Modeling Problems That Bring the Common Core to Life

Maria Hernandez

The NC School of Science and Mathematics
hernandez@ncssm.edu

NCTM Conference
New Orleans
Spring 2014

Image from *jazz.com*

COMMON CORE STANDARDS FOR MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. **Model with mathematics.**
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

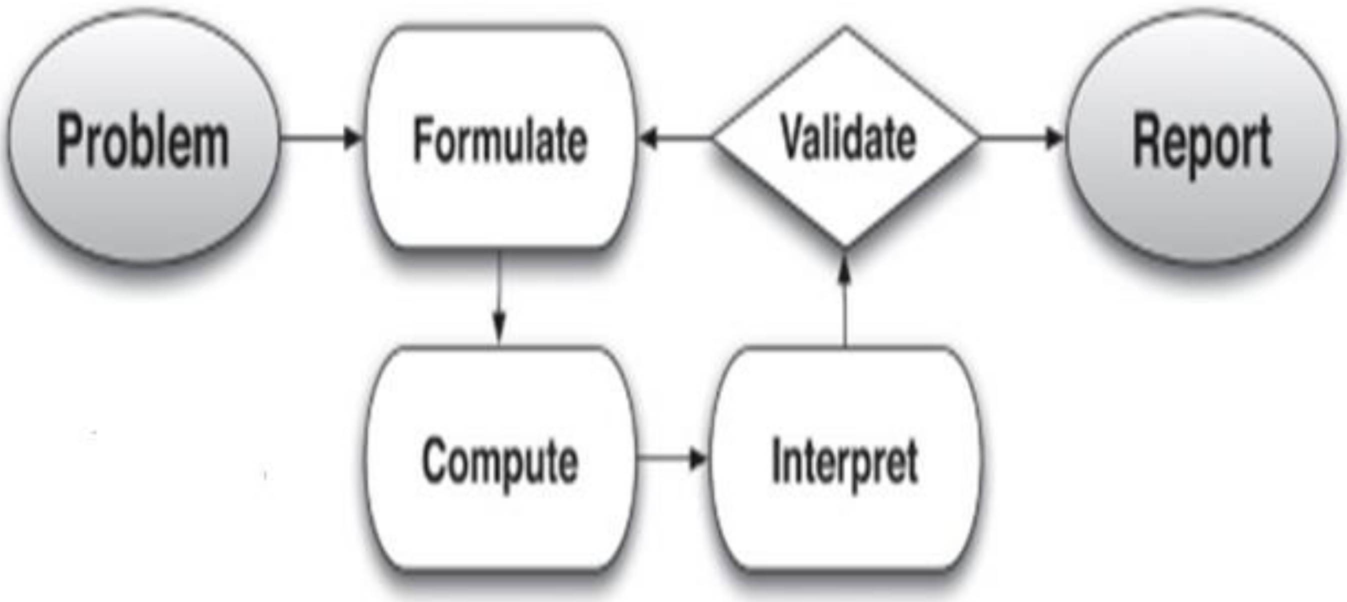
WHAT IS MATH MODELING? (CCSS)

“Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions.

Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.”

MATH MODELING CYCLE

From CCSS



THIS SESSION: WHAT I PROMISED YOU …

- Mathematical Modeling
- Data collection via probes
  and videos

- Analysis of data using piecewise, quadratic and exponential functions

TWO REAL-WORLD SCENARIOS

- Water Jug - Video Data Collection via LoggerPro
- Ball Bounce Data Collection via
  Motion Detectors

Mathematical modeling goals:

Depend on the questions YOU want to answer.

WATER JUG COLLECT DATA USING VIDEOS AND LOGGERPRO

Click here to show the movie

Will demo how to collect data:

- Mark axes,
- provide some scale measure, and
- synchronize the “start” of the movie to
- begin data collection.

Can also use Video Physics App to collect data from videos.

SOME QUESTIONS TO CONSIDER

1. Does the water flow out of the jug at a constant rate?
2. When is it flowing the fastest? The slowest?
3. How does the volume of the liquid vary over time?
4. How does the volume of the liquid change as a function of the height of the liquid?

