

In this lab you will explore how the height of a liquid in container changes as the water drains from the container through a hole. After collecting some data from a video clip, you will use your knowledge of functions and data analysis to explore the height of the liquid over time and find a model for this data set.

1. Opening the movie clip in Logger Pro.....

Open the Logger Pro software. Under **Insert**, select **Movie**, and then navigate to the folder in which you saved your movie. The first frame of the video clip should appear in a window.

2. Viewing the movie clip.....

View the movie by pushing the play button on the left of the bar at the bottom of the movie window. Several frames may be played at the start before the water is poured in the glass. Rewind the movie by dragging the slider at the bottom of the window all the way back to the left. Also try using the forward and backward step buttons to view the video frame-by-frame. Note that on the top left of the movie window the frame number (or the frame time) is indicated for the frame that is currently displayed in the window.

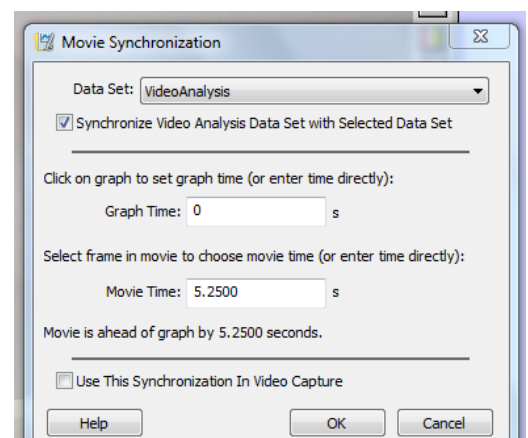
STOP after viewing the movie clip and answer Questions A and B.

- Do you think that the liquid drains at the same rate throughout the time period? Explain.
- Make a sketch of the height of the liquid over time with time in seconds on the horizontal axis and height on the vertical axis. How does your graph correspond to your answer to part A?

3. Synchronizing the time on the video....

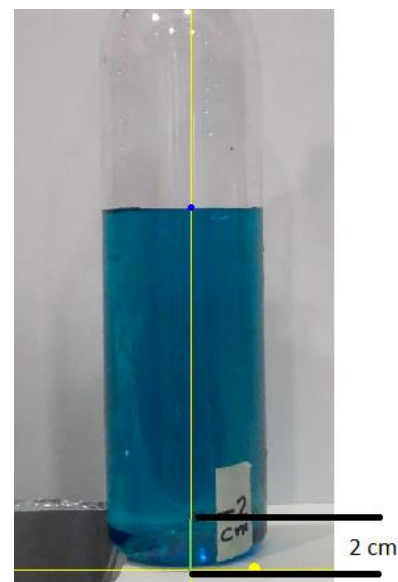
You will now analyze the video clip by marking the top of the liquid in each frame of the video.

- Drag the corner of the movie window to make the clip as large as possible for more accurate data collection.
- Click the button in the lower right corner with three red dots. This will open video analysis buttons.
- Advance the movie to the point when the liquid first starts draining out.
- Click the button with the arrows to the left of the button with three red dots.
- Change the Graph Time to 0 and then click **OK**. This will sync your data points with a graph so that you start time just when the liquid begins to drain. See the window to the right.



4. Marking your origin and scaling the position data.....

- Click the third button down on the right side with the graph to **set an origin**. You should set the origin at the bottom approximate middle of the container.
- You need to create a scale for your video.
- The mark on the container '2' measures 2 cm from the bottom of the container. As shown to the right.
- Scale your graph by clicking the **horizontal ruler button** on the right panel. Click and hold the mouse at the first point you measure from, then while holding the mouse down, drag to the other point and release. You should see a green line appear on the video screen as you mark your scale. A box will then come up allowing you to set the actual distance.



5. Measuring position data from the movie clip.....

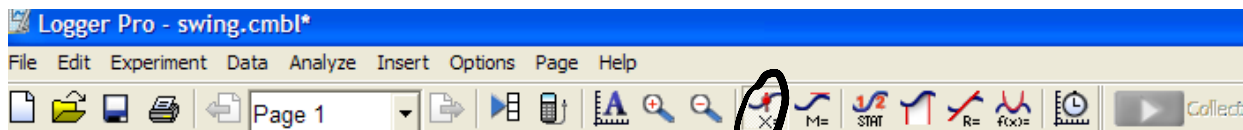
- Now you will want to mark the points on the video. To do this, click the second button down on the right side (red dot with the crosshairs).
- As the liquid is draining from the container, you will mark points at the top of the liquid in the container.
- Advance the video and then mark your points as the water rises. Mark between 15 and 25 data points. You can advance the video by dragging the marker across the bottom of the video window.

Note: To delete any marked points, click on the top button on the right side (white arrow) to select the point and then press the delete key.

6. Viewing the data table for the clip.....

You may notice that the points are being plotted on the graph behind the video. Drag the right edge of the movie window towards the left, so the graph is visible. The data table is shown on the left of your screen. Click on that data table and scroll to the left. You should see time values in the first column (in seconds), and x-, and y-coordinates of the marked points in the second and third columns, respectively. The table also contains dx/dt and dy/dt at each t value. The values shown in red correspond to the red points on the graph, and the values shown in blue correspond to the blue data points on the graph.

