or only in certain cases.

4. Here is an algorithm for turning a scalene triangle into a right triangle. Justify each step.



   a. Why do pieces match up?

   b. Are segments straight?

   c. Why are there right angles?

   d. Is this a specific case, or is it general enough for the algorithm to work for all scalene triangles?

On Your Own

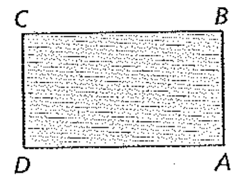
5. Write an algorithm for dissecting a parallelogram into a triangle.

6. Justify each step you used in the dissection of the parallelogram into a triangle.

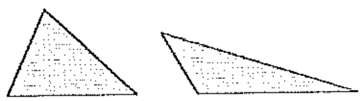
Mathematical Reflections 3A

In this investigation, you used dissection to make scissors-congruent shapes. You also used the Midline Theorem. These questions will help you summarize what you have learned.

1. Describe and justify each step in the dissection that turns any parallelogram into a rectangle.
2. Describe how you would dissect this rectangle into a right triangle. Justify each step. Draw pictures as needed.



3. Is it possible to dissect each of these triangles into a parallelogram? If you think it is possible in either or both of the cases, describe the steps.



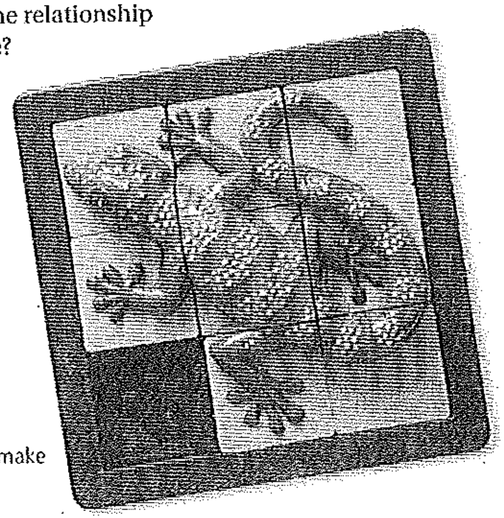
4. Copy and dissect the trapezoid at the right into a triangle.

5. Draw $\triangle ABC$. Join the midpoints of its three sides to form $\triangle DEF$. What is the ratio of the perimeter of $\triangle DEF$ to the perimeter of $\triangle ABC$?
6. What is an algorithm?
7. Why is it important to justify each step in an algorithm?
8. What does the Midline Theorem say about the relationship between a midline and the sides of a triangle?

Vocabulary

In this investigation, you learned these terms. Make sure you understand what each one means and how to use it.

- **algorithm**
- **hypotenuse**
- **scissors-congruent**



Some arrangements of parts make more sense than others.

Investigation 3B

Area Formulas

In *Area Formulas*, you will formalize the ideas of cutting and rearranging. You will derive and prove the area formulas for triangles, parallelograms, and trapezoids.

By the end of this investigation, you will be able to answer questions like these.

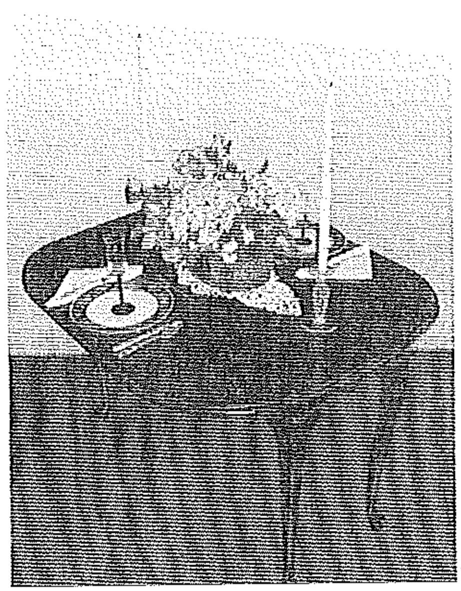
1. If two figures are scissors-congruent, do they have the same area? Explain.
2. Are all squares with the same area congruent?
3. What is the area formula for a parallelogram? For a triangle? For a trapezoid?

You will learn how to

- understand and apply basic assumptions about area
- identify critical measurements that can be used to find the areas of different types of polygons
- use dissection algorithms to develop area formulas for parallelograms, triangles, and trapezoids

You will develop these habits and skills:

- Understand what area is and what kinds of transformations preserve area.
- Calculate areas of rectangles, parallelograms, triangles, and trapezoids.
- Reason by continuity to relate different area formulas and polygons.



You can disassemble the tabletop and the table leaves. The total area remains the same.

3.6 Getting Started

Activating Prior Knowledge
Exploring New Ideas

When you think of measurement, you probably think of feet or inches, or maybe centimeters or miles. These all are measures of length. Sometimes it is more important to know how much area a figure covers rather than how long or wide it is.

For You to Explore

1. a. What is the area formula for a rectangle?

   b. Draw several pictures of rectangles that have area 12 square units. How many are there in all?

2. Use half sheets of standard $8\frac{1}{2}$ in.-by-11 in. paper as your starting rectangle. Cut that rectangle into the following number of equal-area rectangles.

   a. two b. four c. five

3. Suppose you want to cut a rectangle into pieces having equal areas. Is there any number of pieces that is *not* possible? Explain your answer.

Exercises Practicing Habits of Mind

On Your Own

4. Make a parallelogram. Dissect it into a rectangle. Use an algorithm that you know will work with any parallelogram.

   a. Measure and record the length and width of the rectangle.

   b. Carefully rearrange your pieces into your original parallelogram. Note the measurements of the parallelogram that correspond to the rectangle's length and width.

5. Draw a triangle. Dissect it into a rectangle. Use an algorithm that you know will work with any triangle.

   a. Record the length and width of the rectangle.

   b. Carefully rearrange your pieces into your original triangle. Note how the rectangle's length and width relate to measurements of the triangle.

