# Instructional Strategies for Students Who are Struggling in Mathematics 

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 Tech Pred Teamwork Tech Prep

Mira Mira!

Rules: Choose any card and reflect the object or letters using a mira. If you want to reflect your name, choose the blank card and have the teacher write your name for you to reflect.
Math Skills: Practice reflecting a figure over a line. Check to see if the figure reflects over a line of symmetry.

Extra Practice: http://www.math-drills.com/geometry.shtml scroll down to the reflections section

## gRODUCTIVE RRODUCTS:

## Rules:



Materials Needed: playing cards (Ace=1)-9 + King (King=0), one game board per player, pencil The object of the game is to make the greatest product. The deck is placed face down. A card is drawn and placed face up. Each player selects a space on their board and writes the number of the card on it. Five more cards are drawn and players proceed to fill in their game boards. Once all spaces are filled, players complete the multiplication. The player with the greatest product is the winner. As players have more experience with this game, they will develop strategies to maximize their chances.

## Math Skills:

Multiplying Multi-Digit Numbers, Problem Solving
Extra Practice:
www.aaamath.com
www.mathisfun.com

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## Rules:

Participants will first be asked to quickly solve an addition and multiplication problem without calculator or paper and pencil. We will then discuss two methods to solve the same problems using decomposition in conjunction with "10's buddies" for the addition problem and with Distributive Property on the multiplication problem. Participants will them try the strategies on their own

## Math Skills:

Mental Addition and Multiplication

## Extra Practice:

Addition with 10's Buddies:
http://www.theteachersguide.com/pages/printouts/math/addition/additionfindingsumsoften.pdf
Adding with decomposition: http://www.homeschoolmath.net/teaching/a/adding_2_digit.php
Multiplying with Distributive Property: http://www.themathpage.com/arith/Ar_Pr/mul1_2.htm

## Rules:

Students will make their best estimation of how many pieces of candy are in the jar and record it on the paper provided. The student with the closest guess wins the candy!

## Math Skills:

Students will sharpen their estimation skills.

## Extra Practice:

http://www.mathsisfun.com/numbers/estimation-game.php
http://www.mangahigh.com/en_us/games/iceicemaybe
http://www.estimation180.com/


## Rules:

Students will play a game (Cisco BinaryGame) where they must decipher the binary code on the iPad. BinaryGame requires students to be able to add and subtract numbers to "break the code".

1) Turn "on and off" numbers to add up to the correct answer.
2) Based on the numbers turned "on and off", add up numbers to find the correct answer.

The game is similar to Tetris in that the binary problems appear in rows. If the student fails to answer a problem in time, another row containing a new binary problem will appear above the current problem. Like Tetris, if the student lags behind in answer the problems, the "wall" of problems will grow.

## Math Skills:

Students become comfortable in using another number base other than base ten (binary).
Students must quickly add $1,2,4,8,16,32,64$, and 128 selectively to represent decimal numbers from 1 to 255.

## Extra Practice:

Students can practice by loading the Cisco Binary game on iPhones or iPads. The Cisco Binary Game is a free app that can be downloaded by anyone with an apple product from the iTunes App Store. If the student does not have an iPhone or iPad, the game can be practiced at the following website:
http://forums.cisco.com/CertCom/game/binary_game_page.htm

## SHIFTING SHAPES!

## Rules:

Pick from the triangle, parallelogram, trapezoid, hexagon, or star. Create each shape using the smaller pattern blocks provided. Keep the pattern blocks inside the lines. How many different ways can you create each shape?

## Math Skills:

Problem solving, critical thinking

## Extra Practice:

http://illuminations.nctm.org/ActivityDetail.aspx?ID=27
http://www.philtulga.com/patternblock.html
http://www.patternblocktemplates.com/

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## Rules:

Boothvisitors will be able to view a variety of math picture books to make connections between math and literacy. David Schwartz's G is for Googol will be highlighted as a vocabulary activity. A 26 -sided die (icosahedron) will be rolled to determine a random letter. Adults and children will then have thirty seconds to list as many mathematical vocabulary words that begin with that letter as possible.

## Math Skills:

- Connections between math and literacy
- Various books for each strand of mathematics
- Development of mathematical language - Reinforce mathematical vocabulary for appropriate application in problem solving, critical thinking, and written expression


## Extra Practice:

- http://childrenspicturebooks.info/articles/picture_books_for_math.htm
- http://www.scholastic.com/teachers/top-teaching/2012/11/teaching-math-picture-books-part-1
- http://www.mathsisfun.com/definitions/
- http://www.broward.k12.fl.us/studentsupport/ese/PDF/MathWordWall.pdf
- Visit the local library to check out math picture books!

Rules:
Students will find the theoretical probability of choosing a duck that has a penny, a nickel, a dime, a quarter, or a dollar coin attached. They will then pick a duck from the pond.

Math Skills:
Theoretical and Experimental Probability

Extra Practice:
http://mrnussbaum.com/probfair-play/
http://jmathpage.com/JIMSProbabilitypage.html
http://www.kidsmathgamesonline.com/numbers/probability.html

\%OL TO EOLD


Rules:
Fold a sheet of paper in such a way that a shape can be cut out of the center of the paper with a single cut.
Math Skills:
Recognizing and using properties of angle bisectors.
Extra Practice:
How to Fold It by Joseph O'Rourke - http://howtofoldit.org/


Rules:
Deal 6 cards from the deck to each of two players.
Players can choose to keep all six cards or pick two to discard and draw two new cards.
From the hand of 6 cards the player plays the best four to obtain the highest score.
Red cards are negatives and Black cards are positive.

Math Skills:
Adding and Subtracting Integers
Extra Practice:
www.math-play.com/integers-game.html
Rules:

| Each player starts with 11 chips. Each roll to see who goes first. Each player places their 11 chips on one side of |
| :--- |
| the crossing the river playing board. Taking turns, students roll the positive and negative cube and add the two |
| integers. If they have that number on the other side of the river they move one of their chips over. The first |
| player to move all their chips "Across the River" wins! |
| Math Skills: |
| Provides practice in adding integers and promotes critical thinking skills. |
| Extra Practice: |

www.digitallesson.com

## Rules:

Come and create your own straight line curve. Use a ruler and your choice of design to create a curved figure from straight lines.

## Math Skills:

Tangent Lines to Curves

## Extra Practice:

~ Create your own using a figures or lines and have your friends make some straight line curves.
$\sim$ http://mathcraft.wonderhowto.com/how-to/create-parabolic-curves-using-straight-lines-0131301/

## WHATS IN A BAME?

| Rules: |
| :---: |
| Let each n represent 1 square unit. Find the area of each letter in your name then calculate the total area of |
| your name. |
| Math Skills: |
| Calculating the area of complex figures (letters in your name). |
| Extra Practice: |
| http://freemathresource.com/lessons/general-math/91-areas-of-complex-shapes <br> $\underline{\text { http://www.schooltube.com/video/56e31f52b430b234bfa3/ }}$ |

## QIVIDE AND EONQUER!

## Rules:

Students pluck a petal off a large flower. On the petal is a four-digit number. They have to determine if the number is evenly divisible by $2,3,4,5,6,9$, and/or 10 .

Math Skills:
Multiplication and division skills

## Extra Practice:

~ http://www.mathsisfun.com/divisibility-rules.html
~http://www.basic-mathematics.com/divisibility-rules-game.html

## 

## Rules:

We will be highlighting some great math apps for the iPad. Students will get the chance to try out these apps at our booth. Our two highlighted apps are Buzzmath and Coin Math. Come have fun and enjoy hands-on fun! Buzzmath has over 200 Common Core aligned activities and Coin Math allows students to practice real life money concepts.