Our Goal: Find a model for the height of the water in the jug as a function of time.

GRAPH

Sketch a graph of the height of the water in the jug over time.

- What do you notice about the graph?
- How will we define the domain of the function that we use to model the data?

WATER JUG SOLUTION

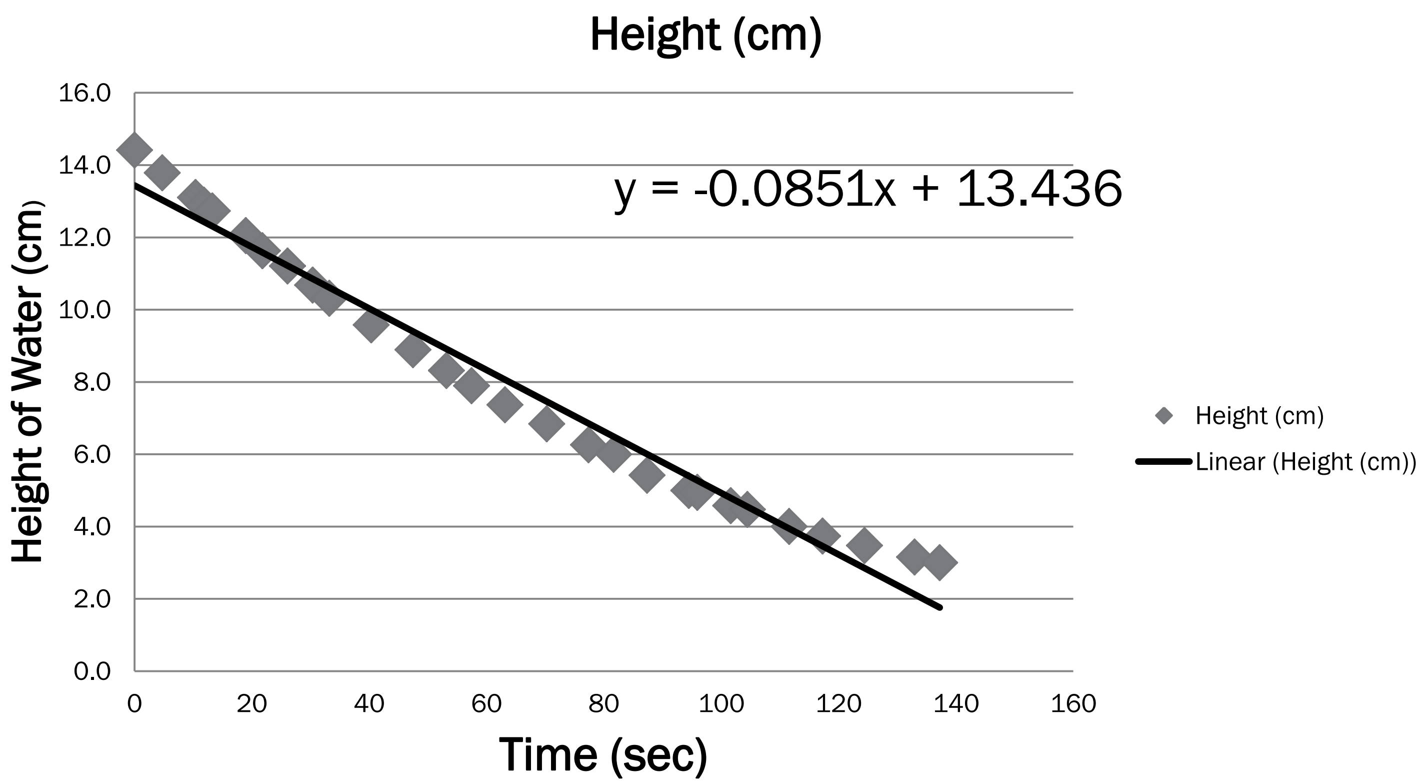
Questions to consider as we move ahead:

- Are the data linear? How do you know?