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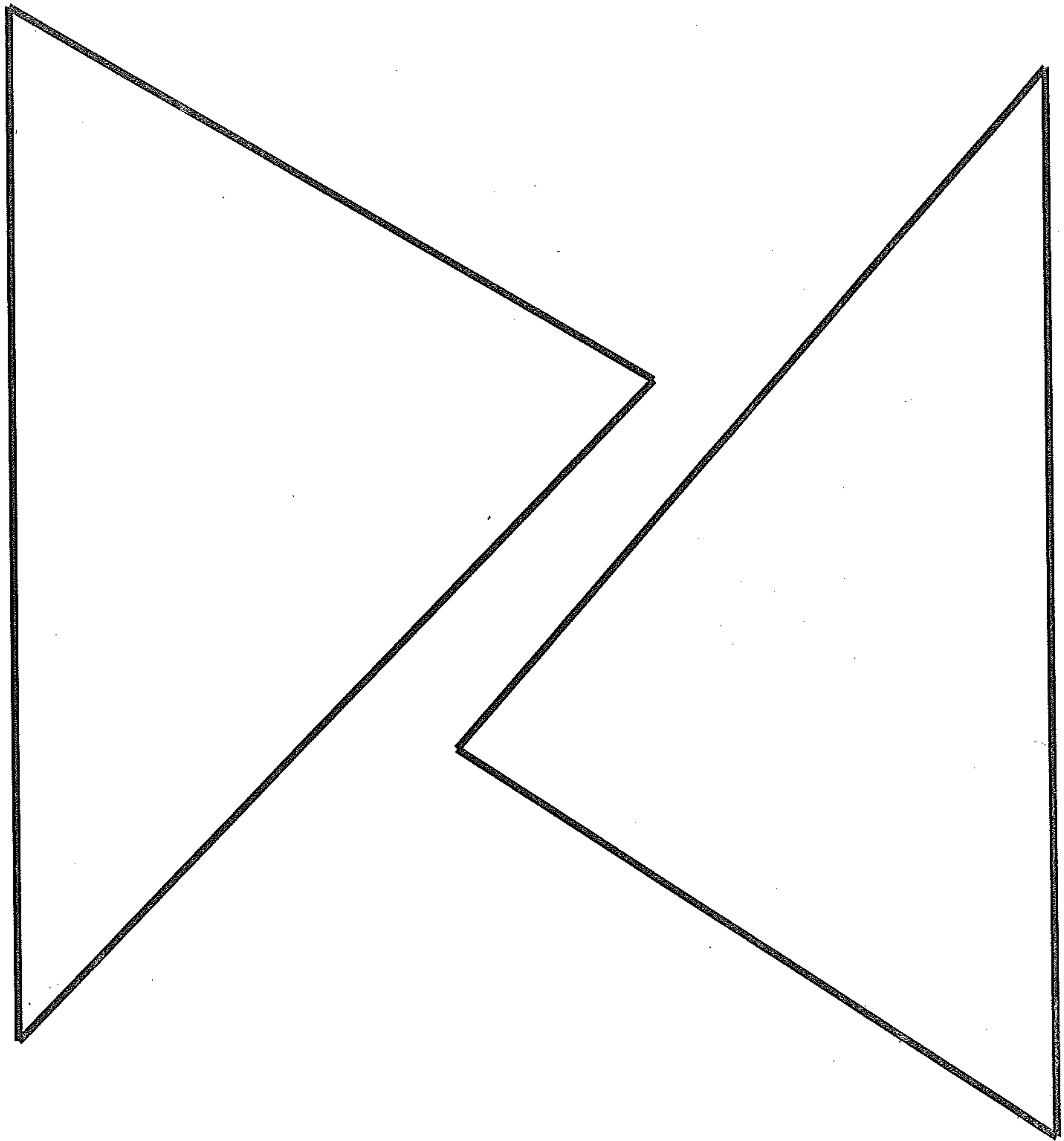
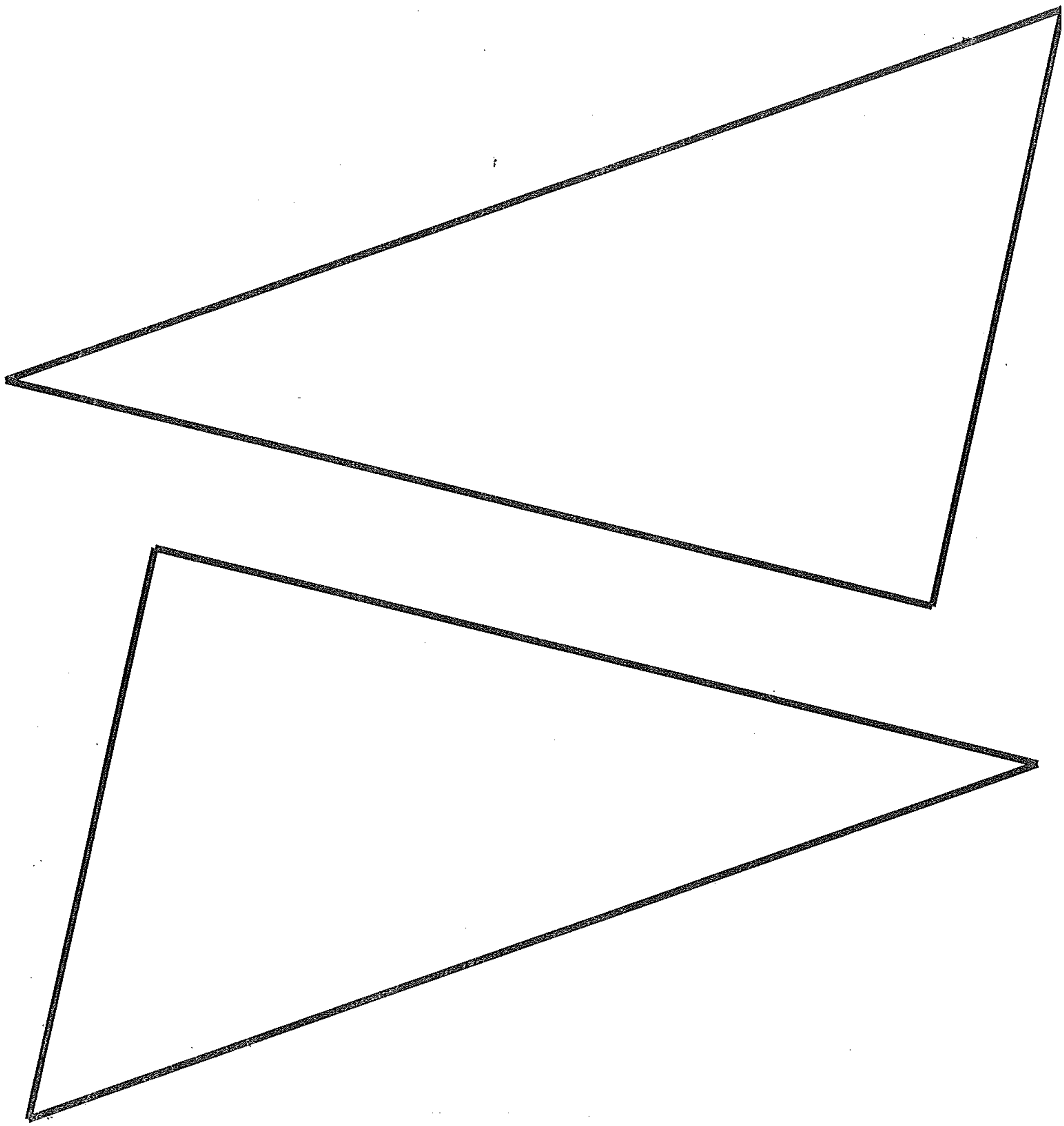
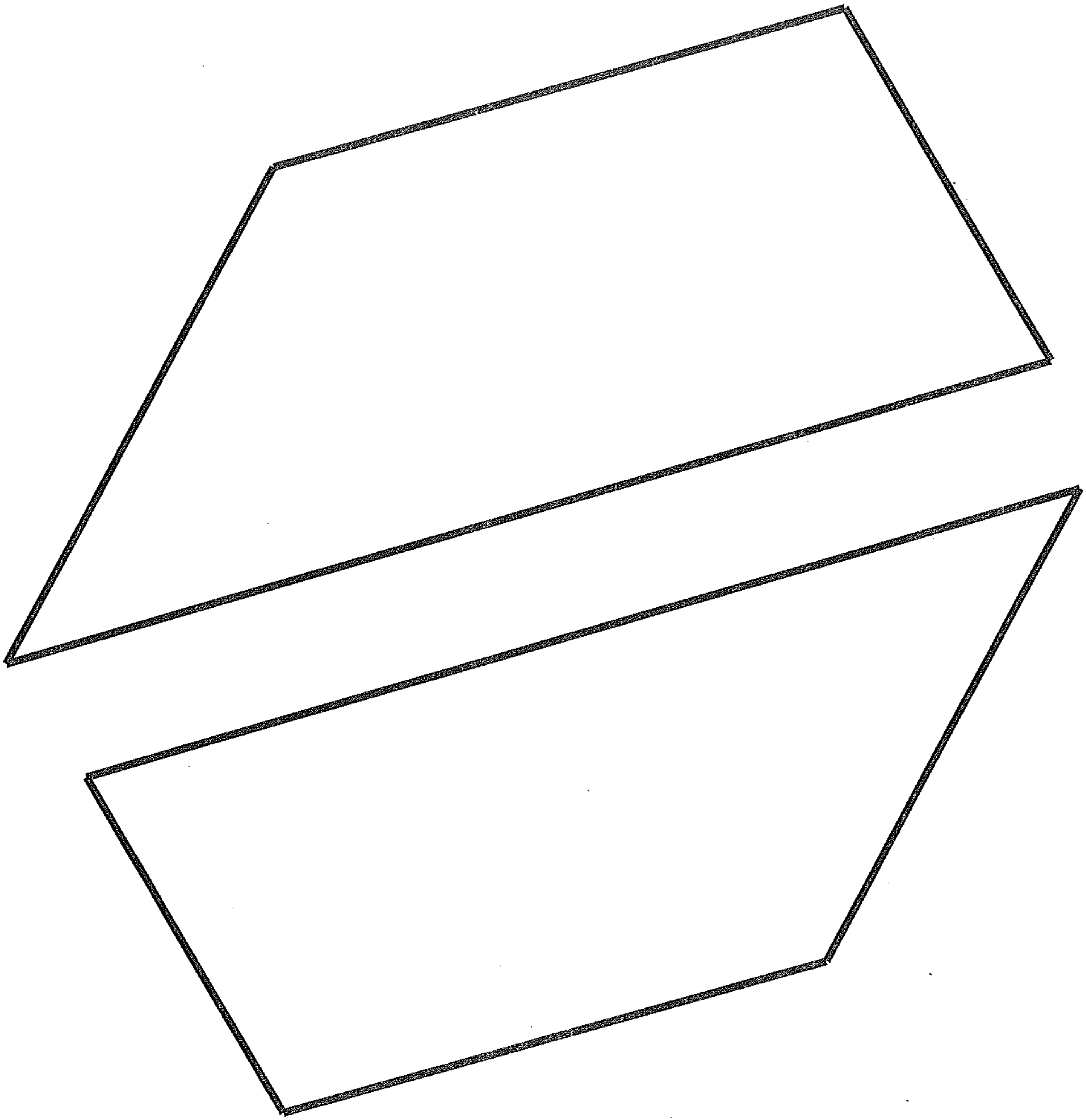
Scalene Triangle 1: adapted from CME Geometry Blackline Master 3.1B