## Math Skills:

Math concepts covered in these apps vary from NCTM standards for money, geometry, algebra, fractions, and much more.

## Extra Practice:

Link to one of the highlighted apps.....Buzzmath
https://itunes.apple.com/app/buzzmath-middle-school/id593186620
Link to our other highlighted app....Coin Math
https://itunes.apple.com/us/app/coin-math/id296596459?mt=8

## EIRTHDAY \&LAST

## Rules:

Students will be provided a sticker to mark their birthday on our super-sized dot plot!

## Math Skills:

Student will construct a dot plot and evaluate the measures of central tendency and variability of their birthdays.
Extra Practice:
http://www.shodor.org/interactivate/activities/Ploplt/
http://www.shodor.org/interactivate/activities/Measures/
http://www.bbc.co.uk/education/mathsfile/shockwave/games/train.html

## ERACTION EAR

Rules:

1) You decide whether you want to add, multiply, or divide fractions to make the biggest value.
2) You will play 3 rounds.
3) The point of the game is to win as many rounds as you can.
4) The game is played with 2 people.
5) Each person will spin the fraction spinner once.
6) Add/Multiply/Divide the two fractions the spinner lands on.
7) The person with the biggest correct answer wins the round.

Math Skills:

The fraction war game builds fluency in adding, multiplying and dividing and extends this learning to fractions. This basic skill is used in all upper levels of math and is useful for students to practice.

## Extra Practice:

Games:
http://www.funbrain.com/fractop/index.html -
http://www.mathplayground.com/fractions_mult.html
Other Resources:
http://www.ixl.com/math/grade-6 (lots of sections for multiplying/dividing, adding/subtracting fractions). http://www.bigideasmath.com/students/?level=7-tutorials, practice pages, quizzes, etc.

## QKK OND

## Rules:

1) Students choose between three and six numbered ducks from a "pond".
2) Students take the numbers from each duck and find the mean, median, and mode. They then select and report the highest measure of the three.
3) If the student chooses the correct measure with an accurate result, they are a winner.

Math Skills:
Measures of central tendency - mean, median, and mode.

## Extra Practice:

http://www.kidsmathgamesonline.com/numbers/meanmedianmode.html
http://www.mathgoodies.com/lessons/toc_vol8.html
http://www.khanacademy.org/math/probability/descriptive-
statistics/central_tendency/e/mean_median_and_mode
http://www.mathscore.com/math/practice/Mean,\ Median,\ Mode/
http://www.regentsprep.org/Regents/math/ALGEBRA/AD2/Pmeasure.htm
http://www.education.com/study-help/article/averages-median-mode_answer/
$Y$
MUBEER

Rules: Students will choose level I or II
Make My Number is a fun, fast-paced game that puts students' and parents' quick thinking arithmetic skills to test. Students and parents will face off and race each other with the same goal: to make my number. Points will be rewarded to the winner of each round, and the team with the highest overall points wins.
*Student and Parent are placed on separate teams.

1) The Make My Number die will be rolled 4 times, each number recorded. These are the numbers that will be used to Make My Number.
2) Contestants then have to use those numbers only ONCE applying order of operations in any combination to create the Make My Number.
Example: 2,5,3,4 are rolled and the Make My Number is 26 . Correct answer: $2 \times 5 \times 3-4=26$
Remember, each number must be used, and used only ONCE. You can use addition, subtraction, multiplication, division, and apply appropriate properties of association, commutative, etc.
Math Skills:
Order of Operations, properties, computation speed and accuracy

## Extra Practice:

http://www.xpmath.com/forums/arcade.php?do=play\&gameid=100
http://www.xpmath.com/forums/arcade.php?do=play\&gameid=99
http://www.mathplayground.com/calculator_chaos.html
http://www.mathplayground.com/order_of_operations.html
http://www.mathplayground.com/quick_calculate.html

## ※NTEGER MOTBALL!

## Rules:

1) Students will draw a card to determine what happens to them during the game. For example a card may say gain 10 yards or lose 5 yards.
2) Students will use the card to find their position on the field. The student closest to the end zone at the end wins!

## Math Skills:

This activity is designed to improve student's fluency at adding and subtracting integers. In addition, the activity should also help students understand integers and their distance from zero.

## Extra Practice:

$7^{\text {th }}$ grade - number operations E.1-E. 5
www.ixl.com
http://www.xpmath.com/forums/arcade.php?do=play\&gameid=94
http://www.mathplayground.com/ASB_OrbitIntegers.html

## Rules:

1) Place cards "TAPS" side-up
2) Students race to be the first to figure out the 'rule' of the $x / y$ table.
3) When a student knows the answer, he/she taps the card with two fingers, says the answer, and checks the answer on the back of the card.
4) If the student is correct, he/she keeps the card. If the student is incorrect, he/she may not re-answer. Other students are given an opportunity to answer.
5) Best $2 / 3$ wins.

Math Skills:
Writing a rule from a table
Extra Practice:
http://www.functiongame.com/play.html
http://letsplaymath.net/2008/05/13/game-function-machine/
http://www.shodor.org/interactivate/activities/FunctionMachine/

## SALUTE!

## Rules:

Students will choose level I or II
Level I

- One player game.

1) Game play begins with the dealer (teacher) shuffling the cards and placing a card, face down, in front of the player.
2) The dealer will then calls "Salute" and the player will raise the card in their hands to their foreheads in a saluting position.
3) The player will not see his own card.
4) The dealer is able to the card and will call out clues about the number to the player.
For example: The card in play is a 10. The dealer might give the following clues:
This number is a whole number.
This number is an integer
This number is a multiple of 10
This number is the square root of 100
This number is composite
Level II - In level II the deck of cards used will include numbers greater than 10

## Math Skills:

Number sense and basic fact mastery

## Extra Practice:

http://www.xpmath.com/forums/arcade.php?do=play\&gameid=60
http://www.xpmath.com/forums/arcade.php?do=play\&gameid=101
gLOLER OTS

## Rules:

1) Students will use beans our counters to determine how many plants go each pot.
2) Move the beans until the sum (total) and the difference are correct.
3) For each puzzle, there is only one answer for the number of plants in the circular pot and one answer for the number of plants in the square pot.
To play at home:
4) Have one person select a value for $\square$ and $O$.
5) Create and addition and subtraction problem to show the sum and difference of the two numbers.
6) The other person can use beans, counters, marbles, etc. to try and determine the value for and $O$.

## Math Skills:

Solving for two unknowns in a given situation, numeracy concepts.

## Extra Practice:

http://illuminations.nctm.org/ActivityDetail.aspx?id=33
http://www.mathplayground.com/algebraic_reasoning.html
http://www.mathplayground.com/algebra_puzzle.html
http://www.mathplayground.com/wangdoodles.html

## Rules:

1) Students will roll dice, add for the total and determine if the number is composite or prime.
2) If composite, they will need to state two factors of the number. If prime, they will state the reason.

## Math Skills:

Processing addition, knowledge (definition) of prime and composite numbers, factoring of numbers.
Extra Practice:

- http://www.xpmath.com/forums/arcade.php?do=play\&gameid=60
- http://www.xpmath.com/forums/arcade.php?do=play\&gameid=109
- http://www.xpmath.com/forums/arcade.php?do=play\&gameid=101




















## KOLL

## Rules:

1) Two students will be given one die each.
2) The students will roll the die at the same time and an operational die to determine which operation to perform.
3) The first student to call out the correct number is declared the winner of that round.
4) The first player to win two games is the champion

## Math Skills:

Basic fact mastery, fraction computation, decimal computation
Extra Practice:
Jamit Fractions - http://www.jamit.com.au/fraction-games.htm
http://www.mathplayground.com/calculator_chaos.html
http://www.mathplayground.com/practice.php

## \% MAPE NORTER!

Rules:
Students will sort a set of shapes into a Venn Diagram. There will be 3 levels.
Math Skills:
Definitions of various geometry polygons. (Squares, rectangles, trapezoids, parallelograms, quadrilaterals, triangles, pentagons, hexagons)
Venn Diagrams
Extra Practice:
www.shodor.org/interactivate/activities/ShapeSorter/
www.illuminations.nctm.org/ActivityDetail.aspx?ID=34
www.mathsframe.co.uk/resources/Sort_shapes_Venn.aspx

## Rules:

1) Tangrams will be placed in baggies on the table.
2) Different tangram animal shapes and different number shapes will be placed on the table for students to put all seven pieces correctly on a shape or a number.
3) How quickly can you complete the shape?

