

Problem Posing

Bringing Logic to Problem Solving for At-Risk Algebra Students

NCTM New Orleans
April 10, 2014



Word Problems

- There are 30 students in a classroom. All but 20 of them leave. How many are left in the classroom?
- A cake is exactly \$9 more than a cup of coffee. Together, they cost \$10. How much is a cup of coffee?
- Three cars travel on a highway at 75 miles per hour. What is the speed of one car?

A Different Approach

- Develop mathematical habits of mind
- Be *producers*—not just consumers—of mathematical language

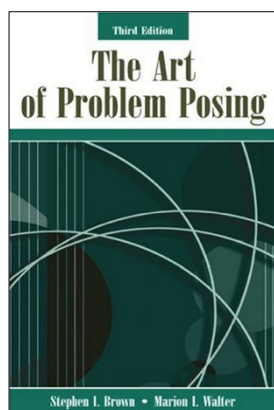
“Education should not merely provide completed information; it should leave children with some unanswered questions, which they can follow up for themselves later.”

– W.W. Sawyer

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The Art of Problem Posing



- Problem posing is necessary in problem solving
- Generates deeper understanding and knowledge
- Shifts mathematical authority

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Focus

- Focus on the **given information** and figure out what one can deduce from it.
- Focus on the **question** and figure out what information one would need to answer it.
- Focus on a potential **answer** and figure out the process needed to test it.

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What Can You Say For Sure? (Tail-less)

	pennies	nickels	dimes
• I have 17¢ in my hand.			
You have no quarters.	17	0	0
You could have 17 pennies.	12	1	0
You must have at least 2 pennies.	7	2	0
You can have at most one dime.	7	0	1
	2	1	1
	2	3	0
The number of pennies ends in 2 or 7.	Any more?		
You will never have more dimes than pennies.			
You must have at least four coins.			

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What Kind of Questions Can You Ask?

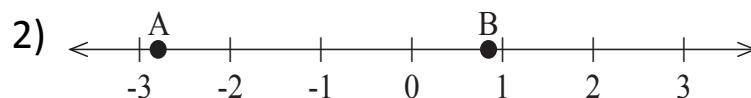
- How many/how much...
- How many ways...
- What is the largest/smallest...
- Will ____ work for the value of...
- Are there any other ways...
- What is better...

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What Can You Say For Sure? (Tail-less)

1) The sum of three consecutive numbers is 63.



3) Rodney is starting a horse-grooming business. His initial expense will be a one-time cost of \$200 for equipment. His earnings will be \$40 per horse groomed. (MCAS 2007 Grade 10 #17)

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What If Not

- I have 17¢ in my hand.
 - What if not our standard currency?
 - 3¢ and 5¢ coins?
 - 2¢ and 4¢ coins?
 - What if not exactly 17¢, but less than 17¢?
 - With 7 pennies and 2 nickels, what other values can be made?
 - With 2 pennies and 3 nickels?

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What If Not (Brown and Walter)

- Pythagorean Equation $a^2 + b^2 = c^2$
 - What if not an exponent of 2?
 - $a^3 + b^3 = c^3$
 - $a^n + b^n = c^n$
 - What if not equal?
 - $a^2 + b^2 > c^2$
 - $a^2 + b^2 = c^2 + 1$

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What Information Do I Need? (Headless)

- 1) How long will it take Mary to drive to Chicago?
- 2) Can I afford this car?
- 3) A math club is planning to sell candles to pay for the cost of attending the regional math competition... How many candles does the club need to sell? (TAKS 2009 Grade 10 #24)

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What Information Do I Need? (Variation)

- *Problems Without Figures* (Gillan, 1909)
 - 1) If you know how many pickets are in a fence that surrounds a square garden, what two things must you know besides, and what will you do to find how many square feet are in the garden?
 - 2) How can I find how many times a wagon wheel will turn in going three miles?
 - 3) A boy bought oranges, apples, and grapes. The oranges cost twice as much as the grapes. If you know the cost of the apples and the total cost, how will you find the cost of the oranges?

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Guess-Check-Generalize

- Alpie is one-third of the age of Betsy. In 14 years, Alpie will be half the age of Betsy. How old is Alpie now?

Guess: Alpie is 8

Betsy: $8 \cdot 3 = 24$

In 14 years:

$$8 + 14 = 22$$

$$24 + 14 = 38$$

$$22 \stackrel{?}{=} \frac{1}{2} (38)$$

Guess: Alpie is 100

Betsy: $100 \cdot 3 = 300$

In 14 years:

$$100 + 14 = 114$$

$$300 + 14 = 314$$

$$114 \stackrel{?}{=} \frac{1}{2} (314)$$

Alpie is a

Betsy: $a \cdot 3 = 3a$

In 14 years:

$$a + 14$$

$$3a + 14$$

$$a + 14 = \frac{1}{2} (3a + 14)$$

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Guess-Check-Generalize

- Driving from Boston to New Orleans, we drove an average of 70 mph on the way down and 60 mph on the way back up. Our return trip took 3.5 hours longer. How far away is Boston from New Orleans?

Guess: Distance is 500

Down: $500/70$

Up: $500/60$

$$500/60 \stackrel{?}{=} 500/70 + 3.5$$

~~Guess~~ Distance is d

Down: $d/70$

Up: $d/60$

$$d/60 \stackrel{?}{=} d/70 + 3.5$$

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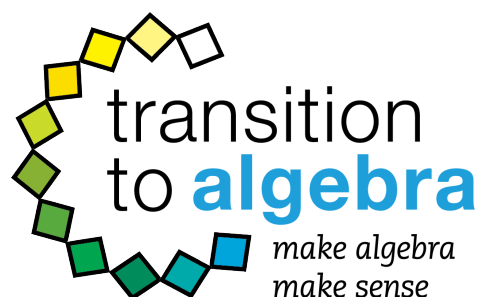
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Transition to Algebra



Full-year algebra-support curriculum with student & teacher materials that supports the Common Core Standards for Mathematical Practice

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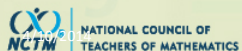
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Bonus!

- How many different 6-story towers can you make with two blue blocks and the rest white?
- Given two equilateral triangles, find a third one whose area is equal to the sum of the areas of the other two.