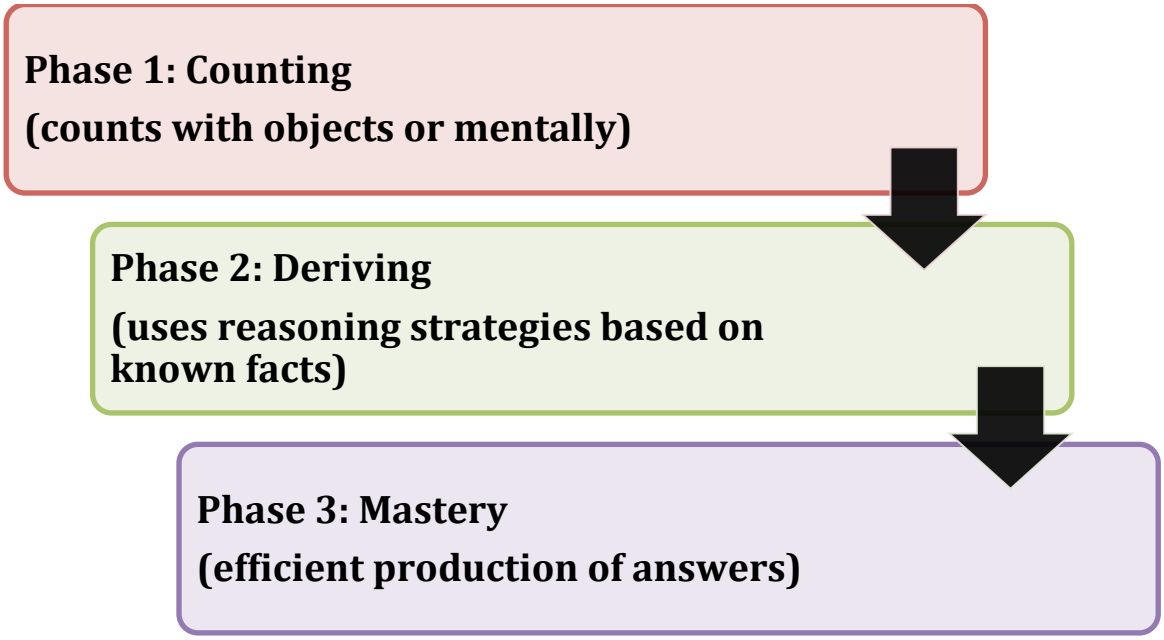
Developing and Assessing Addition Fact Fluency

National Council of Teachers of Mathematics
(NCTM) Annual Meeting
New Orleans, LA
April 12, 2014

Gina Kling,
Western Michigan University
&
Jennifer Bay-Williams,
University of Louisville

Procedural Fluency is "skill in carrying out procedures flexibly, accurately, efficiently and appropriately."
(CCSSO, 2010, p. 6)

Phases of Basic Fact Mastery (Baroody, 2006)



CCSSM Expectations Related to Basic Facts

Grade K Standard K.OA.A.4: "For any number from 1 to 9, find the number that ***makes 10*** when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation."

Grade 1 Standard 1.OA.C.6: "Add and subtract within 20, ***demonstrating fluency**** for addition and subtraction within 10. ***Use strategies*** such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known sums."

Grade 2 Standard 2.OA.B.2 : "***Fluently*** add and subtract within 20 using mental strategies (reference to 1.OA.6). By end of Grade 2, ***know from memory*** all sums of two one-digit numbers."

Note: This list illustrates that CCSS-M recognizes the importance of reasoning strategies (Phase 2) before expecting automaticity with their facts (Phase 3).

Addition and Subtraction GAMES across the Phases

(Bay-Williams & Kling, in press)

"Roll and Total"
(from *Everyday Mathematics*, 2016)

Children take turns rolling two dice (1 labeled with numbers 3-8, 1 six-sided die) and finding the resulting sum. They shade in the first open box in the corresponding column and repeat.

4	5	6	7	8	9	10	11	12	13	14

"Tens Go Fish"
(from *Investigations in Number, Data, and Space*, 2008)

This 2-4 player game is played like the card game "Go Fish," only instead of looking for matching cards, children look for combinations of ten. For example, if a child has a 4 in his hand, he would ask another player "Do you have a 6?" Use numeral cards or playing cards, Ace-10 + Jack (Ace = 1, Jack = 0). Children can continue to draw extra cards as needed, and play continues until all cards are used. Children can then be encouraged to share or record the number sentences for some of the pairs that they found to reinforce the combinations of ten.