- Let’s take a look at a linear model and the residual plot. Can do this in excel, calculator, GeoGebra…

We will do it in Excel

DATA WITH LINEAR MODEL



RESIDUAL PLOT FOR LINEAR MODEL

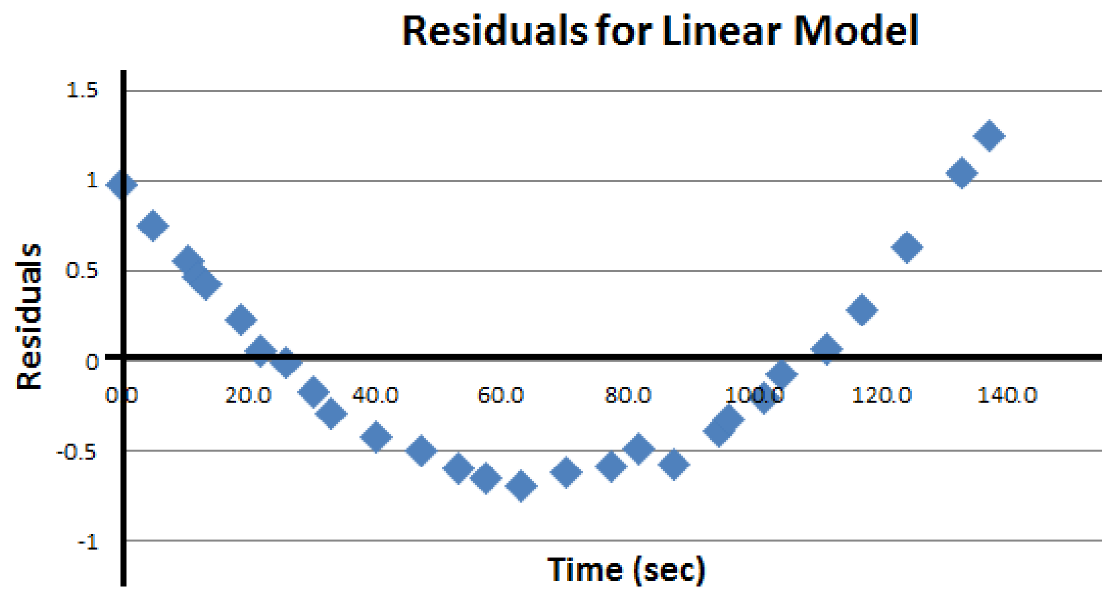
Residuals are defined to be the difference in the actual y-value of the data and the model 's predicted value:

Residual =

Actual - Predicted

Model

sy = -0.0851x + 13.436



DATA ANALYSIS –
HOW CAN INVERSE FUNCTIONS HELP US FIND A MODEL?

What do we know about inverse functions?

If our original function is $f(x)$,

- What f “does”, the inverse, f^{-1} , “undoes”.
- The domain of f is the range of f^{-1} .
- The graph of f is the graph of f^{-1} reflected about the line $y = x$.
- When we compose a function and its inverse we get the original input value. Or in function notation:

$$\boldsymbol{f\left(f^{-1}(x)\right) = x}$$

IN FACT WE KNOW MORE…

Consider a data set that appears to "be" quadratic. I claim if the data can be modeled by a function of the form $y = (ax + b)^2$ for a limited domain, then we can take the square root of the y-values and the ordered pairs $(x, \sqrt{y})$ should produce a linear data set.

This method is called **re-expressing the data.**

OTHER EXAMPLES:

1. Model: $y = (ax + b)^n$

   The ordered pair $(\mathrm{x}, \sqrt[n]{y})$ should be linear data set.

2. Model: $y = a * e^{bx+c}$

   The ordered pair $(x, \ln(y))$ should be linear data set.

We can refer to these as linearization or re-expression techniques and the data as the linearized data.

WHAT DO YOU NOTICE ABOUT ALL OF MY EXAMPLES?

The toolkit (or parent) function is NOT vertically shifted.

