

**TEACHING FOR LEARNING:  
BEST INSTRUCTION BEGINS  
WITH INTENTION**

**NCTM New Teacher Strand 2014  
PK-2, 3-5 Gallery Workshop**

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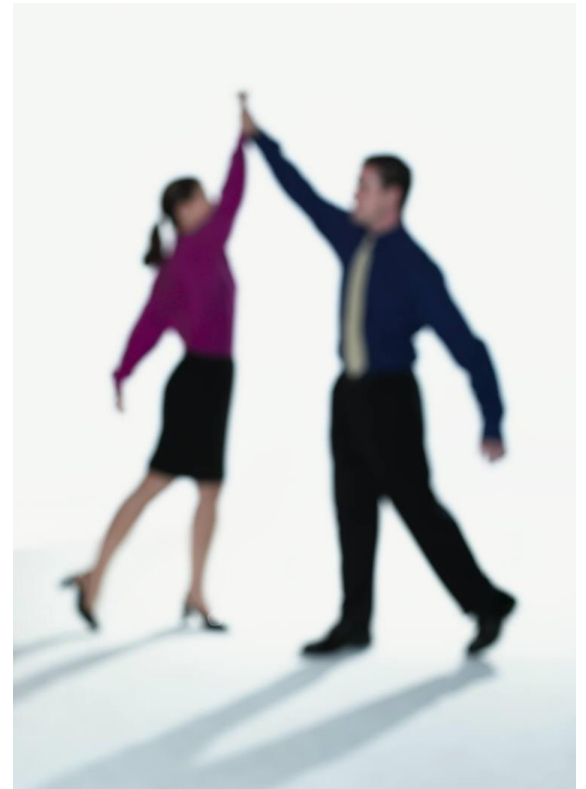
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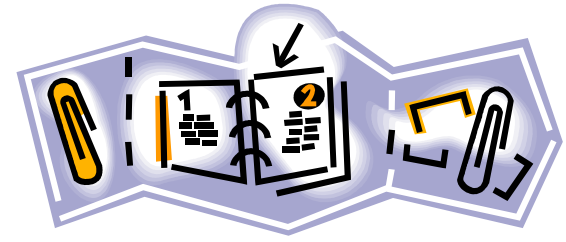


# INTRODUCTIONS: RAISE HAND & SAY “THAT’S ME!”

- Pre-service teacher
- 1st year teacher
- 2<sup>nd</sup> – 3<sup>rd</sup> year teacher
- Veteran teacher
- Grade level is PK-2
- Grade level is 3-5
- Grade level is 6-12



# LESSON PLANNING



- Use students' mathematical thinking as a critical ingredient in developing key mathematical ideas.
- Anticipate what students will do when solving a problem.
- Generate questions to ask during exploration and discussion to promote student engagement and learning.

# THINKING THROUGH A LESSON PLAN



## Protocol Framework

Part 1: Selecting a mathematical task

Part 2: Setting up a mathematical task

Part 3: Supporting students' exploration of task

Part 4: Sharing & discussing task

# THINKING THROUGH A LESSON PLAN



## Protocol Framework

**Part 1: Selecting a mathematical task**

**Part 2: *Setting up a mathematical task***

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# MATHEMATICAL TASKS



## **A Critical Starting Point for Instruction**

- *Not all tasks are created equal – different tasks will provoke different levels and kinds of student thinking.*

Stein, Smith, Henningsen, & Silver, 2000

- *The level and kind of thinking in which students engage determines what they will learn.*

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

# WORTHWHILE MATHEMATICAL TASKS

*According to research studies:*



Having the opportunity to work on  
challenging tasks in  
a supportive classroom environment  
translated into  
substantial learning gains in  
student thinking, reasoning,  
problem solving, and communication.