"Double It"
(from *Investigations in Number, Data, and Space*, 2008)

Children take turns flipping over a number card and making a doubles fact with that number. They shade in the first open box in the column above that sum and repeat.

2	4	6	8	10	12	14	16	18

"Addition Top-It"
(from *Everyday Mathematics*, 2012)

Using a deck of cards with numbers 0–10 (Ace = 1, Jack = 0), partners begin by splitting the deck roughly in half so that they each have their own deck of cards. Then each partner turns over 2 cards and calls out the sum of the numbers. Players check each other's sums, and the player with the largest sum wins the round and takes all of the cards. Adaptations include turning over 3 addends (3 cards each), "Subtraction Top-It," and "Multiplication Top-It."

"Close to 20"
(from *Investigations in Number, Data, and Space*, 2008)

Using a deck of cards with numbers 0–10 (Ace = 1, Jack = 0), each partner turns over 5 cards. Each player then selects from her 5 cards the 3 cards which, when added together, produce a sum as close to 20 as possible. Players record their score as how far away they are from 20. For example, if the player turns over 3, 5, 7, 8, and 6, choosing 5, 7, and 8 would produce a sum of exactly 20, yielding a score of 0. They then discard and replace the used cards and play again. After 5 rounds they find their total score, and the winner is the player with the lowest overall score (like golf!).

"Name that Number"
(from *Everyday Mathematics*, 2012)

Using a deck of cards with numbers 0–10 (Ace = 1, Jack = 0), children play in groups of 2-4. The first player turns over 5 cards, placing them in a line on the left side of the deck, and then turns over one additional card to serve as the "target number," placing it on the right side of the deck. That player then tries to name the target number by adding and/or subtracting the numbers on 2 or more of the 5 cards on the left side of the deck. If the player can name the target number, he records his number sentence and takes all the cards used, including the target card. The next player then draws from the deck to replace the cards and begins her turn. If the first player cannot name the target number, play passes to the next person who draws a new target number card and begins her turn. Play continues until all cards are used, and the player with the most cards wins.

CLASSIC GAMES (Addition or Multiplication)

TABLE 10.1

CLASSIC GAMES ADAPTED TO BASIC FACT MASTERY

Classic Game	How to Use It with Basic Fact Mastery	Suggestions for Differentiation
Bingo	Each bingo card has a fact problem (e.g., 2 × 3) in each box. The same fact will be on multiple bingo cards but in different locations on each card. You will call out an answer (e.g., 6), and the students will find a matching problem (or more than one problem) on their card.	Create bingo boards that focus on different clusters of facts (e.g., doubles or doubles + 1 on some boards, and Up Over 10 on other boards). Be sure that the answers you call out are an even mix of the clusters so that everyone has the same chance to win.
Concentration	Create cards that have a fact problem (e.g., 3 × 5) on one half and the answers (e.g., 15) on the other half. Shuffle the cards and turn them face-down in a 6 × 4 grid. (If you like, you can make the grid larger to use more cards.)	Select cards that focus on a particular cluster of facts (e.g., +1 and ×5 facts) for each round of the game. Multiple groups can play the game simultaneously—each group will use the parts of the deck that contain the facts they are working on. Also, consider making cards that show the ten-frames below the numbers to help provide a visual for students.
Dominoes	Create (or find online) dominoes that have a fact on one side and an answer (not to that fact) on the other. Each student gets the same number of dominoes (around eight). On his or her turn, they can play one of the dominoes in their hand only if they have an answer or a fact that can connect to a domino on the board.	As with other games, select the dominoes that focus on a particular clusters of facts.
Four in a Row	Create a 6 × 6 square game board with a sum (or product) written on each square. Below, list the numbers 0 through 9. Each of the two players has counters of a different color to use as their game pieces. On the first turn, Player 1 places a marker (paper clip) on two addends/factors and then gets to place his or her colored counter on the related answer. (If you have repeated the same answer on different squares of the board, the player only gets to cover one of them.) Player 2 can only move one paper clip and then gets to place his or her colored counter on the related answer. The first player to get four in a row wins.	Rather than list all the values below the chart, just list the related addends or factors. For example, use 1, 2, 6, 7, 8, 9 if you want to work on +1 and +2, or use 3, 4, 5, 6 if you are working on these multiplication facts.
Old Maid (retitled as Old Dog)	Create cards for each fact and each answer. Add one card that has a picture of an old dog (or use your school mascot). Shuffle and deal cards. On each player's turn, they draw from the person on their right, see whether that card is a match to a card in their hand (a fact and its answer), and, if so, lay down the pair. Then the person to their left draws from them. Play continues until all matches are found and someone is left with the Old Dog. Winner can be the person with (or not with) the Old Dog, or the person with the most pairs.	See Concentration (above).