## Math Skills:

Identify the seven geometrical shapes: Five triangles (2 large, 1 medium, 2 small), one square and one rhombus.

## Extra Practice:

http://pbskids.org/cyberchase/math-games/tanagram-game/
http://www.gieson.com/Library/projects/games/matter/
http://www.mathplayground.com/tangrams.html
http://www.primarygames.com/math/tangram/


## Rules:

2-8 players

1) A number between 2 and 10 is rolled on a die. That becomes the "buzz" number.
2) Players take turns counting $1,2,3,4 \ldots$ when a multiple of the buzz number comes, the word " buzz" must be said, not the number.
3) Miss a "buzz" and you are out.

- Challenge - players say the word "bang" when any number containing in the buzz digit comes.
- Challenge - play with 2 numbers

Math Skills:
Multiples
Mental math
Multiplication facts
Extra Practice:
http://www.math-play.com/
http://hoodamath.com/games/factorfeeder.php
http://www.reasoningmind.org/
The factor game - http://illuminations.nctm.org/Activity.aspx?id=4134
The product game - http://illuminations.nctm.org/Activity.aspx?id=4213

## The Magic Number

Card 1

| 1 | 3 | 5 | 7 |
| :---: | :---: | :---: | :---: |
| 9 | 11 | 13 | 15 |
| 17 | 19 | 21 | 23 |
| 25 | 27 | 29 | 31 |
| 33 | 35 | 37 | 39 |
| 41 | 43 | 45 | 47 |
| 49 | 51 | 53 | 55 |
| 57 | 59 | 61 | 63 |

Card 4

| 8 | 9 | 10 | 11 |
| :---: | :---: | :---: | :---: |
| 12 | 13 | 14 | 15 |
| 24 | 25 | 26 | 27 |
| 28 | 29 | 30 | 31 |
| 40 | 41 | 42 | 43 |
| 44 | 45 | 46 | 47 |
| 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 |

Card 2

| 2 | 3 | 6 | 7 |
| :---: | :---: | :---: | :---: |
| 10 | 11 | 14 | 15 |
| 18 | 19 | 22 | 23 |
| 26 | 27 | 30 | 31 |
| 34 | 35 | 38 | 39 |
| 42 | 43 | 46 | 47 |
| 50 | 51 | 54 | 55 |
| 58 | 59 | 62 | 63 |

Card 5

| 16 | 17 | 18 | 19 |
| :---: | :---: | :---: | :---: |
| 20 | 21 | 22 | 23 |
| 24 | 25 | 26 | 27 |
| 28 | 29 | 30 | 31 |
| 48 | 49 | 50 | 51 |
| 52 | 53 | 54 | 55 |
| 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 |

Card 3

| 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: |
| 12 | 13 | 14 | 15 |
| 20 | 21 | 22 | 23 |
| 28 | 29 | 30 | 31 |
| 36 | 37 | 38 | 39 |
| 44 | 45 | 46 | 47 |
| 52 | 53 | 54 | 55 |
| 60 | 61 | 62 | 63 |

Card 6

| 32 | 33 | 34 | 35 |
| :---: | :---: | :---: | :---: |
| 36 | 37 | 38 | 39 |
| 40 | 41 | 42 | 43 |
| 44 | 45 | 46 | 47 |
| 48 | 49 | 50 | 51 |
| 52 | 53 | 54 | 55 |
| 56 | 57 | 58 | 59 |
| 60 | 61 | 62 | 63 |

## LER 7252

## Fast Track Learning ${ }^{\text {TM }}$



The Fast Track Learning ${ }^{\text {TM }}$ series is a collection of books aimed at introducing children to mathematics. The goal of this series is to engage children in lively math activities that are fun, interesting, and challenging, but most of all allow kids to feel successful. This series has been developed keeping the most current research in mathematics education in mind. The National Council of Teachers of Mathematics recommends the use of hands-on materials to teach math of hands- in the early years.

## About This Book

## In Fast Track Learning ${ }^{\text {TM }}$ - Pattern

 Blocks Patterns and Shapes, children develop thinking and reasoning skills through the use of pattern blocks.Pattern blocks are a set of 6 shapes. Each shape has a different color. Pattern blocks are used to create patterns and figures in art, and to study mathematics, in classrooms throughout the world.

In this book, children use pattern blocks to solve puzzles by creating space objects from the pieces, as well as play games.


Green Triangle


## How to Use This Book:

To get started, you will need to pop out the clear plastic case with the pattern blocks from the book cover. The book then will lie flat so children can work directly on the activity pages. When pattern block play is finished, the pieces can be returned to the case and snapped back into the book for safe storage.

The activities in this book are coded by level of difficulty. Look for the stars in the corners of the pages.

## Level 1

The first 5 activities include the outlines of each pattern block shape and can be solved easily even by a beginner. Children must slide, turn, and flip pieces in order for them to fit. Even very young learners can feel successful with the first part of the book.

## Level 2 =

The next 7 activities include partial outlines to make the activities more challenging, while still giving a hint. Encourage beginners to try their best, and to look at the whole puzzle for hints. Each of these pages includes a hint as to how many pieces kids must use to solve the puzzle. Ask children to find other solutions to each puzzle. There is often more than one way to solve many of these problems.

Level 3 The next 6 activities incorporate the idea of symmetry. If children have a hard time grasping the concept, hold a hand mirror down on the line of symmetry so they can see what the completed shape will look like. For young children, have them complete one side while you complete the other.

## Level 4 元

 The final 3 activities do not include any interior lines and are the most challenging. Encourage children to approach the puzzles systematically. Remember, there are often multiple solutions.

At the end of this book, on pages 29 and 30, you'll find Rainy Day Extensions.
These ideas for additional fun will help you extend your pattern block activities.
Possible solutions for selected puzzles are found on pages 31 and 32 .

## Things You Can Do

Work alongside your children as they solve the puzzles. Children love to work with an adult on activities such as these.

Encourage and praise. Kids love to feel successful at whatever they do.

Use this book as a coloring book. With crayons, colored pencils, or markers, children can add color to the pattern block pages as they complete them.

Give hints. If you see your child is struggling with some of the more challenging puzzles, take a peek in the back and give him/her hints. You can also draw in lines for younger children.


Most important of all—have fun!
er the asteroids using your pattern blocks.


4



Make the Space Creature using your pattern blocks.


## Build the Space Capsule.



Create the Space Bug using 11 pattern block pieces.


行


Build the U.F.O. using 14 pieces.


# Use 13 pieces to create this Martian Pet. 



Make the Loading Craft using 14 pieces.


Create the Space Bunny using 16 pieces.


Use 17 pieces to build the Star Ship.
is





Complete the Moon Crawler. Use the dotted line as a mirror. It is called the line of symmetry.


Complete the Rocket. The dotted line is the line of symmetry.

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Finish the Comet.

Complete the 2-headed Space Monster.