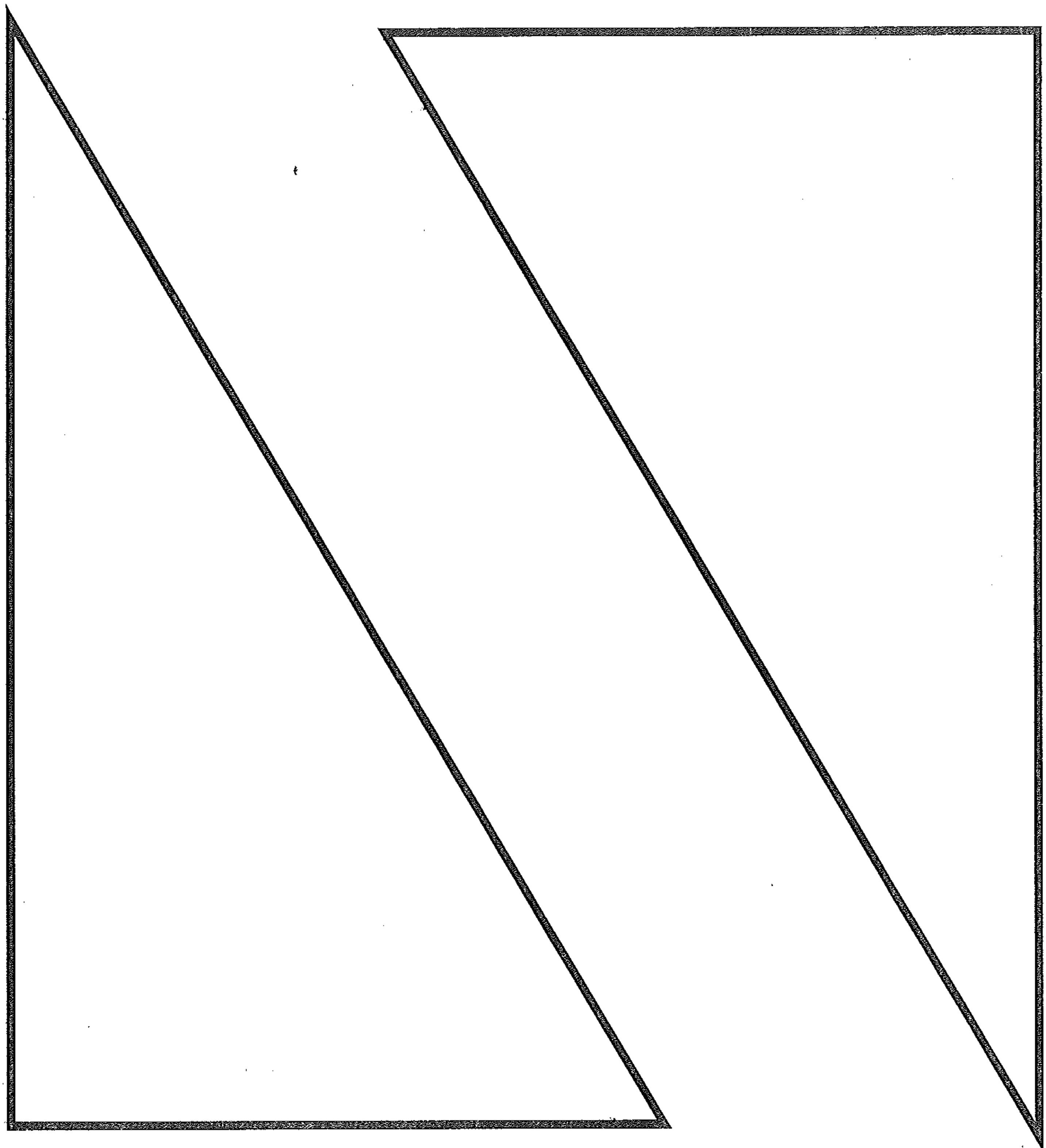
Scalene Triangle 2: adapted from CME Geometry Blackline Master 3.1C



Trapezoid: adapted from CME Geometry Blackline Master 3.1D



Right Triangle: adapted from CME Geometry Blackline Master 3.1A



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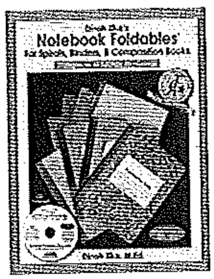