Source: Adapted from Forbringer & Fahsl, 2010, and Kamii & Anderson, 2003.

From Van de Walle, Karp, & Bay-Williams (2013). *Elementary and Middle School Mathematics: Teaching Developmentally.* New York, NY: Pearson.

Addition Fact Fluency Quiz

Solve these problems and tell how you solved out.

4 + 5 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

10 + 6 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

6 + 2 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

5 + 3 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

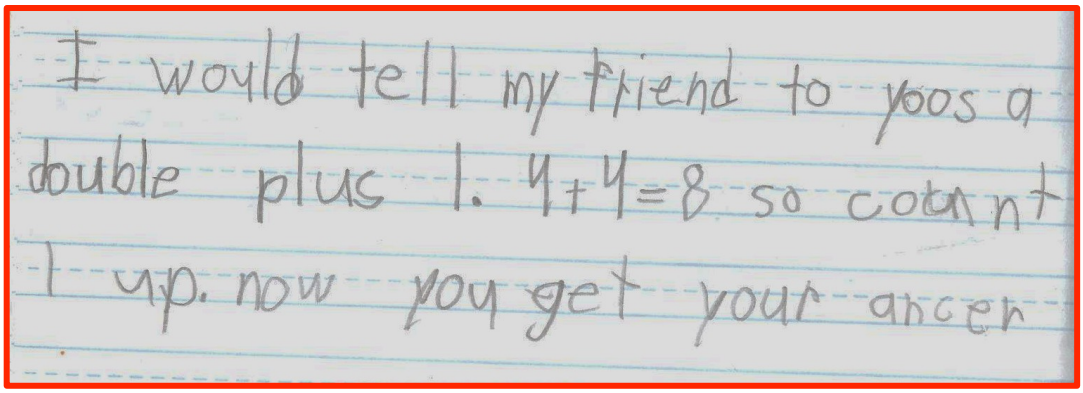
2 + 9 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

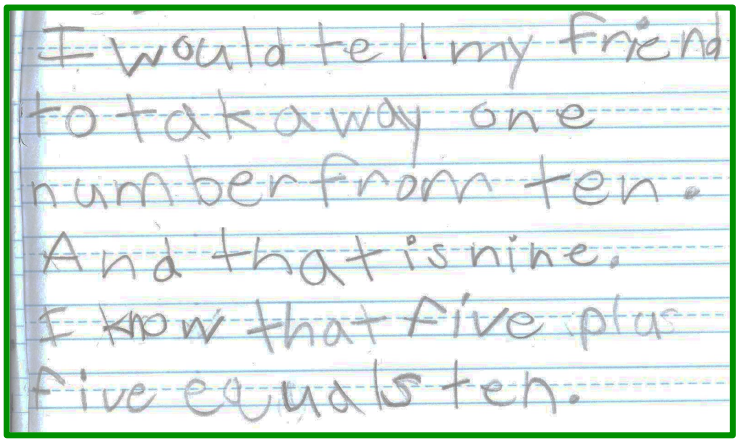
3 + 10 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

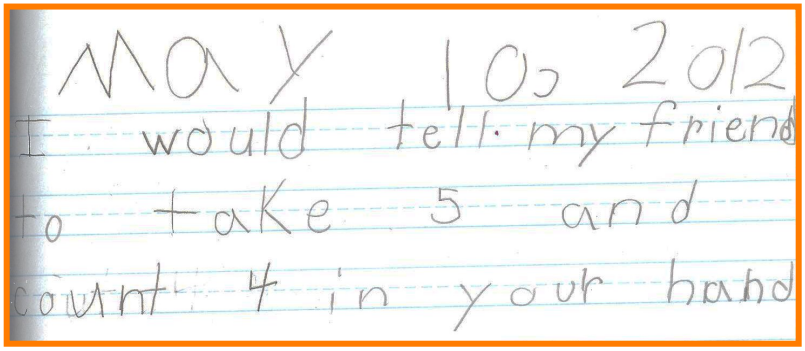
5 + 7 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