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## Space Race

Players: 2
Object: To be the first player to reach the "Finish" space.
Rules:

1) Players begin on the "Start" spaces and continue, in order, along their roads.
2) Players take turns drawing pieces from a paper bag. If the piece matches the next shape on the road, the player places it on the road. If not, the piece is returned to the bag. No pieces are placed on "Free" spaces.
3) The game ends when one player reaches the
"Finish" space.

4) is Use pattern blocks to make


Make the Asteroid using pattern blocks.


Create the Spiral Galaxy.


## Rainy Day Extensions

1) Ask your child to make up his or her own space asteroid belt. Build asteroids with pattern blocks similar to those found on pages 4 and 5 .
2) Create the same shapes using different pattern
block pieces. After your child has completed pages 6,7 , and 8 , explore which pieces can be substituted for others.

For example, ask your child, "How many red pieces make up a yellow piece? (2) How many green pieces make up a red piece? (3) Can you combine green and red pieces to make a yellow piece? (Yes)"
3) Have your child build different shapes for each object on pages 9 through 15.
4) Create more symmetric shapes. Ask your child to show you the line of symmetry.

5) Take turns with your child creating "halfsymmetric" shapes. Challenge each other to complete the missing halves.
6) Ask your child to make up a story using pattern blocks to create shapes. Your child can trace and color the shape on a piece of paper. Write the story below the shape. Hang the picture story in a special place.
7) Play a pattern block word game. Modify the rules of the game Hang-Man. Instead of drawing a limb whenever a player guesses a letter incorrectly, place a pattern block piece on the outline of a figure in this book. Players try to guess the word before the figure is complete.




## CONCEPT DEVELOPMENT



Mathematics Assessment Resource Service University of Nottingham \& UC Berkeley Beta Version

For more details, visit: http://map.mathshell.org © 2012 MARS, Shell Center, University of Nottingham

## Interpreting Distance-Time Graphs

## MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to interpret distance-time graphs and, in particular, to help you identify students who:

- Interpret distance-time graphs as if they are pictures of situations rather than abstract representations of them.
- Have difficulty relating speeds to slopes of these graphs.


## COMMMON CORE STATE STANDARDS

This lesson relates to the following Standards for Mathematical Content in the Common Core State Standards for Mathematics:
8.F Construct a function to model a linear relationship between two quantities.

Describe qualitatively the functional relationship between two quantities by analyzing a graph
This lesson also relates to the following Standards for Mathematical Practice in the Common Core State Standards for Mathematics:
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

## INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students work on a task designed to reveal their current understandings and difficulties. You review their work and create questions for students to answer in order to improve their solutions.
- A whole-class introduction provides students with guidance on how to work through the first task. Students then work in small groups on a collaborative discussion task, matching verbal interpretations with graphs. As they do this, they translate between words and graphical features, and begin to link the representations.
- This is followed by a whole-class discussion about applying realistic data to a graph.
- Students next work in small groups, matching tables of data to the existing matched pairs of cards. They then explain their reasoning to another group of students.
- In a final whole-class discussion, students draw their own graphs from verbal interpretations.
- Finally, students return to their original task and try to improve their individual responses.


## MATERIALS REQUIRED

- Each student will need two copies of the assessment task Journey to the Bus Stop, a miniwhiteboard, a pen, and an eraser.
- Each small group of students will need copies of Card Set A: Distance-Time Graphs, Card Set B: Interpretations, Card Set C: Tables of Data, a large sheet of paper, and a glue stick for making posters. The cards should be cut up beforehand.
- You will also need a supply of graph paper to give to students who request it. There are some projector resources to support your teaching.


## TIME NEEDED

15 minutes before the lesson, a 90 -minute lesson (or two 45 -minute lessons), and 10 minutes in a following lesson (or homework). Timings are approximate and will depend on the needs of the class.

## BEFORE THE LESSON

## Assessment task: Journey to the Bus Stop (15 minutes)

Set this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of Journey to the Bus Stop.

Briefly introduce the task and help the class to understand the problem and its context.

## Read through the task and try to answer it as carefully as you can.

It is important that, as far as possible, students are allowed to answer the questions without your assistance.

Students should not worry too much if they cannot
 understand or do everything because in the next lesson they will engage in a similar task that should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

## Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas that follow. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

## Suggested questions and prompts:

## Student interprets the graph as a picture

For example: The student assumes that as the graph goes up and down, Tom's path is going up and down.

Or: The student assumes that a straight line on a graph means that the motion is along a straight path.
Or: The student thinks the negative slope means Tom has taken a detour.

## Student interprets graph as speed-time

The student has interpreted a positive slope as speeding up and a negative slope as slowing down.

- If a person walked in a circle around their home, what would the graph look like?
- If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?
- In each section of his journey, is Tom's speed steady or is it changing? How do you know?
- How can you figure out Tom's speed in each section of the journey?
- If a person walked for a mile at a steady speed, away from home, then turned round and walked back home at the same steady speed, what would the graph look like?
- How does the distance change during the second section of Tom's journey? What does this mean?
- How does the distance change during the last section of Tom's journey? What does this mean?
- How can you tell if Tom is traveling away from or towards home?
- Can you provide more information about how far Tom has traveled during different sections of his journey?
- Can you provide more information about how much time Tom takes during different sections of his journey?
- Can you provide information about Tom's speed for all sections of his journey?
- Can you write his speed as meters per second?
Or: The student has written the speed for a section as the distance covered in the time taken, such as " 20 meters in 10 seconds."


## Student misinterprets the scale

For example: When working out the distance the student has incorrectly interpreted the vertical scale as going up in 10s rather than 20s.

## Student adds little explanation as to why the graph is or is not realistic

- What is the total distance Tom covers? Is this realistic for the time taken? Why?/Why not?
- Is Tom's fastest speed realistic? Is Tom's slowest speed realistic? Why?/Why not?


## SUGGESTED LESSON OUTLINE

If you have a short lesson or you find the lesson is progressing at a slower pace than anticipated, we suggest you break the lesson after the first sharing of posters and continue it at a later time.

## Whole-class introduction: interpreting and sketching graphs ( 10 minutes)

Throughout this activity, encourage students to articulate their reasoning, justify their choices mathematically, and question the choices put forward by others. This introduction will provide students with a model of how they should work with their partners in the first small-group activity.
Show the class the projector resource Matching a Graph to a Story:

## Matching a Graph to a Story

| A. Tom took his dog for a walk |
| ---: | ---: |
| to the park. He set off |
| slowly and then increased |
| his pace. At the park Tom |
| turned around and walked |
| slowly back home. |

Ask students to match the correct story to the graph. They are to write down at least two reasons to support their decision.

After two or three minutes ask students who selected option A to raise their hands. Ask one or two to justify their choice. You may wish to use some of the questions on the sheet Suggested questions and prompts to encourage students to justify their choices and others to challenge their reasoning.

Repeat this with options B and C. Even if explanations are incorrect or only partially correct, write them next to the appropriate section of the graph. Encourage students to challenge these interpretations.

Slide P-2 of the projector resource allows you to write three different student explanations on the board at the same time.

A graph may end up looking like this:


This is how students should annotate their graphs when working on the collaborative task.

## Collaborative activity: matching Card sets $\boldsymbol{A}$ and $\boldsymbol{B}$ (20 minutes)

Ask students to work in small groups of two or three students.
Give each group the Card Set A: Distance-Time Graphs, and Card Set B: Interpretations together with a large sheet of paper, and a glue stick for making a poster.

You are now going to continue exploring matching graphs with a story, but as a group.
You will be given ten graph cards and ten story cards.
In your group take a graph and find a story that matches it. Alternatively, you may want to take a story and find a graph that matches it.
Take turns at matching pairs of cards. Each time you do this, explain your thinking clearly and carefully. If you think there is no suitable card that matches, write one of your own.

Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.


Write your reasons for the match on the cards or the poster just as we did with the example in class. Give explanations for each line segment.

Make sure you leave plenty of space around the cards as, eventually, you will be adding another card to each matched pair.

The purpose of this structured group work is to encourage students to engage with each other's explanations and take responsibility for each other's understanding.

Slide P-3 of the projector resource summarizes these instructions.

You have two tasks during the small-group work: to make a note of student approaches to the task, and to support student reasoning.

## Make a note of student approaches to the task

Listen and watch students carefully. Note different student approaches to the task and any common mistakes. For example, students may interpret the graph as a picture or students may read the graph from right to left. Also notice the ways students check to see if their match is correct and how they explain and justify a match to each other. You can use this information to focus a whole-class discussion.

Support student reasoning
Try not to make suggestions that move students towards a particular match. Instead, ask questions to help students to reason together. If you find one student has produced a solution for a particular match, challenge another student in the group to provide an explanation.

John matched these cards. Sharon, why do you think John matched these two cards?
If you find students have difficulty articulating their decisions, then use the sheet Suggested questions and prompts to support your own questioning of students.

In trials of this lesson some students had difficulty stating where home is on the graph.
For this graph, where does the journey start? Is that home?
Give me a graph that shows a journey starting away from home.
For this graph, does the journey end at home? How do you know?
If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold an interim, whole-class discussion. You could ask students who performed well in the assessment to help struggling students.

Some of the cards are deliberate distracters. For example, a student who matches Card 2 and E indicates that they think that graphs are pictures of the situation.

```
2 Opposite Tom's home is a
    hill. Tom climbed slowly up
    the hill, walked across the
    top, and then ran quickly
    down the other side.
```



Allow students time to match all the cards they can.

## Sharing posters (5 minutes)

As students finish matching the cards, ask one student from each group to visit another group's poster.

You may want to use Slide P-4 of the projector resource to display the following instructions.
If you are staying at your desk, be ready to explain the reasons for your group's matches.
If you are visiting another group, write your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.
If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

When you return to your own desk, you need to consider as a group whether to make any changes to your own poster.

Students may now want to make changes to their poster. At this stage there is no need for students to glue the cards onto their posters as they may decide to make further changes.

If you need to extend the lesson over two days:
Once students have finished sharing posters, organize a whole-class discussion. Invite pairs of students to describe one pair of cards that they think they have matched correctly and the reasoning they employed. Encourage other students to challenge their explanations.

Finally, ask students to note their matches on the back of their poster and to use a paperclip to attach all cards to the poster.

At the start of the second lesson, spend a few minutes reminding the class about the activity:
Can you remember what we were working on in the last lesson?
Return the posters to each group. The whole-class discussion on interpreting tables can serve as an introduction to the lesson.

## Whole-class discussion: Interpreting tables (15 minutes)

Bring the class together and give each student a mini-whiteboard, a pen, and an eraser. Display Slide P-5 of the projector resource:


On your whiteboard, create a table that shows possible times and distances for Tom's journey.
After a few minutes, ask students to show you their whiteboards. Ask some students to explain how they created their tables. Write their figures on the board. Ask the rest of the class to check these figures.

Is Tom's speed slower or faster in this section compared to that section?
How do you know from the graph? From the table?
Is this speed constant? How can you tell? Do the figures in the table show a constant speed for this section of the journey?

What units might these be measured in?
Are these figures realistic?

## Collaborative activity: matching Card Set C ( 20 minutes)

Hand out Card Set C: Tables of Data and ask students to match these cards with the cards already on their poster.

You are now going to match tables with the cards already on your desk. In your group take a graph and try to find a table that matches it, or take a table and find a graph that matches it.

Again take turns at matching cards you think belong together. Each time you do this, explain your thinking clearly and carefully.

Write your reasons for the match on the poster.
Students may also wish to suggest suitable units for the distances and times on the cards.
The tables should help students confirm or modify existing matches.
As they work on the matching, support the students as in the previous matching activity.
In the past, some students have had difficulty understanding the repetition in Table R. The table is intended to show the impossibility of Graph H.

| $R$ | Time |
| :---: | :---: |
| 0 | Distance |
| 1 | 0 |
| 2 | 36 |
| 3 | 54 |
| 3 | 84 |
| 5 | 120 |



Some teachers have found that it helps students to look at the average speeds between consecutive times, by calculating differences. For example, average speeds for Table of Data Q would look like this.

| $Q$ | Time |
| :---: | :---: |
| 0 | Distance |
| 0 | 0 |
| 1 | 10 |
| 2 | 20 |
| 3 | 40 |
| 4 | 60 |
| 5 | 120 |

Average speed
10
10
20
20
60

This may help students to understand that the table on Card Q matches Tom's hill walk, and that the correct distance-time graph should therefore be Card D.


If some students finish quickly, encourage them to devise their own pairs of cards.

## Sharing posters (5 minutes)

When students have completed the task, the student who has not already visited another pair should share their work with another pair of students. Students are to share their reasoning as they did earlier in the lesson unit.

Students may now want to make final changes to their poster. When they are completely satisfied, ask them to glue their cards onto the large sheet of paper.

## Whole-class discussion ( 10 minutes)

Using mini-whiteboards, make up some journeys and ask the class to show you the corresponding graphs.

On your whiteboards, draw a distance-time graph to show each of the following stories:

- Sam ran out of his front door, then slipped and fell. He got up and walked the rest of the way to school.
- Sara walked from home up the steep hill opposite her house. She stopped at the top to put her skates on, then skated quickly down the hill, back home again.
- Chris cycled rapidly down the hill that starts at his house. He then slowed down as he climbed up the other side.

Ask students to show their whiteboards to the whole-class. Select some to explain their graph to the class. Encourage others in the class to challenge their reasoning.

## Improving individual solutions to the assessment task ( 10 minutes)

Return to the students their original assessment task Journey to the Bus Stop, as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.
Using what you have learned, try to improve your work.
If you have not added questions to individual pieces of work then write your list of questions on the board. Students should select from this list only those questions they think are appropriate to their own work.

If you find you are running out of time, you could set this task in the next lesson or for homework.

## SOLUTIONS

## Assessment task: Journey to the Bus Stop

1. The graph shows Tom's journey is split into four sections. The straight lines indicate that Tom moves at a constant but different speed in each section.

A In this section of the journey Tom walks away from home at a speed of 2 meters per second ( $100 \div 50$ ) for 50 seconds.

B The negative slope here means a change in direction. At 100 meters from home Tom starts to walk towards home. He walks for 60 meters at a speed of 3 meters per second ( $60 \div 20$ ).

C At the start of this section Tom changes direction. He is now
 walking away from home at a fast pace. His speed is 4 meters per second ( $120 \div 30$ ). He moves at this speed for 30 seconds and covers 120 meters.

D Here the slope is zero. This means at 160 meters from home Tom stops. It has taken him 100 seconds to get to this point.
2. The speeds provided in the answer to question 1 are realistic. A speed of 2 meters per second is a brisk walk. A speed of 4 meters per second means Tom is running.

Collaborative activity

| Graph | Interpretation | Table | Graph | Interpretation | Table |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | 5 | W | B | 10 | S |
| C | 4 | V | D | 2 | Q |
| E | 6 | T | F | 3 |  |
| G | 1 | P | H | 8 | R |
| I | 7 | U | J | 9 | X |



## Journey to the Bus Stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.

Distance from home in meters


1. Describe what may have happened.

You should include details like how fast he walked.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
2. Are all sections of the graph realistic? Fully explain your answer.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Card Set A: Distance-Time Graphs



## Card Set A: Distance-Time Graphs (continued)



## Card Set B: Interpretations

1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.