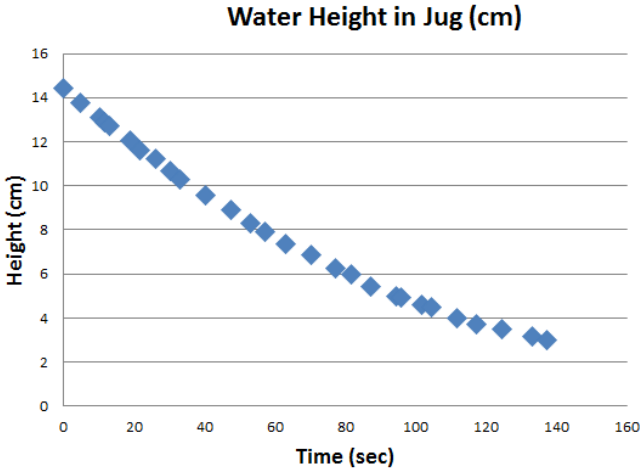
- For $y = (ax + b)^n$, this means the vertex is on the x-axis.
- For $y = a * e^{bx+c}$, this means the horizontal asymptote is the x-axis.

BE CAREFUL WHEN USING THESE RE-EXPRESSION TECHNIQUES

You may need to "adjust" the data before performing the linearization technique.

This may involve only shifting the data, but it may also involve both shifting and reflecting the data.

IF A NONLINEAR MODEL IS A BETTER CHOICE.



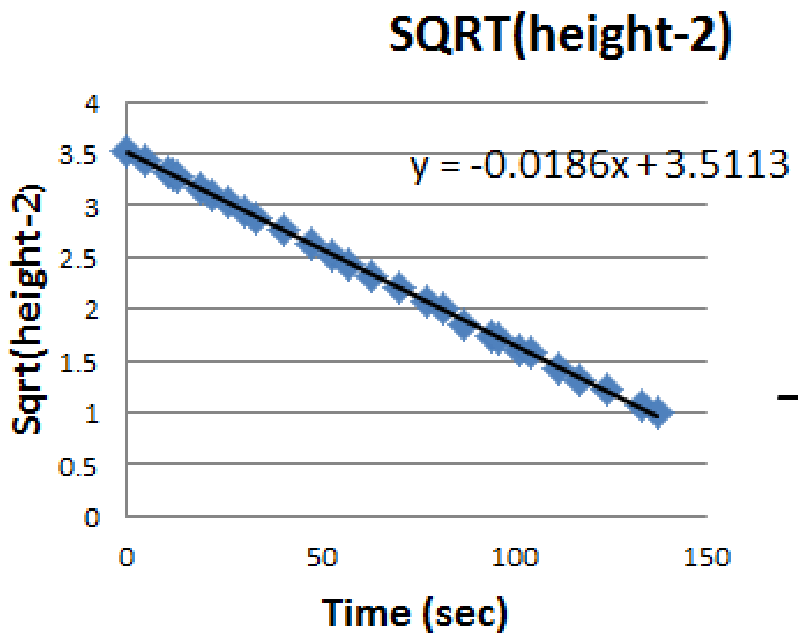
- What is an appropriate model?
- If a quadratic function is an appropriate model, is the vertex on the x-axis (time axis)? How do you know?
- How can we linearize the data using inverse functions?
- How can we find the nonlinear model after linearizing the data?