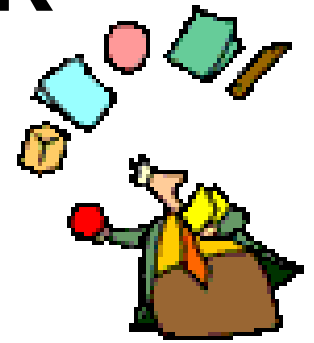
# WHAT MAKES A TASK WORTHWHILE?

A worthwhile task is a project, question, problem, construction, application, or exercise that **engages students** to reason about mathematical ideas, make connections, solve problems, and develop mathematical skills.

(NCTM)

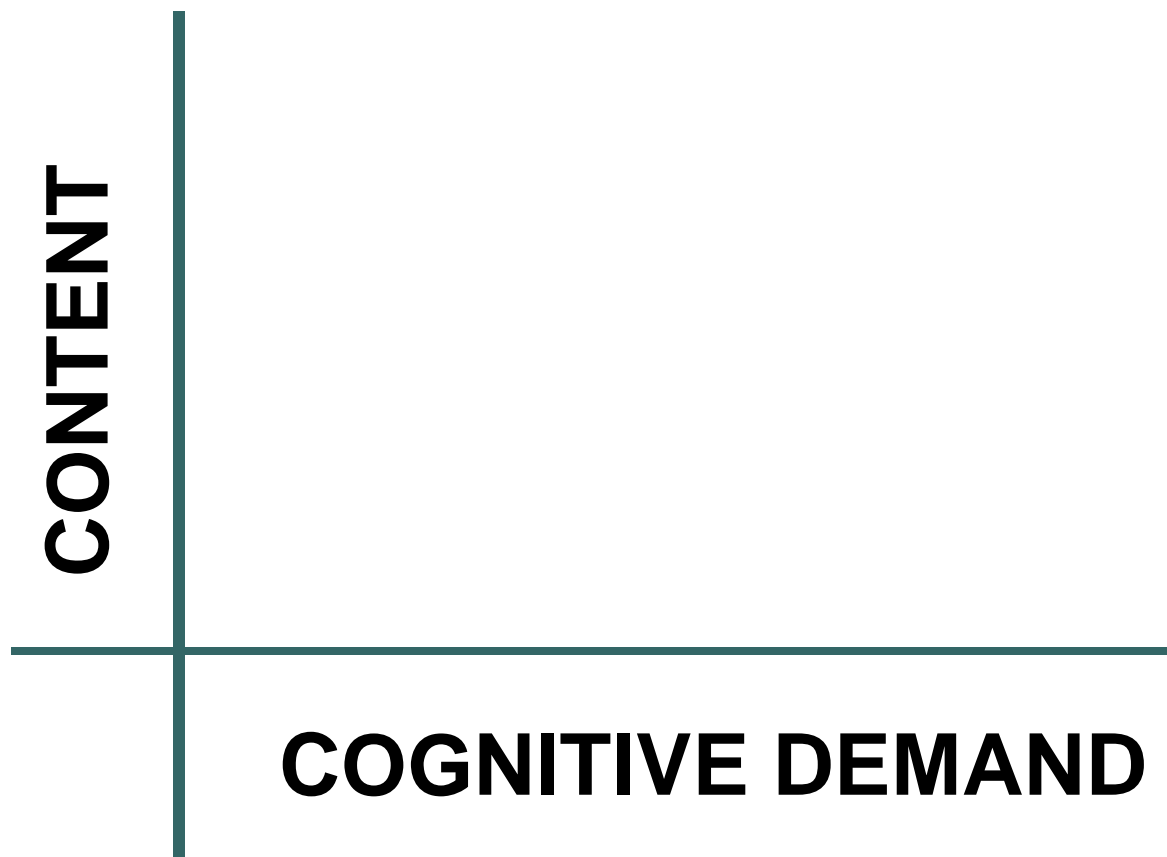


# WHAT MAKES A TASK WORTHWHILE?



- Allows for connections to previous knowledge, experiences, and interests.
- Incorporates multiple approaches and solutions.
- Requires higher-level cognitive demand.
- Facilitates reasoning and communicating mathematically.

