

# Conjecture and Proof in Geometry: A Practical Unit of Study

In this activity, we use a simple geometric construction to deliver a number of interesting conjectures, followed by the proofs of these conjectures. The vision is to have the class share this construction, and divide into groups to pursue group conjectures and proofs. Instead of the typical, more artificial triangle proofs, these proofs involve quadrilaterals.

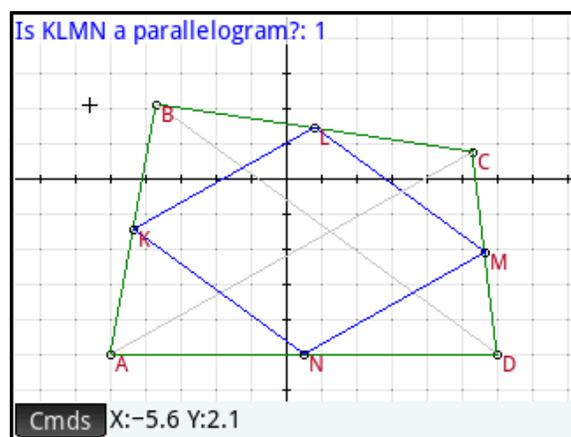
The geometric construction that forms the basis of this activity consists of a quadrilateral ABCD. The midpoints of the sides of this quadrilateral are found and named KLMN. Then quadrilateral KLMN is constructed. Rather than have the students create this construction themselves (which is entirely possible but not done here to save time), we created an app named *MidpointQuad*, based on the built-in HP Prime Geometry app. Because of this, the app uses the primary app keys as described in the table below.

Key	View Name	Description
	App Library	Displays the library of apps
	Symbolic View	Displays the definitions of each geometric object
	Plot View	Displays the geometric construction
	Numeric View	Displays any calculations or measurements in the construction
	Info View	Displays a note explaining the purpose of the app and how to use it

## Getting Started

1. Press  and tap on the icon for the MidpointQuad app

The app opens in Plot view. Press  at any time to return to this view. The green quadrilateral ABCD is the basis of the construction. You can tap and drag to move any of its vertices. You can also select and move any of the sides. The blue quadrilateral is formed by connecting the midpoints of the sides of ABCD. The diagonals AC and BD are also drawn to aid in conjecture and proof. At the top of the view is a test for whether or not KLMN is a parallelogram.



The test can return any of the following five values:

- 0: KLMN is not a parallelogram
- 1: KLMN is a parallelogram
- 2: KLMN is a rhombus
- 3: KLMN is a rectangle
- 4: KLMN is a square

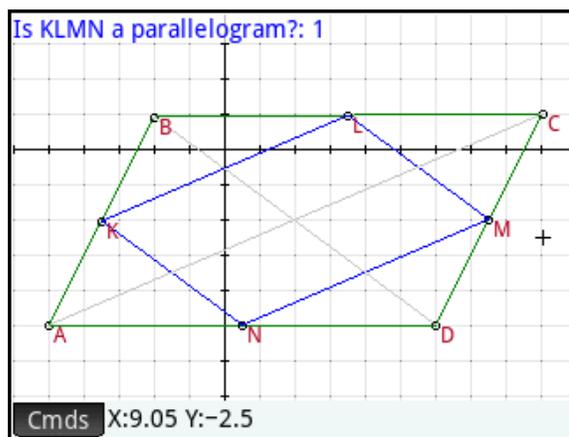
Take a moment and experiment with the construction by moving points A, B, C, or D. You will see that the test never returns zero. Even if you make ABCD concave, the test will continue to return a value of at least one. This leads us to our first conjecture:

The quadrilateral formed by the midpoints of the sides of any other quadrilateral is a parallelogram.

### The Technique

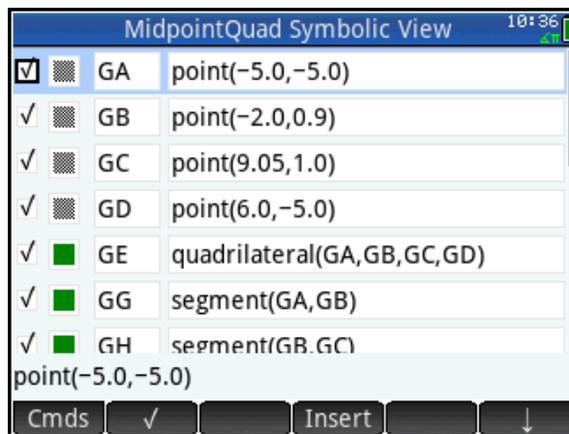
Now let's make ABCD a parallelogram.

2. Move points A, B, C, and D so that they appear to form something close to a parallelogram. In the figure to the right, the opposite sides of ABCD appear to be roughly equal in length and parallel.
3. Press  to open the Symbolic view of the app and observe the definitions of points A, B, C, and D.