SHIFT AND RE-EXPRESS THE WATER JUG DATA

Let’s see the results in our excel file

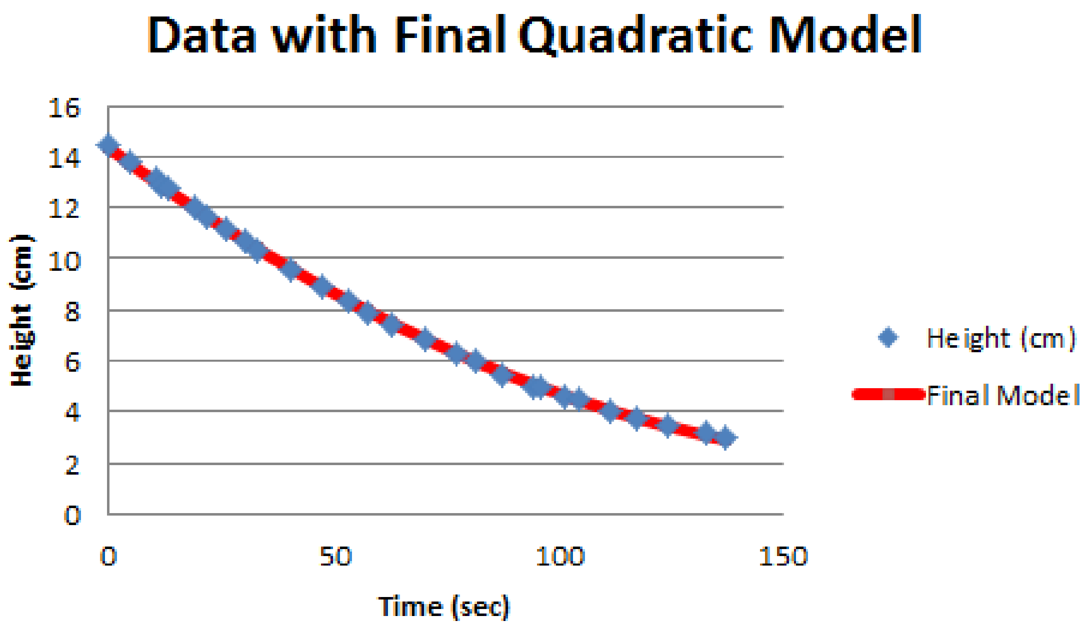
LINEAR MODE FOR THE RE-EXPRESSED DATA

Need to work our way back to the quadratic model



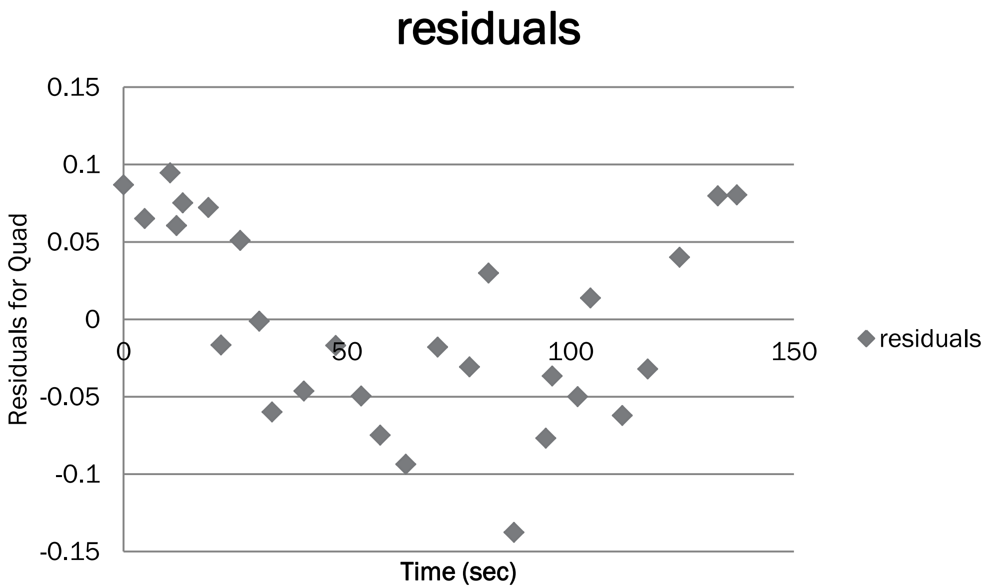
FINAL NONLINEAR MODEL

Look at residuals to assess goodness of fit



RESIDUALS FOR NONLINEAR DATA

Can work with vertex to improve these some.