● ● ● | **MATHEMATICAL TASKS**



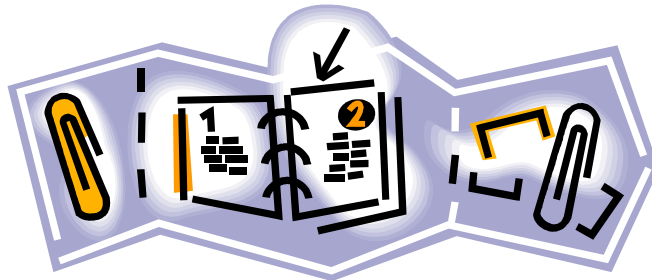


# FRAMEWORK FOR LEVELS OF COGNITIVE DEMAND

<i>Lower-Level Demand</i>	<i>Higher-Level Demand</i>
<b>Memorization</b>	<b>Procedures With Connections</b> <ul style="list-style-type: none"><li>• Represented in multiple ways</li><li>• Focuses on development of deeper concepts</li></ul>
<b>Procedures Without Connections</b> <ul style="list-style-type: none"><li>• No connections to concepts or meaning</li><li>• Focuses on producing correct answers</li></ul>	<b>Doing Mathematics</b> <ul style="list-style-type: none"><li>• Complex, non-algorithmic</li><li>• Explore to understand the nature of concepts or relationships</li></ul>

# WORTHWHILE TASKS

1. Individually, complete all parts of the task.
2. Solve the task in a variety of ways.
3. Work with a partner to compare your work and possible other approaches to problem.



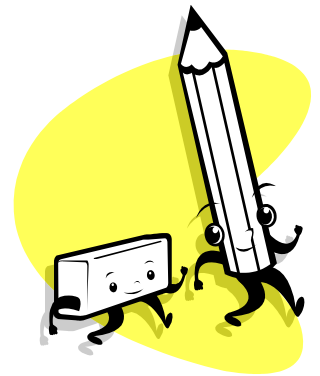
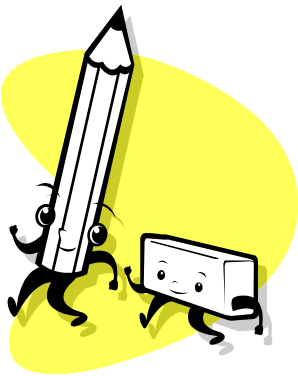
# PENCILS AND ERASERS

The school store sells pencils and erasers.

Pencils sell for  $10\text{¢}$ .

Erasers sell for  $5\text{¢}$ .

You have  $40\text{¢}$  to spend.



How many pencils and erasers  
can you buy?

# TRICYCLES AND WAGONS

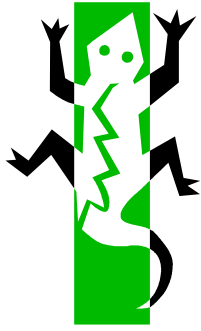


Children go to the park riding their tricycle or in a wagon.

Susie counts 26 wheels at the park today.

Rich counts 8 vehicles at the park today.

**How many tricycles and wagons are at the park today?**



# LIZARDS & BEETLES



Lizards have 4 legs. Beetles have 6 legs.  
There are lizards and beetles in a container.

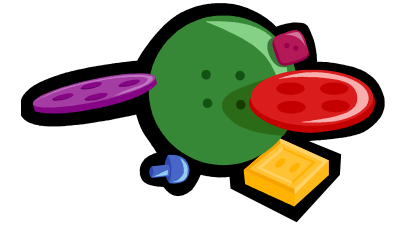
Amy counted 36 legs.

Scott counted 8 critters.