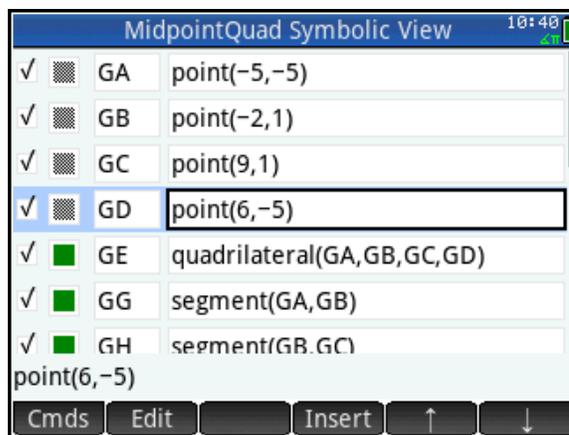
We shall edit the coordinates of these points so that they are more exactly the coordinates of a parallelogram. Specifically, in our case (but not necessarily yours), we will make the following changes:

- GA: point(-5, -5)
- GB: point(-2, 1)
- GC: point(9, 1)
- GD: point(6, -5)



4. Tap on the definition of GA to select it.
5. Tap **Edit** to edit the definition
6. Delete the current coordinates and enter your own. Press **Enter** or tap **OK** when you are done.

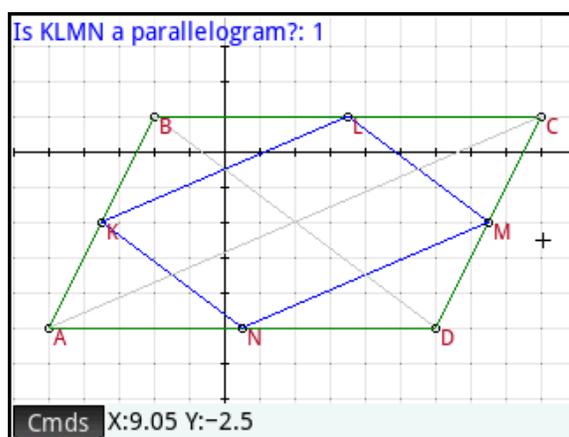
In the figure to the right, the editing has been done to the definitions of A, B, C, and D.



7. Press **Plot** to return to Plot view to see what has changed.

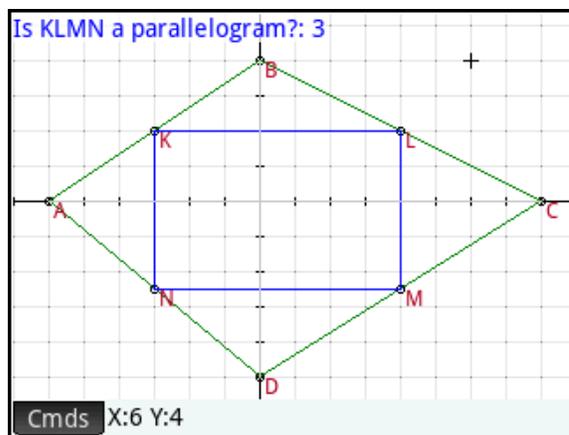
In this case, nothing has changed. Making ABCD a parallelogram has had no effect on KLMN. This example was selected merely to illustrate the technique involved:

- Move A, B, C, or D to form what appears to be a special quadrilateral
- Go to Symbolic view and edit the definitions to make the coordinates exact
- Return to Plot view to see the effects



### Try it yourself

Make ABCD a kite. Once it is a kite, select one of the vertices and press **Enter** to select it. Now you can move the vertex carefully using the directional pad, one pixel at a time. Move the vertex so that ABCD continues to be a kite. In the figure to the right, you could move A or C horizontally. Or you could move points B and D vertically.



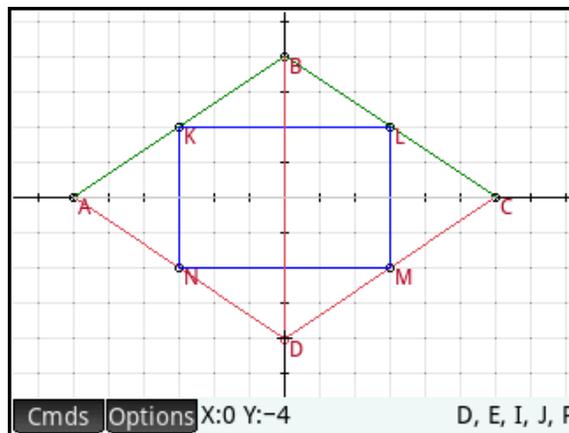
Make a second conjecture based on your experience.

### Do these on your own

Now that you have practiced the technique, try each of the following and make a conjecture for each one.

Make ABCD a rhombus.