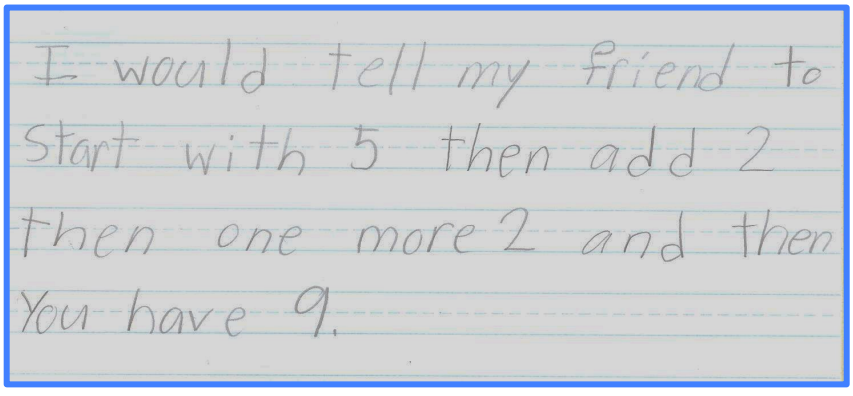
8 + 10 = _____ Check one: ____ I used this strategy: _____
____ I just knew.

Facts Assessment: First-grade journal responses to
"If your friend didn't know the answer to 4 + 5, how could he figure it out?"


I would tell my friend to yoos a double plus 1. 4+4=8 so count 1 up. now you get your ancer


I would tell my friend to take away one number from ten. And that is nine. I know that five plus five equals ten.


MAY 10, 2012
I would tell my friend to take 5 and count 4 in your hand


I would tell my friend to start with 5 then add 2 then one more 2 and then you have 9.

Salute!

(Bay-Williams & Kling, in press)

Place students in groups of three, and give each group a deck of cards (omitting face cards and using aces as ones, jacks as zeros). Two students draw a card without looking at it and place it on their forehead facing outward (so the others can see it). The student with no card tells the sum. The other two players determine the value of their cards. Once both players have done so, they look at their cards and then students rotate roles before starting the next hand. This can be differentiated by including only certain cards (e.g., addition facts using only the numbers 1 through 5).

Questions to Ask While Playing Facts Games

Use questions such as the following to encourage good mathematical thinking during game play:

- *How did you figure it out?*
- *Can you say out loud how you thought about it in your head?*
- *Is there another way you could figure it out?*
- *Can you think of another fact that strategy would work well for?*
- *If someone didn't know the answer to ____, how would you tell them to figure it out?*

Assessing Basic Fact Mastery

(Kling & Bay-Williams, 2014)

Interviews

Focus on fluency:

1. Write 4 + 5 on a card. [point at card] What does 4 + 5 mean?
2. What is the answer to 4 + 5?
3. How did you find the answer to 4 + 5? Could you find it another way?
4. If your friend was having trouble remembering this fact, what strategy might you suggest to her/him?

Focus on flexibility and **strategy selection.**

1. What is 8 + 5?
2. How can you use 8 + 2 to help you solve 8 + 5?

Writing Prompts

Flexibility	Accuracy
Solve 8 + 7 using one strategy. Now try solving it using a different strategy.	What is the answer to 9 + 4? How do you know it is correct (how might you check it)?
Efficiency	**Appropriate Strategy Selection**
Which facts do you "just know"? Which facts do you use a strategy to solve?	Emily solved 6 + 8 by changing it in her mind to 4 + 10. What did she do? Is this a good strategy? Tell why or why not.

How might we assess these components of Procedural Fluency?

Flexibility	Accuracy
Efficiency	**Appropriate Strategy Selection**

Strategy Tracking Table: Addition Facts

Names	Addition Fact Strategies						
	1 more/ 2 more	Combinations that Make 10	Making 10	Doubles	Near Doubles	Find Fives	Applies Commutativity

Kling, G. & Bay-Williams. J. M. (2014). Assessing basic fact fluency. *Teaching Children Mathematics.*