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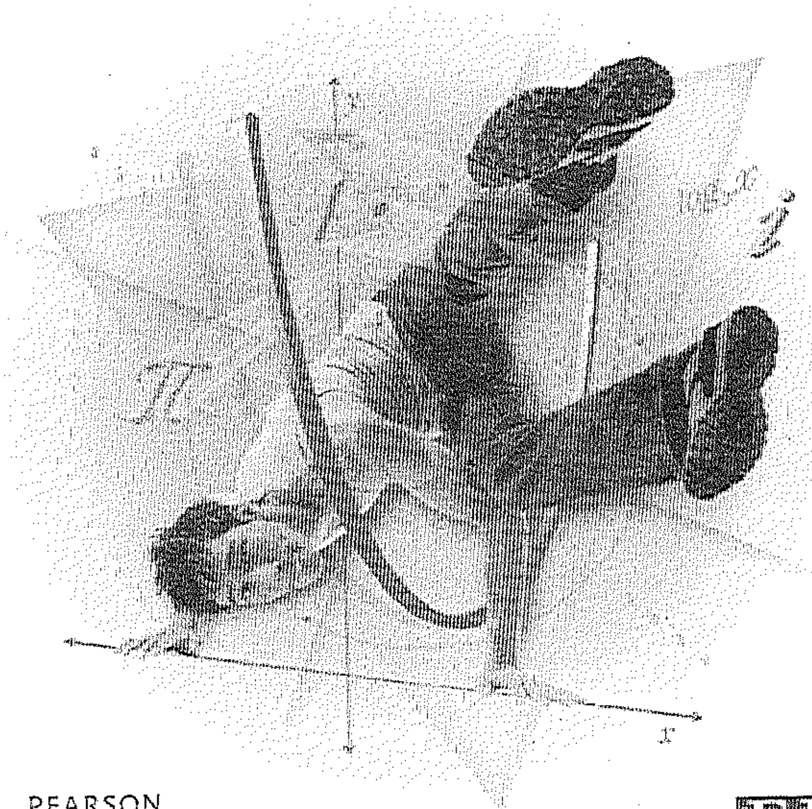
CME PROJECT

Algebra 2



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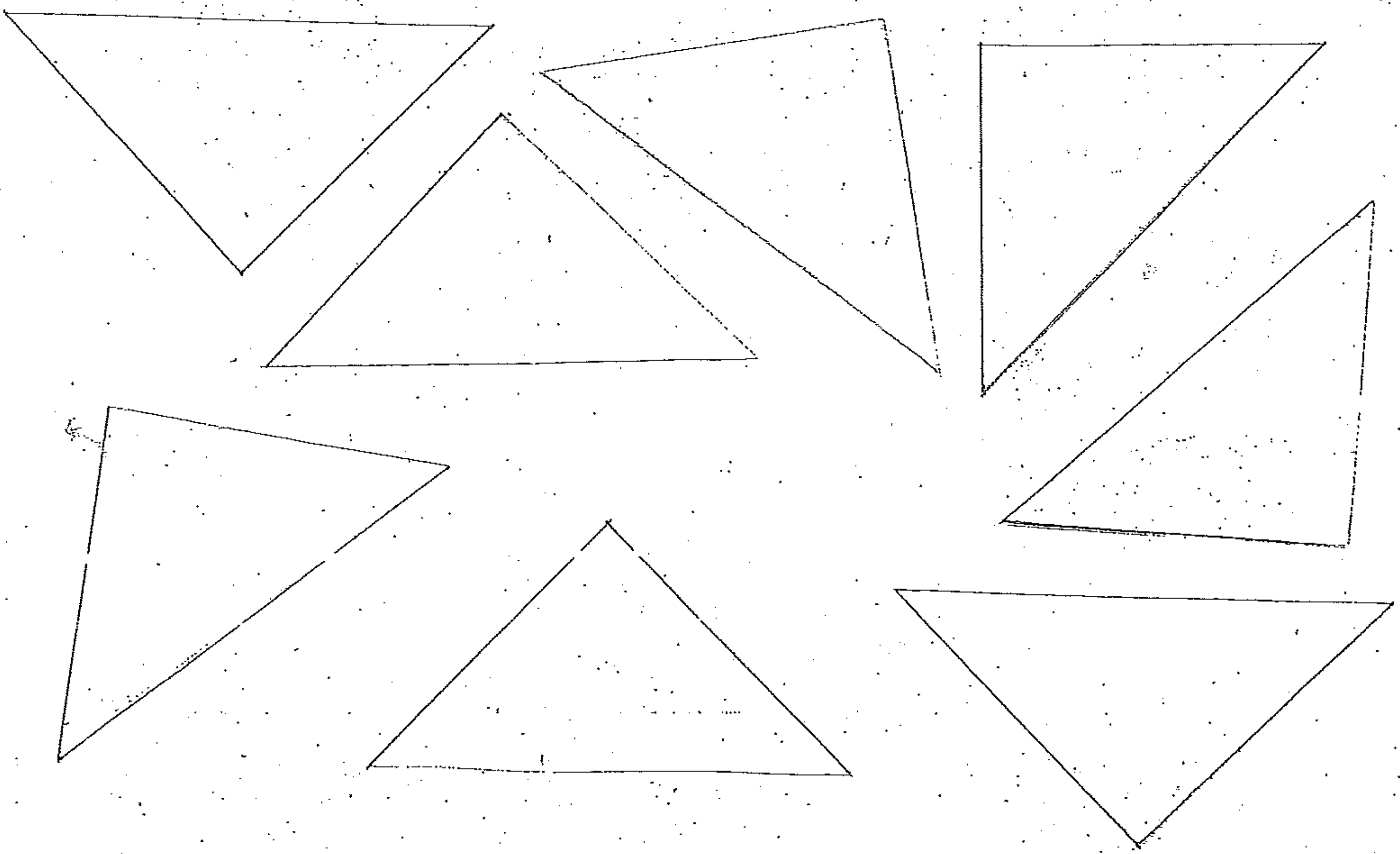
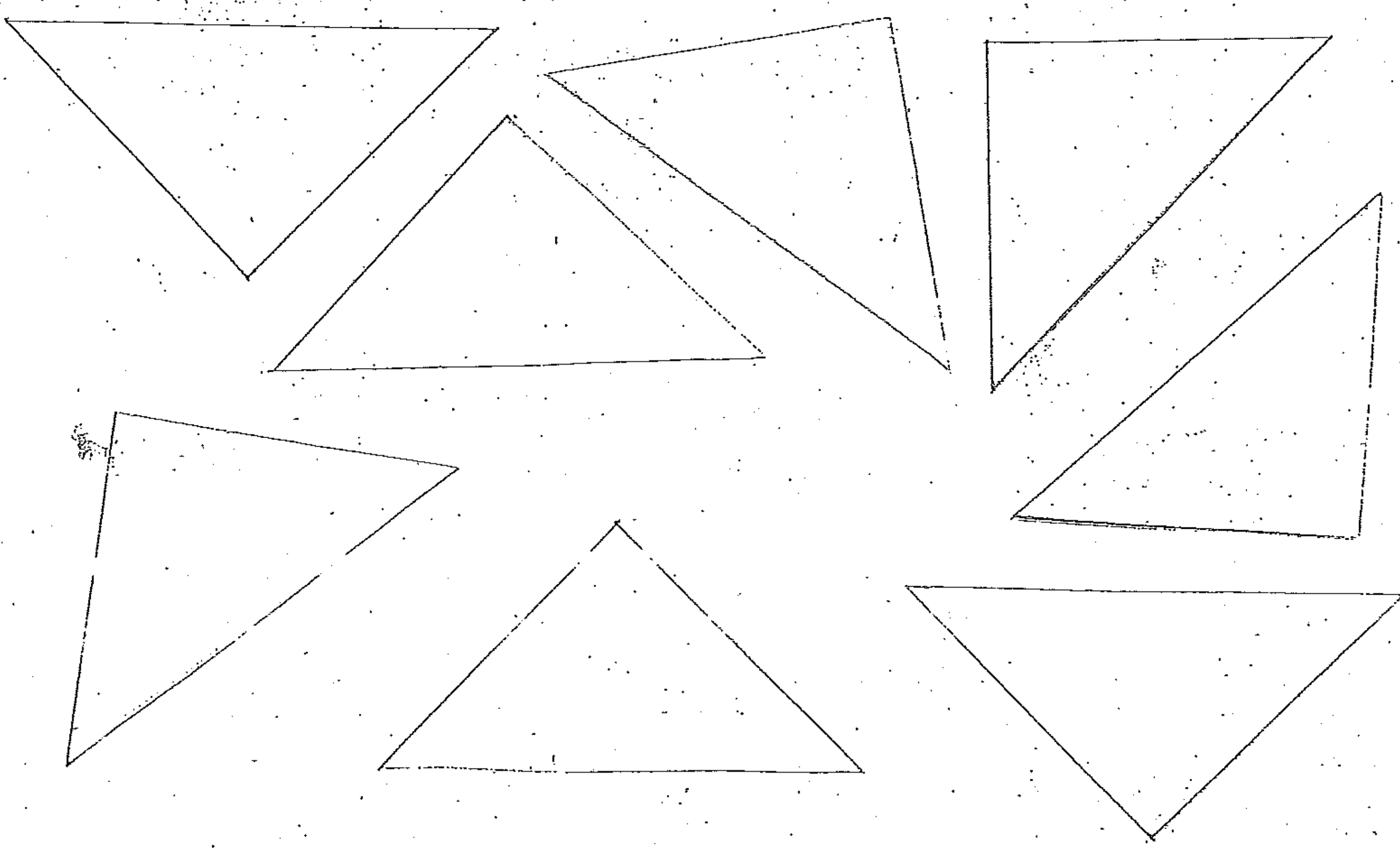
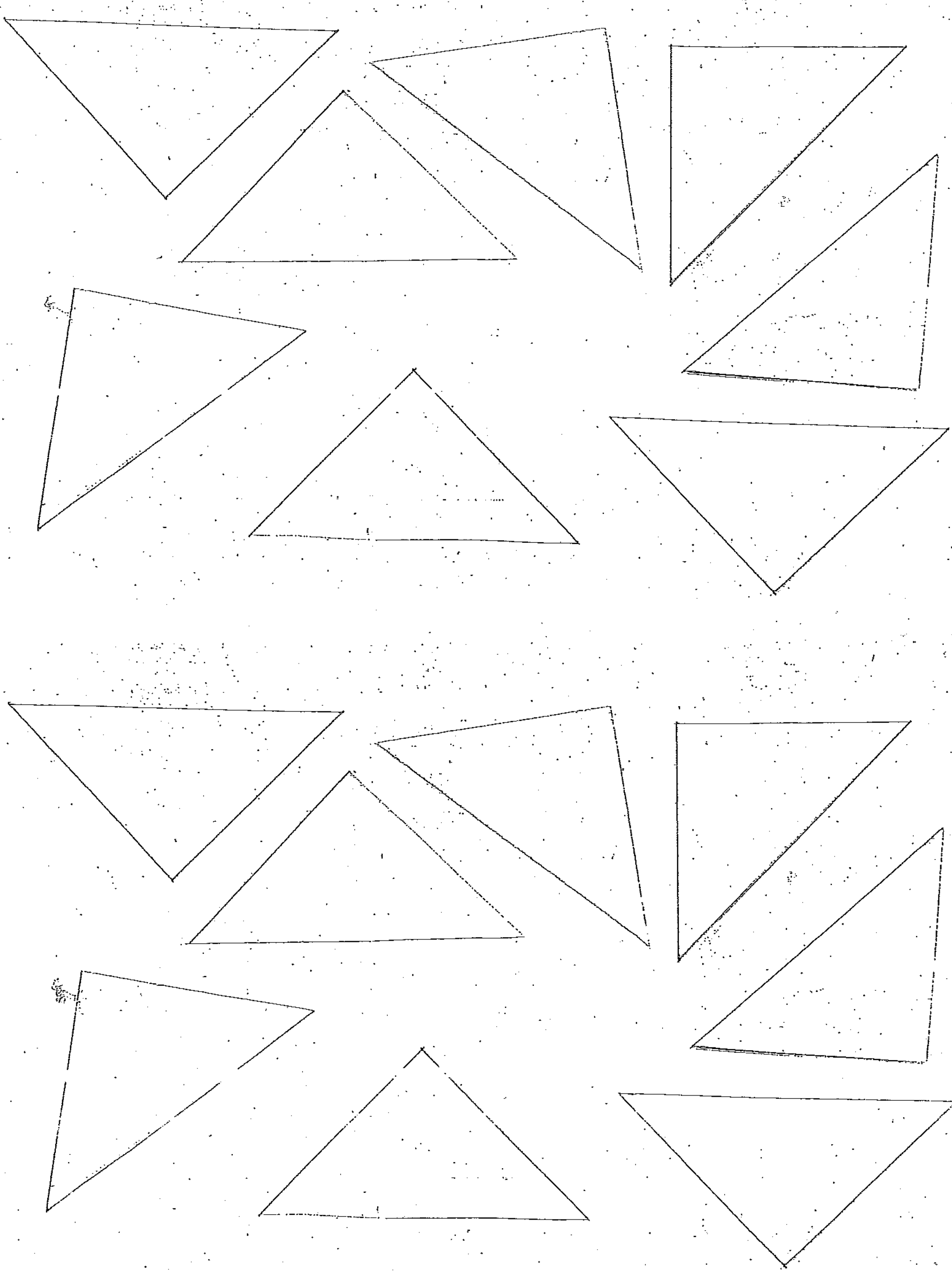