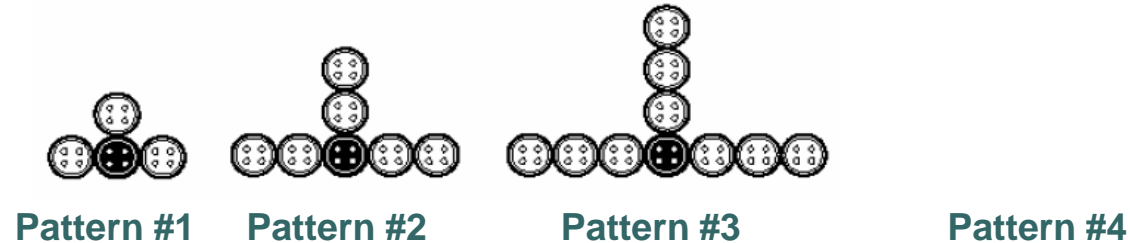
**How many beetles and lizards  
are in the container?**



# BUTTONS TASK



Gita plays with her grandmother's collection of black & white buttons. She arranges them in patterns. Her first 3 patterns are shown below.



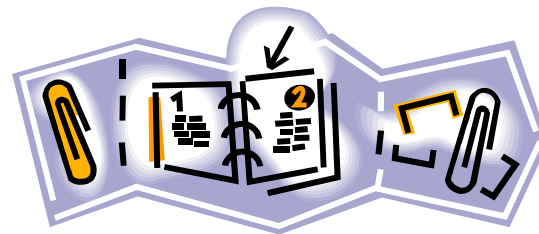
1. Draw Pattern 4 next to Pattern #3.
2. How many white buttons does Gita need for Pattern 5 and Pattern 6? Explain how you figured this out.
3. How many buttons in all does Gita need to make Pattern 11? Explain how you figured this out.
4. Gita thinks she needs 69 buttons in all to make Pattern 24. How do you know that she is **NOT** correct? How many buttons does she need to make Pattern 24?

# WORTHWHILE TASKS

## Question:

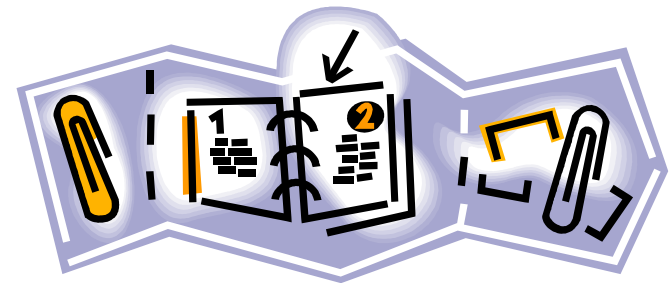
What cognitive demand level would you give to the mathematical task?

Justify your thinking.



# PART 1: SELECTING A MATHEMATICAL TASK

- What are your goals (*content and mathematical practices*) for the lesson?
- In what ways does the task build on students' prior knowledge and experience?
- What are all the ways the task can be solved?
- What challenges might the task present to struggling or ELL students?







# SELECTING TASKS: MATHEMATICAL PRACTICES

“The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.”

The Mathematical Practices describe what ***students*** should be ***doing*** as they learn the mathematics content standards.