2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

3 Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.

4 Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.

5 Tom left his home for a run, but he was unfit and gradually came to a stop!

6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.

7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.

8 This graph is just plain wrong. How can Tom be in two places at once?

9 After the party, Tom walked slowly all the way home.

10 Make up your own story!

## Card Set C: Tables of Data

| P | Time | Distance | Q | Time | Distance | R | Time | Distance |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 |  | 0 | 0 |  | 0 | 0 |
|  | 1 | 40 |  | 1 | 10 |  | 1 | 18 |
|  | 2 | 40 |  | 2 | 20 |  | 2 | 36 |
|  | 3 | 40 |  | 3 | 40 |  | 3 | 54 |
|  | 4 | 20 |  | 4 | 60 |  | 3 | 84 |
|  | 5 | 0 |  | 5 | 120 |  | 5 | 120 |
| S | Time | Distance | T | Time | Distance | U | Time | Distance |
|  | 0 | 0 |  | 0 | 0 |  | 0 | 0 |
|  | 1 | 40 |  | 1 | 20 |  | 1 | 30 |
|  | 2 | 80 |  | 2 | 40 |  | 2 | 60 |
|  | 3 | 60 |  | 3 | 40 |  | 3 | 0 |
|  | 4 | 40 |  | 4 | 40 |  | 4 | 60 |
|  | 5 | 80 |  | 5 | 0 |  | 5 | 120 |
| V | Time | Distance | W | Time | Distance | X | Time | Distance |
|  | 0 | 0 |  | 0 | 0 |  | 0 | 120 |
|  | 1 | 20 |  | 1 | 45 |  | 1 | 96 |
|  | 2 | 40 |  | 2 | 80 |  | 2 | 72 |
|  | 3 | 40 |  | 3 | 105 |  | 3 | 48 |
|  | 4 | 80 |  | 4 | 120 |  | 4 | 24 |
|  | 5 | 120 |  | 5 | 125 |  | 5 | 0 |
| Y | Make this one up! |  | Z | Make this one up! |  |  |  |  |
|  | Time | Distance |  | Time | Distance |  |  |  |
|  | 0 |  |  | 0 |  |  |  |  |
|  | 1 |  |  | 1 |  |  |  |  |
|  | 2 |  |  | 2 |  |  |  |  |
|  | 3 |  |  | 3 |  |  |  |  |
|  | 4 |  |  | 4 |  |  |  |  |
|  | 5 |  |  | 5 |  |  |  |  |
|  | 6 |  |  | 6 |  |  |  |  |
|  | 7 |  |  | 7 |  |  |  |  |
|  | 8 |  |  | 8 |  |  |  |  |
|  | 9 |  |  | 9 |  |  |  |  |
|  | 10 |  |  | 10 |  |  |  |  |

## Matching a Graph to a Story

A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.
B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.
C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.
A. Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.
B. Tom rode his bike east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.
C. Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.


## Matching Cards

- Take turns at matching pairs of cards. You may want to take a graph and find a story that matches it. Alternatively, you may prefer to take a story and find a graph that matches it.
- Each time you do this, explain your thinking clearly and carefully. If you think there is no suitable card that matches, write one of your own.
- Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.
- Write your reasons for the match on the cards or the poster just as we did with the example in class. Give explanations for each line segment.
- Make sure you leave plenty of space around the cards as, eventually, you will be adding another card to each matched pair.


## Sharing Work

- One student from each group is to visit another group's poster.
- If you are staying at your desk, be ready to explain the reasons for your group's matches.
- If you are visiting another group:
- Write your card placements on a piece of paper.
- Go to another group's desk and check to see which matches are different from your own.
- If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.
- When you return to your own desk, you need to consider as a group whether to make any changes to your own poster.


## Making Up Data for a Graph



| Time | Distance |
| :---: | :---: |
| 0 |  |
| 2 |  |
| 4 |  |
| 6 |  |
| 8 |  |
| 10 |  |

# Mathematics Assessment Project CLASSROOM CHALLENGES 

This lesson was designed and developed by the Shell Center Team<br>at the<br>University of Nottingham<br>Malcolm Swan, Nichola Clarke, Clare Dawson, Sheila Evans<br>with<br>Hugh Burkhardt, Rita Crust, Andy Noyes, and Daniel Pead

It was refined on the basis of reports from teams of observers led by David Foster, Mary Bouck, and Diane Schaefer based on their observation of trials in US classrooms along with comments from teachers and other users.

This project was conceived and directed for MARS: Mathematics Assessment Resource Service
by

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and based at the University of California, Berkeley

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## Pattern Block Lessons to Meet Common Core State Standards



The MATH LEARNINGCENTER

Excerpts From Bridges in Mathematics

## Pattern Block Lessons to Meet Common Core State Standards Grades K-2

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Bridses in Mathematics is a standards-based K-5 curriculum that provides a unique blend of concept development and skills practice in the context of problem solving. It incorporates the Number Corner, a collection of daily skill-building activities for students.

The Math Learning Center is a nonprofit organization serving the education community. Our mission is to inspire and enable individuals to discover and develop their mathematical confidence and ability. We offer innovative and standards-based professional development, curriculum, materials, and resources to support learning and teaching. To find out more, visit us at Www.mathlearningcenter.org.

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## Introduction

## Pattern Blocks and the Common Core State Standards

Pattern Blocks are a familiar manipulative available in most elementary schools. We've created this Pattern Block Lessons sampler to help you meet the new Common Core State Standards (CCSS) and organized it in two grade level bands, $\mathrm{K}-2$ and 3-5. The lessons are excerpts from the Bridges in Mathematics curriculum, published by The Math Learning Center. We hope you'll find the free resources useful and engaging for your students.

The Common Core State Standards (2010) define what students should understand and be able to do in their study of mathematics. A major goal of the CCSS is building focus and coherence in curriculum materials. The standards strive for greater consistency by stressing conceptual understanding of key ideas and a pacing the progression of topics across grades in a way that aligns with "what is known today about how students' mathematical knowledge, skill, and understanding develop over time." (CCSSM, p. 4). In addition to the content standards, the CCSSM defines Eight Mathematical Practices that describe the processes-the how teachers will teach, and how students will interact in a mathematics classroom.

Bridges in Mathematics helps teachers meet the challenges of the Content Standards and the Eight Mathematical Practices. During a Bridges lesson, students make sense of mathematics using manipulatives, visual and mental models to reason quantitatively and abstractly. They solve challenging problems daily that develop their stamina to carry out a plan and to present their thinking to their classmates. Students make conjectures and critique the reasoning of others, by asking questions, using tools, drawings, diagrams and mathematical language to communicate precisely. Students develop and use a variety of strategies to become computationally fluent with efficient, flexible and accurate methods that make use of patterns and the structures in operations and properties. They use dimensions, attributes, and transformations to make use of the structures in Number and Geometry. Bridges encourages students to estimate a reasonable answer, and continually evaluate the reasonableness of their solution. This Pattern Block sampler will provide you with examples of lessons from whole group Problems and Investigations and centers called Work Places. In many cases there are suggestions for support and challenge to help you meet the CCSS standards and differentiate your instruction.

## Bridges in Mathematics

Bridges in Mathematics is a full $\mathrm{K}-5$ curriculum that provides the tools, strategies, and materials teachers need to meet state and national standards.

Developed with initial support from the National Science Foundation, Bridges offers a unique blend of problem-solving and skill building in a clearly articulated program that moves through each grade level with common models, teaching strategies, and objectives.