Investigation 5A

Working with Exponents

In *Working with Exponents*, you will use the basic rules of algebra to discover the laws of exponents. You will learn how to extend the rules to work with zero, negative, and rational exponents.

By the end of this investigation, you will be able to answer questions like these.

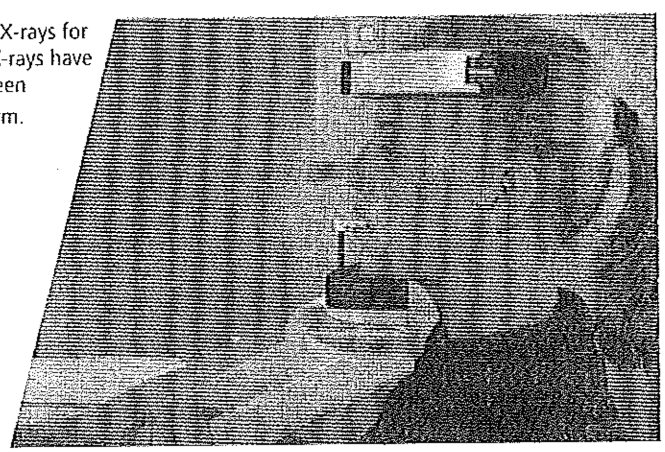
1. What is the Fundamental Law of Exponents? What are some of its corollaries?
2. How do you extend the laws of exponents to define zero, negative, and rational exponents?
3. What are the simplified forms of the expressions 4^0, 7^{-2} and $5^{\frac{27}{3}}$?

You will learn how to

- evaluate expressions involving exponents, including zero, negative, and rational exponents
- find missing terms in a geometric sequence and generate geometric sequences to interpret expressions involving rational exponents
- convert between exponential and radical forms for rational exponents

You will develop these habits and skills:

- Extend the laws of exponents to allow evaluation of zero, negative, and rational exponents.
- Reason logically to verify that a particular interpretation of an exponent follows the laws of exponents.
- Generalize from specific examples to develop and verify identities.



Radiographers use X-rays for medical imaging. X-rays have wavelengths between 10^{-7} and 10^{-10} cm.