There is a very cool extension for calculus students involving difference quotients…

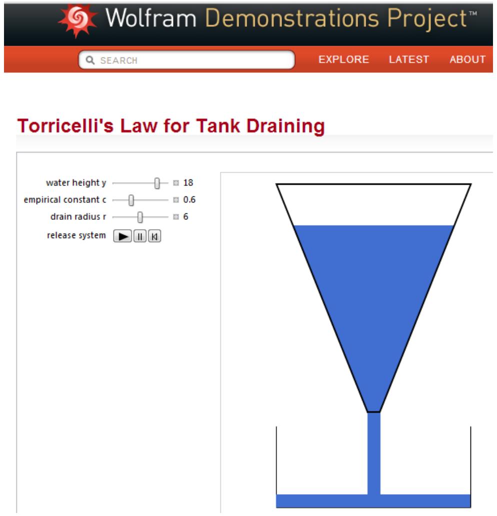
YOUR MODEL

- What is it good for?
- What are its limitations?

Quadratic model can be verified using Calculus methods!

Toricelli's Law

Wolfram's Demo Project



BALL BOUNCE DATA COLLECTION

LoggerPro or TI/Vernier Probes

- Motion detectors
- Temperature probes
- Light sensors
- Microphones
- Pressure sensors

LET'S USE A MOTION DETECTOR

- Hold the motion detector above the ball.
- Drop the ball under the motion detector and record the distance from the ball to the detector as the ball bounces.
- We will collect data for about 5 seconds.
- What will the graph of the data look like if we have time along the horizontal axis and the distance from the detector to the ball along the vertical axis?
- Sketch a graph on your papers.

LET'S DO IT!

Exit to LoggerPro or TI calculator to collect the data.

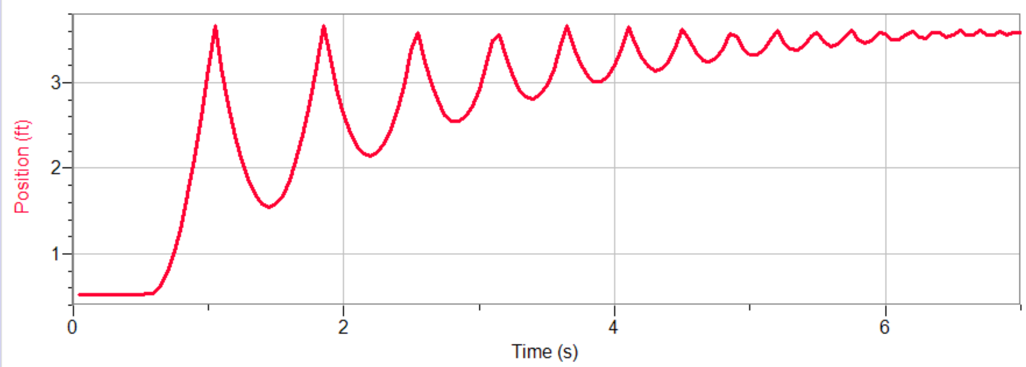
SOME QUESTIONS TO CONSIDER

1. Why does the graph look upside-down?
2. What happens to the lowest points as time increases?
3. How can we transform the data so that its graph looks like our expected graph?
4. Can we find a model for each of the “hoops”?
5. Can we find a model for the max/min points?

We will try to answer Questions 4 and 5.

DIVIDE AND CONQUER

- Assign each group one of the hoops. Using your knowledge of transformations of functions find a model for your hoop.
- Put the models all together – will need to define the domain for each hoop and use a piecewise function to find a model for the entire data set.