A Bridges classroom features a combination of whole-group, small-group, and independent activities. Lessons incorporate increasingly complex visual mod-els-seeing, touching, working with manipulatives, and sketching ideas-to create pictures in the mind's eye that helps learners invent, understand, and remember mathematical ideas. By encouraging students to explore, test, and justify their reasoning, the curriculum facilitates the development of mathematical thinking for students of all learning styles.

Written and field-tested by teachers, Bridges reflects an intimate understanding of the classroom environment. Designed for use in diverse settings, the curriculum provides multiple access points allowing teachers to adapt to the needs, strengths, and interests of individual students.

Each Bridges grade level provides a year's worth of mathematics lessons with an emphasis on problem solving. Major mathematical concepts spiral throughout the curriculum, allowing students to revisit topics numerous times in a variety of contexts.

To find out more about Bridges in Mathematics visit www.mathlearningcenter.org

## Activity 1

## WORK PLACE

## Pattern Block Designs

## Overview

Students use pattern blocks to copy designs from cards, first with real blocks and then if they wish by gluing paper pattern blocks on black construction paper. Students also have the opprtunity to create their own designs.

## Skills

* Describe objects in the environment using geometric shape names (K.G.1)
* Identify shapes in the environment (K.G.1)
$\star$ Identify shapes, regardless of orientation or size (K.G.2)
* Analyze 2-D shapes (K.G.4)
$\star$ Use informal language to describe the similarities and differences between different 2-D shapes (K.G.4)
$\star$ Compose simple shapes to form larger shapes (e.s., compose triangles to form a rectangle) (K.G.6)


## This center will need

* Pattern Block Designs cards (Teacher Masters 1-5, run 1 copy each on cardstock. Color appropriately. Laminate if desired.)
* 3 buckets of pattern blocks
$\star 6$ small containers of paper pattern block shapes
* 20-30 pieces of 6" $\times 9^{\prime \prime}$ black construction paper in a folder or ziplock bas
$\star 6$ small bottles of glue



## Work Place Instructions

1. Choose the pattern block design card that you would like to copy.
2. What do you notice about the design? Which shapes will you need? How many? How can you make them fit together?
3. Use your pattern blocks to copy it. Does your design look just the same?
4. If you'd like to make a copy of your work with the paper shapes, find the shape(s) you need. Glue them carefully to the black construction paper to make it look just like the figure you made.
5. Do you want to take your work home to share with your family or leave it at school for others to see?

Activity 1 Pattern Block Designs (cont.)
6. Can you use the pattern blocks to create some designs of your own?
7. Would you like to make a copy of one of your original designs?

## Instructional Considerations

After years of watching five-year-olds work with pattern blocks, we've concluded that some children need a jump start. If your class has been producing magnificent creations with the pattern blocks, you may choose to omit the design cards and see what happens. Can they use the paper shapes to reproduce their own pattern block figures? We've often seen kindergartners joyfully glue the paper shapes on a piece of paper in random fashion, totally unconcerned about relating the work to their actual pattern block creations. Copying a design card with pattern blocks and then reproducing it with the paper shapes helps some children make the connection better. Most are then able to consider the number of blocks, the particular shapes, and the ways the shapes fit together. Some will lack the fine motor skills required to achieve accurate reproductions-you'll need to celebrate all their efforts, and trust that with time their work will improve. Be sure to display their creations on a wall or in a bound class book.

## Pattern Block Designs Card I



## Pattern Block Designs Card 2



## Pattern Block Designs Card 3



## Pattern Block Designs Card 4



## perky pattern puppy

## Pattern Block Designs Card 5


dancing donkey

## Activity 2

110

## WORK PLACE

## Hungry Caterpillars

## Overview

Students race to be the first to fill up their triple hexason catepillar by placing their pattern blocks in various combinations.

## Skills

ฝ Describe objects in the environment using geometric shape names (K.G.1)
ڤ Identify shapes, regardless of orientation or size (K.G.2)
$\star$ Compare 2-D shapes (K.G.4)

* Model 2-D shapes in the world by drawing them (K.G.5)
$\star$ Compose simple shapes to form larger shapes (e.s., compose triangles to form a rectangle) (K.G.6)


## This center will need

* Hungry Caterpillars gameboards (Teacher Master 6 run 3 copies on cardstock. Cut apart on thin lines. Laminate if desired.)

丸 Hungry Caterpillars spinners (Teacher Master 7 , run 1 copy on cardstock. Cut apart on thin lines.)
ڤ 3 single spinner overlays or paper clip spinners

* 3 containers of pattern blocks (Each container should have 20 of each of the following shapes: triangles, blue rhombuses, and trapezoids.)


## Work Place Instructions

1. You and your partner will need a spinner and a container of pattern blocks to share. Each of you will need your own caterpillar board.
2. Take turns spinning the spinner. Each time you spin a shape, take a pattern block of the same shape and place it on your caterpillar. (We've noticed that some kindergartners tend to place their shapes at random. This is fine, as long as they fit them into the triangular guidelines.)

3. The first person to fill his or her caterpillar wins. The catch is, you have to fill all the hexagons exactly to go out. If all the space you have left is a rhombus and you spin a trapezoid, you miss your turn and have to try for

## Activity 2 Hungry Caterpillars (cont.)

a rhombus or a triangle next time. Continue playing until one person fills his or her caterpillar.

## Instructional Considerations

Here are some things you might look for as you watch students play this game and listen to their conversations.

- Do children refer to the shapes by name or by color?
- Do they attempt to fit their shapes into the triangular guidelines of the caterpillar, or do they just set them loosely on the board? If they're attempting to fit the shapes in accurately, can they do so with relative ease?
- Do they seem aware that some shapes fill the hexagonal sections more quickly than others? Are they able to tell how many triangles, rhombuses, and/or trapezoids it takes to fill a hexagon?
- Are they able take turns and wait patiently as their partner finds his or her blocks and sets them on the gameboard?
Hungry Caterpillars Gameboard
Hungry Caterpillars Gameboard

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## Hungry Caterpillars


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## Hungry Caterpillars



Hungry Caterpillars

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## Paper Clip Spinners

## Option 1

Insert a pencil into the small loop of a paper clip at the mid point of the spinner base. Hold the pencil to anchor the paper clip. Spin the paper clip with your free hand or have your
 partner help you.

## Option 2

You will need a brass paper fastener (brad), a paper clip and a ${ }^{1 / 4} 4^{\prime \prime}$ section of straw. Poke a small hole through the midpoint of the spinner base. Poke the brad though the straw and
 the paper clip and insert it into the hole in the front of the spinner base. Bend each side of the fastener flat against the under side. The section of straw servers as a spacer to ensure the paper clip spins.

## Activity 1

## WORK PLACE

## Pattern Block Reflections

## Overview

Students use hinged mirrors and pattern blocks to create and observe the patterns in the reflections. Then they reproduce what they see in the mirror by gluing pattern block shapes on paper.

## Skills

* Demonstrate an understanding of the difference between the defining and non-defining attributes of a 2-D shape (1.G.1)
* Create a composite shape by composing 2-D shapes (rectangles, squares, trapezoids, triangles) (1.G.2)
$\star$ Identify lines of symmetry (4.G.3)
* Draw lines of symmetry (4.G.3)
* Identify figures with line symmetry (4.G.3)


## This center will need

* pattern blocks
^ 8 double-hinged mirrors
ฝ cut paper pattern blocks
$\star$ glue
* Pattern Block Reflections record sheets (Teacher Master 1, run 30 copies)


## Work Place Instructions

1. Get a hinged mirror, 2 or 3 pattern blocks, a record sheet, some paper pattern blocks, and glue.
2. Tuck one of your pattern blocks into the corner of the hinged mirror and take a peek to see what the reflection looks like. How many of the blocks do you see now? Open the mirror out and then tuck it back up against the block.