5.1 Getting Started

Activating Prior Knowledge
Exploring New Ideas

Expressions or equations may contain variables as exponents.

For You to Explore

1. Copy and complete this table for the function $f(n) = 2^n$.

Input, n	Output, $f(n)$
6	64
5	32
4	16
3	8
2	▦
1	▦
0	▦
−1	▦
−2	▦
−3	▦

2. Solve each equation.
   a. $2^3 \cdot 2^5 = 2^a$
   b. $2^b \cdot 2^8 = 2^{14}$
   c. $3^c \cdot 3^c = 3^{12}$
   d. $(3^d)^2 = 3^8$
   e. $\frac{5^7}{5^f} = 5^6$
   f. $3^g = 9^5$
   g. $5^{3h} = 5^7$
   h. $(5^k)^3 = 5^4$

3. **Write About It** What are some rules of exponents? Give examples.

A *geometric sequence* is a list of numbers in which you get each term by multiplying the previous one by a constant. For example, the sequence below is a geometric sequence, since each term is three times as great as the previous term.

4, 12, 36, 108, 324, . . .

For Problems 4 and 5, find the missing terms in each geometric sequence.

4. a. 4, 8, 16, ▦, ▦, ▦, ...
   b. 4, −8, 16, ▦, ▦, ▦, ...
   c. 2, $2\sqrt{3}$, ▦, ▦, ▦, ▦, ...
   d. a, $2a$, ▦, ▦, ▦, ▦, ...
   e. k, $3k$, ▦, ▦, ▦, ▦, ...
5. a. 1, ▦, ▦, 8, ▦, ▦, ...
   b. ▦, ▦, 1, $\frac{1}{2}$, ▦, ▦, ...
   c. 2, ▦, 18, ▦, ▦, ▦, ...
   d. 1, ▦, ▦, ▦, 9, ▦, ...

Habits of Mind

Look for relationships. What is a closed form for the function g with this sequence of outputs?

Input	Output
0	4
1	12
2	36
3	108
4	324

Exercises Practicing Habits of Mind

On Your Own

6. Decide whether each equation is true for all positive integers a, b, and c.

   a. $a^b \stackrel{?}{=} b^a$ b. $a^{b+c} \stackrel{?}{=} a^b + a^c$ c. $a^{b+c} \stackrel{?}{=} a^b \cdot a^c$

   d. $a^b \cdot a^c \stackrel{?}{=} a^{bc}$ e. $(a^b)^c \stackrel{?}{=} a^{bc}$ f. $(a^b)^c \stackrel{?}{=} a^{(b^c)}$

   g. $\frac{a^b}{a^c} \stackrel{?}{=} a^{b-c}$ h. $(ab)^c \stackrel{?}{=} a(b^c)$

Habits of Mind

Experiment. If the equation is not true for all positive integers, is it true for some values of a, b, and c?

7. Determine whether each expression is equal to 2^{12}. Explain.

   a. $2^{10} + 2^2$ b. $(2^4)(2^4)(2^4)$ c. $2^6 \cdot 2^6$

   d. $2^9 + 2^3$ e. $(2^{10})(2^2)$ f. $2^{11} + 2^{11}$

   g. $(2^4)(2^3)$ h. $4(2^{10})$

Try this without a calculator.

8. Copy and complete the table for the function $g(n) = 3^n$.

Input, n	Output, $g(n)$
5	243
4	81
3	27
2	▒
1	▒
0	▒
−1	▒
−2	▒
−3	▒

9. Problem 1 shows a table for $f(n) = 2^n$. Exercise 8 shows a table for $g(n) = 3^n$. Consider the function $h(n) = f(n) \cdot g(n)$.

   a. Calculate $h(3)$.

   b. Use the completed tables to calculate $h(0)$, $h(1)$, and $h(2)$.

   c. Find a simple rule for $h(n)$.

10. Take It Further

    a. Explain why there are no positive integers a and b such that $2^a = 5^b$.

    b. Find the number x that makes $2^x = 5$. Round to four decimal places.

    c. Find the number y that makes $2 = 5^y$. Round to four decimal places.

    d. What is the relationship between x and y?

Maintain Your Skills

11. Solve each equation.

    a. $3^x = 81$ b. $3^{x+1} = 81$ c. $3^{2x} = 81$

    d. $3^{-x} = 81$ e. $3^{4x-1} = 81$ f. $3^{x^2} = 81$

Investigation 5C

Logarithmic Functions

In *Logarithmic Functions*, you will learn that logarithms are the inverses of exponential functions. You will use logarithms to solve exponential equations.

By the end of this investigation, you will be able to answer questions like these.

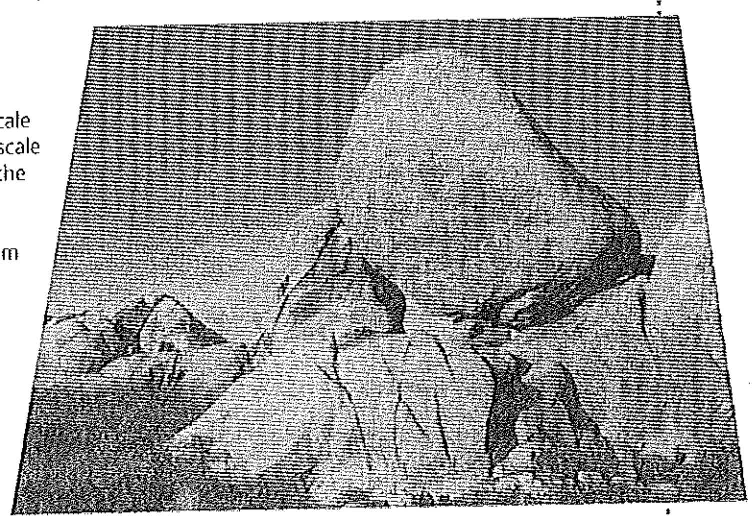
1. What are some reasons to use logarithms?
2. What is a logarithmic scale and when do you use it?
3. If you invest $1000 at 6% interest, compounded annually, how many years will it take until your money grows to $10,000?

You will learn how to

- evaluate logarithms of any base using a calculator
- use logarithms to solve exponential equations
- graph logarithmic functions

You will develop these habits and skills:

- Reason logically from the definition of a logarithmic function and the laws of exponents to develop the laws of logarithms.
- Visualize the graph of a logarithmic function from the graph of the corresponding exponential function.
- Convert flexibly and strategically between logarithmic form and exponential form, and choose the best form to solve problems.



The Krumbein scale is a logarithmic scale used to classify the size of particles. A boulder with diameter 256 mm or greater has a scale value of −8 or less.

5.11 Getting Started

Activating Prior Knowledge
Exploring New Ideas

In this lesson, you will investigate a function on your calculator called LOG. There may be a LOG key on your calculator. If there is not, look in your calculator's function library. You use the notation $\log x$ for this function.

> Although you use parentheses in $f(x)$ function notation, the parentheses are optional for the log function. You can write $\log(x)$ or $\log x$. You still must use parentheses when needed to avoid confusion, for example in expressions such as $\log(x + 1)$.