# SELECTING TASKS: STANDARDS FOR MATHEMATICAL PRACTICE

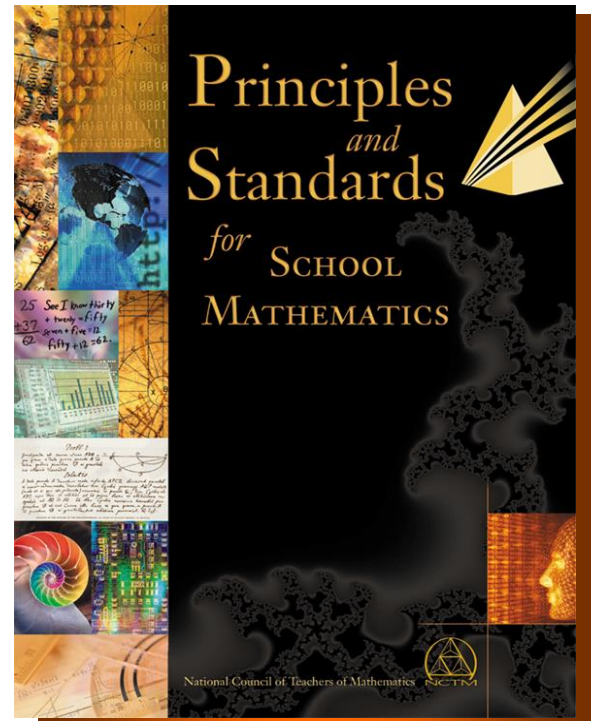
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

# UNDERLYING FRAMEWORKS FOR MATHEMATICAL PRACTICES

National Council of Teachers of Mathematics

Five **Process** Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representations



NCTM (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.



# NCTM PROCESS STANDARDS & STANDARDS FOR MATHEMATICAL PRACTICE

NCTM Process Standards	Standards for Mathematical Practice
Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
Reasoning and Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning.
Communication	3. Construct viable arguments.
Connections	6. Attend to precision. 7. Look for and make use of structure.
Representations	4. Model with mathematics.



● ● ● |

# PART 1: SELECTING A MATHEMATICAL TASK

## MATHEMATICAL GOALS

What are the mathematical goals  
for this task?

What do you want students to know and  
understand about mathematics as a  
result of this task?



# PART 1:

## SELECTING A MATHEMATICAL TASK



### Previous Knowledge:

- In what ways does the task build on students' previous knowledge?
- What definitions and/or concepts do students need to begin work on task?

# PART 1:

## SELECTING A MATHEMATICAL TASK



### Challenges for Students:

- What particular challenges might the task present to struggling students?
- How will you address these challenges?

# THINKING THROUGH A LESSON PLAN



## Protocol Framework

Part 1: *Selecting a mathematical task*

Part 2: **Setting up a mathematical task**

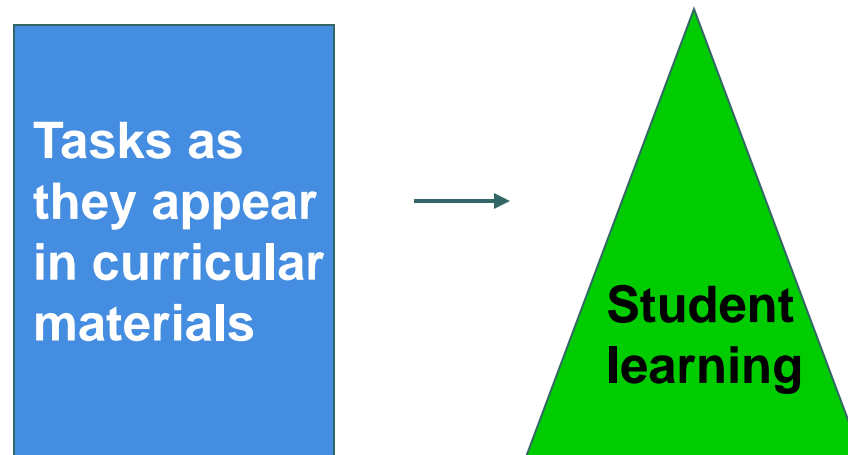
Part 3: *Supporting students' exploration of task*