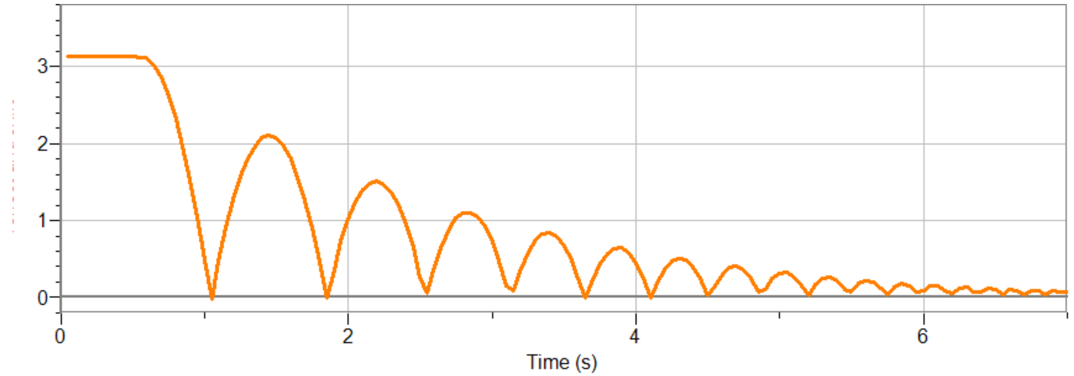


BALL BOUNCE LAB PART II: REBOUND HEIGHTS

You may want to transform the data so that we have rebound heights:
- Requires a reflection about the horizontal axis and a vertical translation. Student handout uses data “as is”.

Then use our knowledge of exponential functions to find a model for the rebound heights as function of the bounce number.

Note: We will call the first bounce, BOUNCE #0



BALL BOUNCE LAB

Can be used over the course of a semester or year:

Mathematical Topics Include:

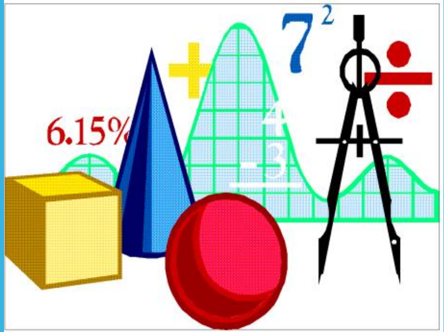
- Transformations of Quadratic Functions
- Piece-wise Functions
- Exponential Functions
- Data Analysis

CC Content Standards are listed in the student handout.

IMPORTANT MATHEMATICAL AND PEDAGOGICAL IDEAS

- Collecting real world data using videos and probes can be powerful for students and can help them see math in the world around them!

- Straightening data using inverse functions can set the stage for future data analysis in Statistics and is a great application of inverse functions. It can also help your students stay away from “shopping for models” using the “magic” regression buttons.
- In fact, it can take some of the magic out of the magic regression buttons.



DATA AND MOVIE ON THE WEB FOR YOU!

- Data is provided in Excel spreadsheet and LoggerPro file
- Handout to be used with calculator or other devices

Information about Tools for Data Collection:

- LoggerPro http://www.vernier.com/support/updates/logger-pro/
- Easy Link http://www.vernier.com/products/interfaces/ez-link/
- Video Physics http://www.vernier.com/products/software/video-physics/
- Graphical Analysis for iPad http://www.vernier.com/

RESOURCES

- Summer Math, Science and Technology Conference at Phillips Exeter https://www.exeter.edu/summer_programs/7325.aspx
- NCSSM Teaching Contemporary Mathematics Conference : Durham, NC, January 2015

http://courses.ncssm.edu/math/tcm/TCM2014/

- NCSSM Post AP Projects http://www.ncssm.edu/courses/math/apcalcprojects/

MORE RESOURCES

- NCSSM Advanced Functions and Modeling/Algebra 2

  Recursion and Swing
  - http://www.dlt.ncssm.edu/AFM/topic.htm
  - http://www.dlt.ncssm.edu/stem/content/lesson-1-introduction-recursion
  - http://www.dlt.ncssm.edu/stem/content/swing-lab-documents
- NCSSM Math I Project http://betterlesson.com/unit/144785/math

- Inspirations Video Cristobal Vila

http://www.etereaestudios.com/docs_html/inspirations_htm/movie_a.htm

THANK YOU!

Enjoy making your own movies and collecting and analyzing "live" data with your students!

Send me email if you have questions or want to share how you use these modeling problems in your classes!

Presentation available on NCTM site.

Send me email if you want handouts and data files and movie. Will send you a website address

hernandez@ncssm.edu

mhernandez@amplify.com