For You to Explore

1. Use your calculator to find the output when you use the LOG key for each integer input from 0 to 10. Copy and complete the table below. Record each output to four decimal places.

x	log x
0	■
1	■
2	■
3	■
4	■
5	■
6	■
7	■
8	■
9	■
10	■

2. Find the value of $\log 2 + \log 3$ to four decimal places.
3. a. Calculate $\log 2 + \log 6$.
   b. Calculate $\log 3 + \log 4$.
   c. Calculate $\log 3 + \log 2 + \log 2$.
   d. Find the number x such that $\log x = \log 2 + \log 6$.
4. Estimate each result using the table from Problem 1 and any patterns you have seen so far. Then use a calculator to confirm the answer.
   a. $\log 15$ b. $\log 24$ c. $\log 36$ d. $\log 63$
5. Find a rule for calculating $\log MN$ in terms of $\log M$ and $\log N$. (*Hint:* Refer to the results from Problem 4.)
6. Estimate each result using the table from Problem 1 and any patterns you have seen so far. Then use a calculator to confirm the answer.
   a. $\log 16$ b. $\log 32$ c. $\log 64$ d. $\log 2^{10}$ e. $\log 3^5$
7. Find a rule for calculating $\log M^p$ in terms of p and $\log M$. (*Hint:* Refer to the results from Problem 6.)
8. **Write About It** Explain how you can use the result from Problem 7 and the table from Problem 1 to estimate $\log \frac{1}{8}$.
9. Determine the domain and range of the function $x \mapsto \log x$.
10. **Take It Further** Use the rules that govern the function $x \mapsto \log x$ to find the solution to the following equation.

$$2^x = 5$$

Exercises Practicing Habits of Mind

On Your Own

Try evaluating these expressions without using a calculator.

11. You have seen that $\log 10 = 1$. Now calculate each of the following values.
   a. $\log 10^2$
   b. $\log 10^3$
   c. $\log 10^6$
   d. $\log 10^{10}$
   e. $\log 10^{-3}$

12. Calculate each value.
   a. $10^{\log 2}$
   b. $10^{\log 3}$
   c. $10^{\log 6}$
   d. $10^{\log 10}$
   e. $10^{\log -3}$

13. Describe the relationship between the functions $x \mapsto \log x$ and $x \mapsto 10^x$. Give examples.

14. Solve each equation for x.
   a. $\log \frac{5}{4} + \log 4 = \log x$
   b. $\log \frac{7}{3} + \log 3 = \log x$
   c. $\log \frac{6}{17} + \log 17 = \log x$
   d. $\log x + \log 2 = \log 5$
   e. $\log x + \log 7 = \log 3$
   f. $\log x = \log 11 - \log 4$

15. Find a rule for calculating $\log \frac{M}{N}$ in terms of $\log M$ and $\log N$.

Refer to the results from Exercise 14.

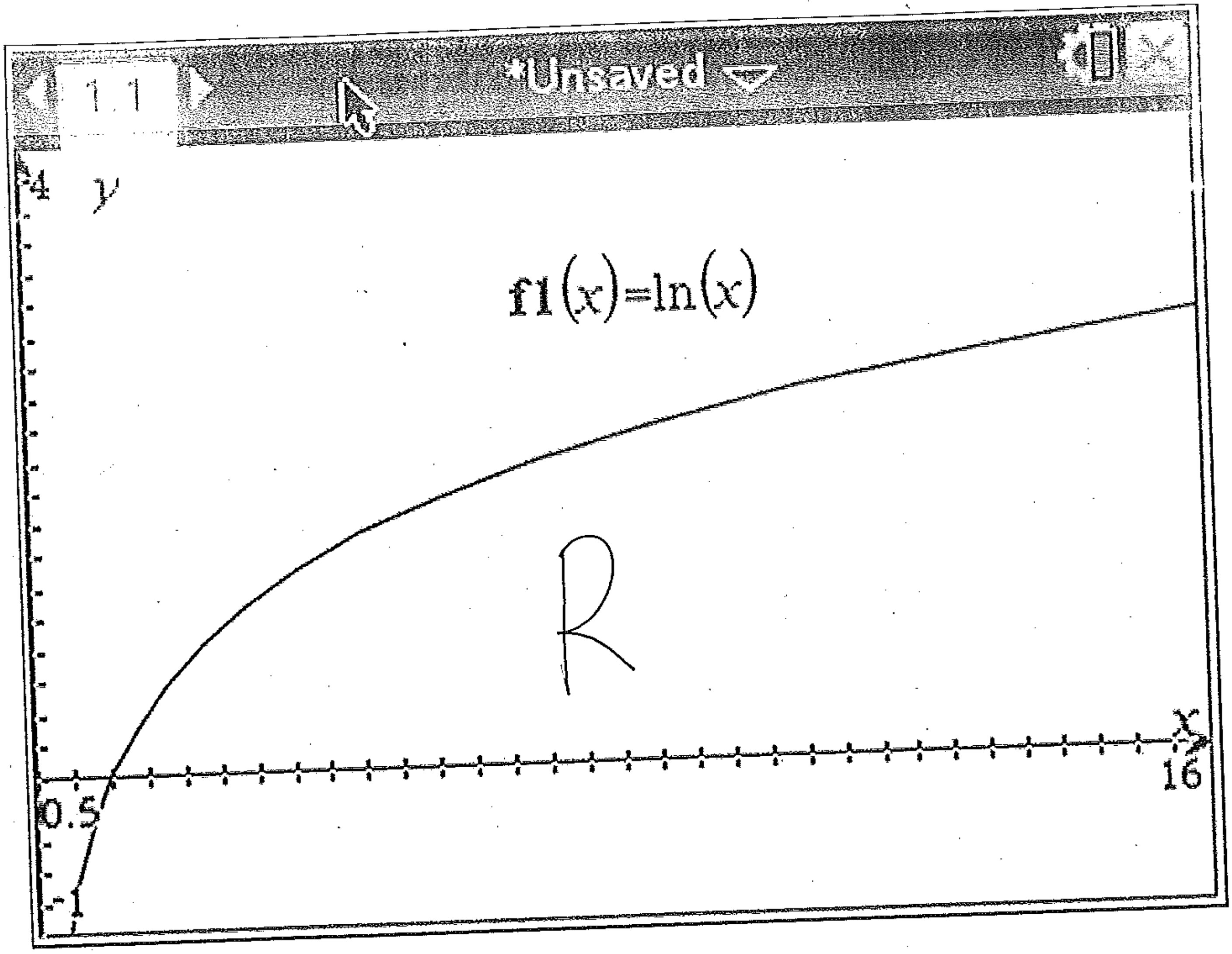
16. On the same axes, sketch the graphs of $f(x) = 3^x$ and its inverse function.