Part 4: *Sharing & discussing task*



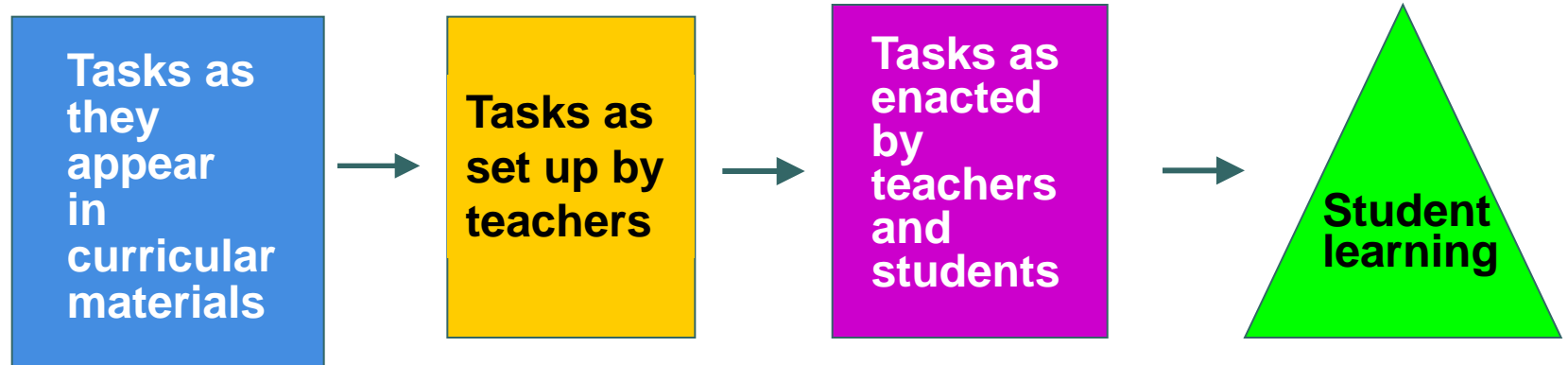
# The Nature of Tasks Used in the Classroom ...

Will Impact Student Learning!



● ● ● | But, WHAT TEACHERS DO  
with the tasks matters too!

## The Mathematical Tasks Framework



*Stein, Grover & Henningsen (1996)*

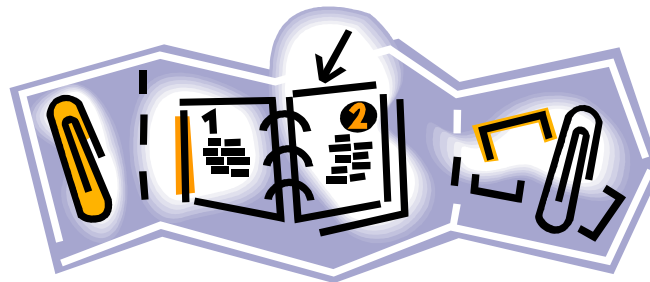
*Smith & Stein (1998)*

*Stein, Smith, Henningsen & Silver (2000)*

# PART 2:

## SETTING UP A MATHEMATICAL TASK

- What are expectations for students?
- What resources or tools will students have to use in their work?
- How will students work (individual / pairs)?
- How will students record and report their work?
- How will we introduce students to the activity?



# FACTORS THAT SUPPORT HIGH-LEVEL DEMAND COGNITIVE TASKS



- Sufficient time to explore – not too much, not too little.
- Scaffolding of student thinking and reasoning by asking thought-provoking questions that preserve task complexity.
- Sustained press for justifications, explanations, and/or meaning.
- Students are provided means of monitoring their own progress.
- Teacher and/or students model high-level thinking and reasoning.



# THINKING THROUGH A LESSON PLAN



## Protocol Framework

Part 1: *Selecting a mathematical task*

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# FACTORS THAT IMPEDE HIGH-LEVEL DEMAND OF COGNITIVE TASKS



- Problematic aspects of task become routinized.
- Emphasis shifts from meaning, concepts, or understanding to correctness/completeness of answer.
- Not enough time to wrestle with demands of task or too much time is allowed (classroom management).
- Students not held accountable for high-level products or processes.