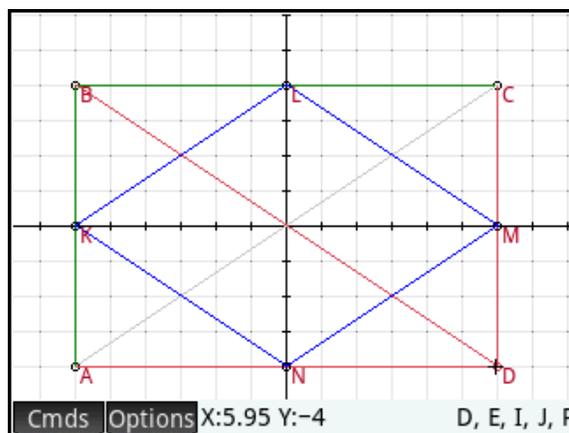
In the figure to the right, the test has been hidden. Press  to go to Numeric view, tap on the check box to select the test, and tap again to toggle it to unchecked (or tap ). Numeric view is used to create tests, measurements, and other calculations. Those that are checked appear in Plot view.



Make Conjecture #3.

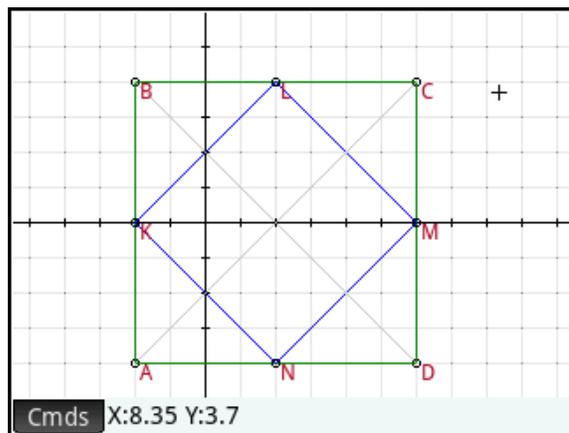
Make ABCD a rectangle

Make Conjecture #4



Make ABCD a square

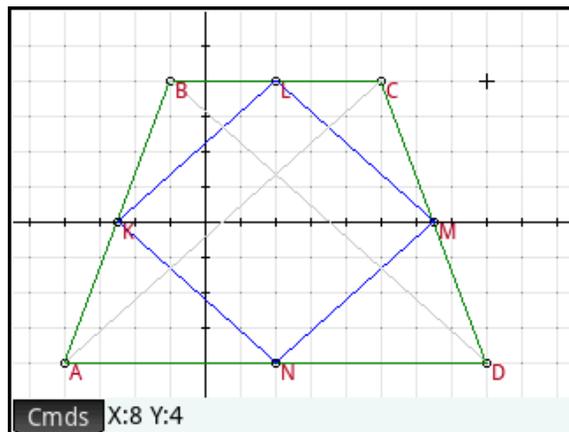
Make Conjecture #5



## Advanced: Trapezoids are Interesting!

Make ABCD an isosceles trapezoid

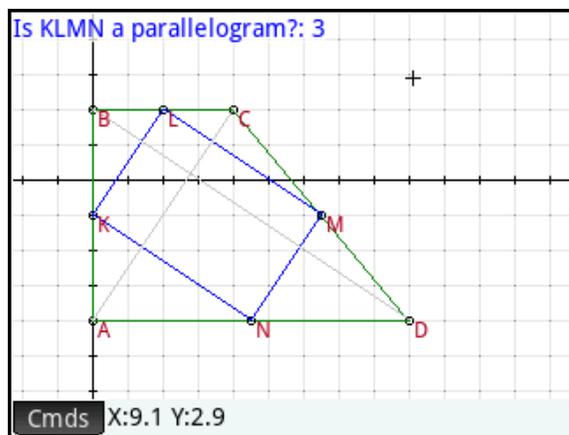
Make Conjecture #6



You can go even further. In the figure to the right, ABCD is a special trapezoid and the test indicates that KLMN is a rectangle.

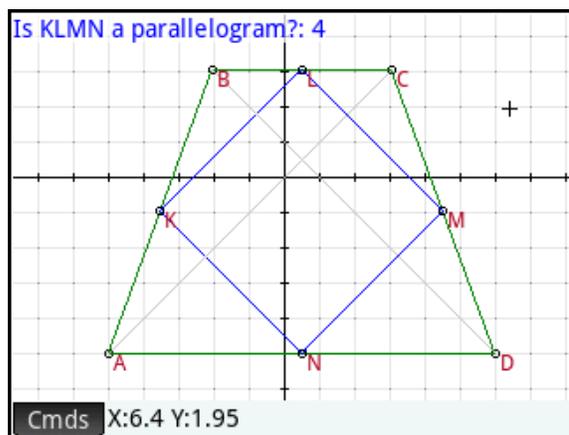
What property or properties must trapezoid ABCD possess in order to make KLMN a rectangle?

Make Conjecture #7



In the figure to the right, ABCD is a very special isosceles trapezoid and the test indicates that KLMN is a square. What special property or properties must isosceles trapezoid ABCD possess in order to make KLMN a square?

Make Conjecture 8



In this set of activities, we used one simple construction to generate 8 conjectures to prove, from the relatively easy to the devilishly tricky. The Numeric, Symbolic, and Plot views of the HP Prime Geometry app encourage open-ended experimentation, the making of conjectures, and the exploration of paths to proofs.