17. On the same axes, sketch the graphs of $f(x) = \log x$ and its inverse function.

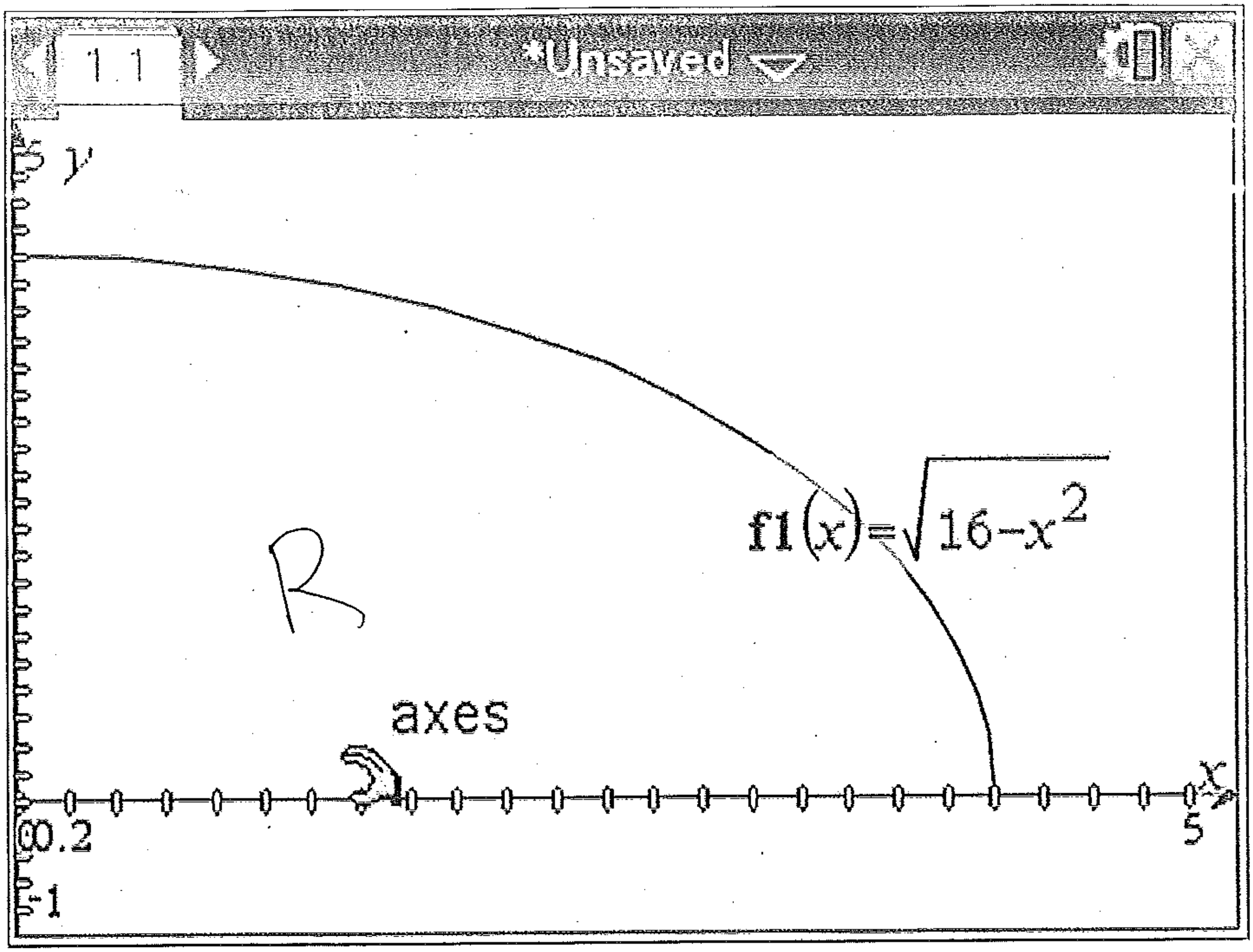
18. Suppose $\log(A) = 1.6$ and $\log(B) = 2.7$. Find each value.
   a. $\log AB$
   b. $\log A^2$
   c. $\log \frac{1}{A}$
   d. $\log \frac{B}{A}$
   e. $\log AB^2$
   f. $\log \sqrt{A}$

Maintain Your Skills

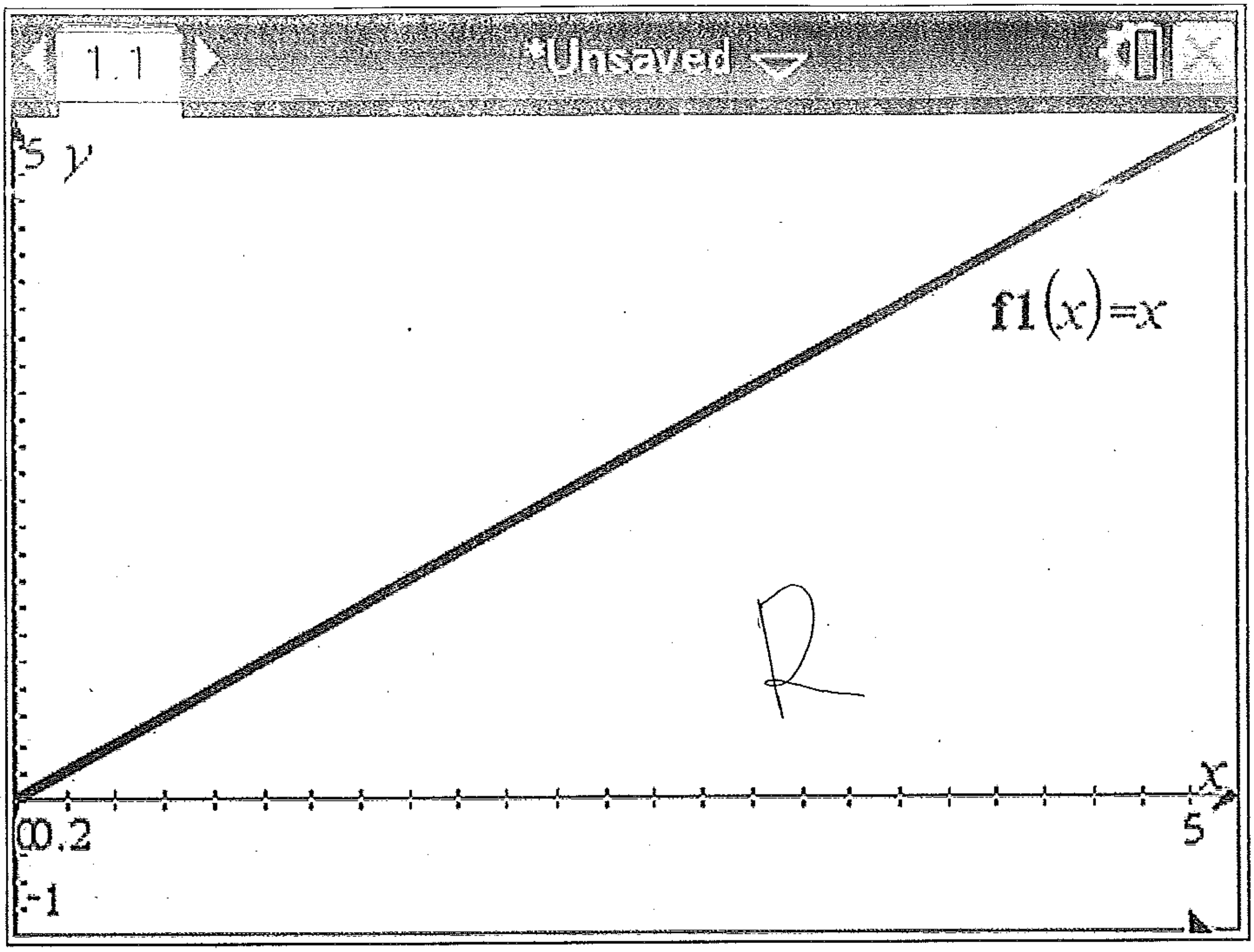
19. Find a pair of consecutive integers j and k that satisfy each inequality.
   - $j < \log 7 < k$
   - $j < \log 70 < k$
   - $j < \log 7000 < k$
   - $j < \log 143{,}265 < k$



2) The region below the curve, above the x axis, and between 1 and 16 has cross sections that are perpendicular to the x axis. Each cross section is a rectangle. The base of the rectangle lies on the region and is ½ the height of the rectangle. Find the volume of the solid.



3) The region below the curve, above the x axis, and between 0 and 4 has cross sections that are perpendicular to the x axis. Each cross section is a semicircle. Find the volume of the solid.



4) The region below the curve, above the x axis, and between 0 and 5 has cross sections that are perpendicular to the x axis. Each cross section is an isosceles right triangle. One leg of the triangle is the height, the other leg lies in the region. Find the volume of the solid.