# PART 3:

## SUPPORTING STUDENTS' EXPLORATION



As students work, what questions do we ask to:

- Help a group get started or make progress on the task?
- Focus students' thinking on the key mathematical ideas in task?
- Assess students' understanding of key mathematical ideas, problem-solving strategies, or representations?
- Advance students' understanding of mathematical ideas?
- Encourage *all* students to share their thinking with others or to assess their understanding of their peers' ideas?

# PART 3:

## SUPPORTING STUDENTS' EXPLORATION



### How will you ensure students remain engaged in task?

- What assistance will you give to a student who becomes frustrated?
- How will you extend task to provide additional challenge to those who finish before others?
- What will you do if students focus on non-mathematical aspect of activity?

# PART 3: SUPPORTING STUDENTS' EXPLORATION

## Additional support for students

- Hint Cards



- Definition of terms in pictorial form
- Question that connects prior learning to new task
- Way to modify task

- Think Beyond Cards

- Modify rules for greater challenge

# THINKING THROUGH A LESSON PLAN



## Protocol Framework

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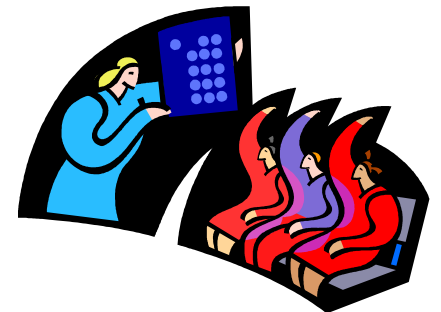
Part 3: *Supporting students' exploration of task*

Part 4: **Sharing & discussing task**

# PART 4:

## SHARING AND DISCUSSING THE TASK

- In what ways will the order in which solutions are presented help develop students' understanding of the mathematical ideas of the task?
- What specific questions will you ask so students will:
  - Make sense of the mathematical ideas to learn?
  - Expand on and/or question the solutions shared?
  - Make connections among the strategies presented?
  - Look for patterns?
  - Begin to form generalizations?





# FACILITATING MEANINGFUL DISCUSSIONS

**Establish appropriate expectations for student participation.**

- Students always supply justifications for why their thinking makes sense.
- Students provide counterexamples.
- Teacher asks other students to repeat peers' responses.
- Classmates pose questions to student making presentation.







# FACILITATING MEANINGFUL DISCUSSIONS

**Teacher decides which aspect will be focus of discussion.**

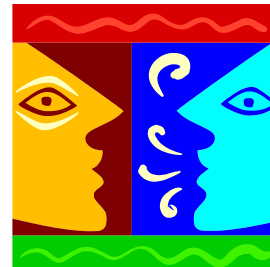
- Use of various strategies
  - Decide if valid, reasonable for context of problem
  - Compare for similarities (differences) or efficiency
- Student errors
- Generate different ideas with which to grapple



# FACILITATING MEANINGFUL DISCUSSIONS

**Teacher encourages all students to communicate their thinking in writing.**

- Create a representation on paper (words, picture, diagram, table)
- Record key discussion points
- Scaffold process until students are able to write independently.
  - Whole class
  - Partner
  - Individual



# THINKING THROUGH A LESSON PLAN



## Useful Tool: Plan, Teach, Reflect

- Think deeply about specific lesson in how to advance students' mathematical understanding
- Shift emphasis to student thinking rather than teacher action
- Collaboration with other teachers

# THINKING THROUGH A LESSON PLAN



A problem-solving atmosphere allows all students at different levels to develop their understanding of mathematics by engaging in relatable problem contexts while also deepening their understanding of mathematics concepts.

# THINKING THROUGH A LESSON PLAN



- Choose problems purposefully.
- Plan questions/support ahead to develop students' understanding of mathematics.
- Expect all students to verify/justify their strategy and answer.
- Choose which strategies and their sequence to share in large group sharing.
- Establish expectations and purposes for listening and understanding classmates' strategies.

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