You may want to “trace” the points on the LoggerPro graph. To do so, choose the Examine feature on the toolbar shown below. Then move your cursor on the data graph to see the specific coordinates associated with a data point.



Now COPY the data for time and vertical position (Y) into an excel file or calculator and use your knowledge of inverse functions to try to linearize the data.

Ball Bounce Activity Part I Transformations of Functions

Common Core Standards: *F.IF.4 Functions as Models, F.BF.3 Transformations to find Models, F.IF.7 Piecewise Functions, S.ID.6 Create a scatter plot from two quantitative variables, S.ID.6a categorize data as quadratic, explain the meaning of the constant and coefficients in context.*

We will collect data using a CBL device (Calculator Based Lab), a motion detector and a calculator. We will hold the motion detector above a large ball, bounce the ball and collect data for about 4-5 seconds.

1. Sketch a graph of what you would expect if you graphed time along the horizontal axes and the distance from the ball to the motion detector along the vertical axis. Distance will be measured in meters and time in seconds.
2. Sketch a graph of the general shape of the actual data below.

If your graph in #1 is different from that in #2, explain your reasoning in #1.

We will divide the data set into sections so that each group will find a function to represent each parabolic curve in the data set. Use your knowledge of transformations of functions as you determine the model for your piece of the graph.

3. Carefully explain how your group found the function for your section of the data.

Write your function here: _____

4. Explain what the 'y'-value of the vertex of your parabola represents in terms of the physical problem and the ball.
5. Using the other groups' functions along with your function, write a piecewise function that fits the entire data set.
6. Explain how would you transform the data set so that it represents the actual **height** of the ball off the ground from the beginning of the data collection cycle to the end of the cycle?
7. Sketch a graph of the transformed data from #6 above. Explain what the x-intercepts represent in the physical problem and the ball.

Ball Bounce Activity GROUP FORM

Names of Group Members:

Transformations of Functions and Data Analysis – TURN IN ONE OF THESE FOR EACH GROUP

We will collect data using a CBL device (Calculator Based Lab), a motion detector and a calculator. We will hold the motion detector above a large ball, bounce the ball and collect data for about 4 seconds.

Distance will be measured in meters and time in seconds.

We will divide the data set into sections so that each group will find a function to represent each parabolic curve in the data set.

1. Carefully explain how your group found the function for your section of the data.

Write your function here: _____

2. Explain what the 'y'-value of the vertex of your parabola represents in terms of the physical problem and the ball.
3. Discuss how well your function represents the data set. Be specific.
4. Using the other groups' functions along with your function, write a piecewise function that fits the entire data set.
5. Explain how would you transform the data set so that it represents the actual **height** of the ball off the ground from the beginning of the data collection cycle to the end of the cycle?
6. Sketch a graph of the transformed data from #6 above. Explain what the x -intercepts represent in the physical problem and the ball.

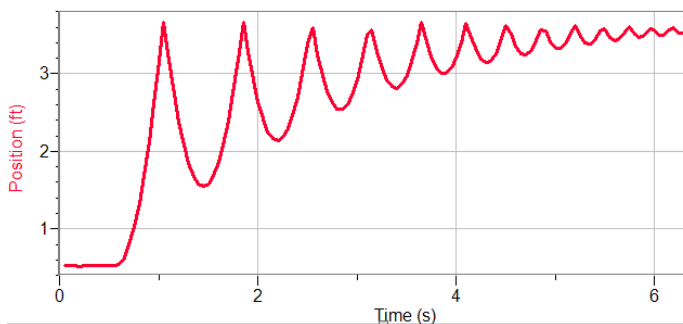
Ball Bounce Activity

Part II – Further Exploration

Common Core Standards: *F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions, F-LE.2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table), F-LE.5. Interpret the parameters in an exponential function in terms of a context.*

Re-Expressing Data

Using the ball bounce data you collected in Part I of this lab, we will explore the minimum values for the graph of the entire data set. The graph of the data you collected in Part I should look something like the graph shown below. The data in the table gives the bounce number and the minimum distance from the ball to the motion detector for the first eleven bounces shown in the graph.



Bounce #	Min Distance(ft)
0	1.55
1	2.141
2	2.547
3	2.814
4	3.01
5	3.14
6	3.241
7	3.323
8	3.381
9	3.427
10	3.466

1. Notice that the minimum values in the graph above are increasing with each bounce. Using the data given in the data table sketch a graph of (bounce#, minimum distance).
2. Describe the long-term behavior of this graph. Explain how this long-term behavior makes sense in the context of the problem.
3. Using re-expression techniques, linearize the data from #1 (bounce #, minimum distance). Note: Since the horizontal asymptote something other than the horizontal axis, you will need to transform the data. Carefully explain how you transformed the data before linearizing it.
4. a. Find the linear regression model for your linearize data.
b. Find the residuals for the linearized data and sketch a scatter plot of those residuals.
5. Using your linear model from #4, find a nonlinear model for the data (bounce #, minimum distance). Show your algebraic work and write your model in the form $D(\text{bounce}) = Pe^{Q*\text{bounce}} + R$, where P , Q and R are constants.
6. Explain the meaning of P , R and e^Q in the context of the problem.
7. How confident are you in using your model to make predictions? Explain.
8. Use your model to predict the minimum distance for bounces #4 and #10.