Activity 1 Pattern Block Reflections (cont.)

3. Copy exactly what you see by gluing down paper pattern block shapes on your record sheet. Create your design in the center of the page so you can add more blocks to it later.
4. Add another block to the one that's already set in the corner of the hinged mirror. Add more paper pattern blocks to your design to match what you see. Your design will grow very quickly. Add another block or two and copy the new reflections each time until you have the pattern block design you want.
5. Set your picture somewhere safe to dry.

## Instructional Considerations

If some children enjoy working with the mirrors and pattern blocks without recording their designs, that's fine. Positioning and gluing the paper shapes may actually get in the way of some students' investigations, although others will adore taking their record sheets home.

When you're talking to children about their work, there are plenty of opportunities to discuss reflection, symmetry, and number. When you pull a hinged mirror snugly around a green triangle, you see 6 of them in the reflection counting the real one. Does that happen with all of the pattern blocks? (No) Do you see 6 squares including the real one when you pull the mirror

Activity 1 Pattern Block Reflections (cont.)
around the square? (No) How many squares do you see? (4) What about the hexagon (3) or the white rhombus (12)? Is there any way to predict how many of a particular shape you'll see? (The number of shapes you see in a hinged mirror depends on the angle of the shape you've snuggled into the corner, If it's $90^{\circ}$, like the corner of the square block, you'll see 4 blocks (including the real one). If it's $60^{\circ}$, like the corner of the green triangles, you'11 see 6 in all-1 real and 5 reflected blocks.


You might notice a pattern to this, which is that the angle of the shape you've snuggled into the corner of the hinged mirror multiplied by the number of blocks (real and reflected) always comes to $360(4 \times 90=360,6 \times 60=360)$. While this is too abstract for first graders, some might notice that the "skinnier" the pattern block is, the more times they'll see it reflected in the mirror.

"Wow! When I put the white rhombus in, I can see 12!"
What happens to the number of shapes when you add more blocks to the first block inside the mirror? What happens if you open and close the doublehinged mirror around a shape? (Try it and find out!)

## NAME

$\qquad$

## Pattern Block Reflections record sheet

$\square$

## Activity 2

## WORK PLACE

## Last Shape In Wins

## Overview

Students pairs take turns placing one pattern block at a time on a large rhombus shaped gameboard, using the guidelines on the board to position the pieces. Each new shape needs to touch one of shapes already on the board. The player who places the piece that completes the rhombus wins.

## Skills

ฝ Demonstrate an understanding of the difference between the defining and non-defining attributes of a 2-D shape (1.G.1)
^ Create a composite shape by composing 2-D shapes (rectangles, squares, trapezoids, triangles) (1.G.2)

## This center will need

^ pattern blocks-hexagons, trapezoids, triangles, and blue rhombuses only (Organize sets of pattern blocks-4 hexagons and 12 each of the other shapes-into 3 ziplock bass so partners can easily get what they need.)

Last Shape In Wins gameboards (Teacher Master 2, run 3 copies. Laminate if desired.)

## Work Place Instructions

1. Get a partner, some pattern blocks, and a gameboard. Decide who will go first and who will go second.
2. Take turns placing blocks on the gameboard. The first block can be placed anywhere; after that each new block has to touch at least one of the blocks already on the gameboard. You may use any of the 4 shapes. You must take your turn every time, down to the very end. The object of the game is to be the person who gets to complete the big rhombus by fitting in the final shape.

Activity 2 Last Shape In Wins (cont.)


## Instructional Considerations

The strategizing that may go on in the last few moves of this game is similar to chess in that a player needs to envision several different possibilities, imagining what will happen if she places a trapezoid on the board instead of a rhombus, or a triangle instead of a hexagon. Not all of your students will spend a lot of time agonizing over the last few moves, although more might if you continue to challenge them to develop winning strategies.

## Last Shape In Wins gameboard



## Activity 1

## WORK PLACE

## Caterpillar Fill \& Add

## Overview

In this game, students take turns rolling dice and adding the two numbers to determine how many triangles they can fill in on their gameboards. Each triangle is worth 1. A roll of 3 and 5 means the player will be able to fill in 8 triangles (or their equivalent in other shapes) on their board. The first player to fill their caterpillar exactly wins.

## Skills

* Recosnize shapes having specified attributes (e.s., a certain number of angles or congruent faces) (2.G.1)
* Identify triangles, quadrilaterals and hexagons (2.G.1)
* Partition a hexagon into 2 equal parts (2.G.3)
* Partition a hexason into 3 equal parts (2.G.3)
* Use the terms halves and half of to talk about the 2 equal parts (2.G.3)
$\star$ Use the terms thirds and a third of to talk about the 3 equal parts (2.G.3)
* Fluently add with sums to 20 using mental stratesies (2.OA.2)
* Recall from memory all sums of two 1-digit numbers (2.OA.2)


## This center will need

* pattern blocks—hexasons, trapezoids, triangles, and blue rhombuses only (You may want to organize sets of pattern blocks into 3 ziplock bags so partners can reach into the Work Place basket easily and get what they need. Each set should have about 10 hexagons and 20 each of the other shapes.)
* 6 Caterpillar Fill \& Add gameboards (Teacher Master 1, run 3 copies on cardstock. Cut apart on thin lines. Laminate if desired)
$\star$ Caterpillar Fill \& Add record sheet (Teacher Master 2, run 30 copies and place in a folder)
* 3 pairs of dice (1 of each pair should be numbered 0-5, the other 1-6) See note.
$\star$ red, green, yellow, and blue crayons

Note $0-5$ dice can be made by numbering a blank wooden cube with a narrowtipped permanent marker.

## Work Place Instructions

1. Get a partner, two gameboards, one record sheet to share between the two of you, a pair of dice, some pattern blocks, and some crayons.
2. Each of you needs a gameboard and some pattern blocks right in front of you; the record sheet should be placed between you so that you can take turns recording your moves. You will both work on the same record sheet.

## Activity 1 Caterpillar Fill \& Add (cont.)

3. Take turns rolling the dice, adding the two numbers that come up, and taking that many pattern blocks in triangles or their equivalent in other shapes (if you roll $2+4$, you can take 6 triangles, 1 hexagon, 3 diamonds, 2 trapezoids, or any combination of blocks equivalent to 6 triangles). Place the blocks on your gameboard, and record your move on the record sheet each time it's your turn. The object of the game is to be the first to fill the caterpillar exactly.

Here's what someone who rolled $5+4$ on his or her first turn might do:

"I got $5+4$ on my first roll. That's 9 . I'm going to take a hexagon-that's 6 , and a trapezoid- that's 3 . I'll put those blocks on my gameboard, and then color in what I did on the record sheet."

Activity 1 Caterpillar Fill \& Add (cont.)
4. Keep playing back and forth until one of you fills the caterpillar exactly. Once you're down to 6 or fewer triangles to fill, you can opt to roll only one die instead of two. If you roll more than the number of triangles you have left to fill, you lose your turn and you don't write anything down. In the end, you may have to trade the dice (or die) back and forth a few times until someone finally gets the number he or she needs. Be sure to record your score each time you can make a play, and total the numbers at the end, even if you don't make it to 24 .

5. When you're finished, dump the pattern blocks off your gameboards and play the game again so that both you and your partner have a record sheet to put in your folders.

## Instructional Considerations

Although children will have played this game once at a whole-group level, you may have to model it more than once as a Work Place. This is partly because there are several steps. Players have to roll the dice, figure the total, and take an equivalent amount in pattern blocks. Then they have to set the

## Activity 1 Caterpillar Fill \& Add (cont.)

blocks on their gameboards, and finally, color in their record sheets each time it's their turn.

It will be very important for children to understand that they don't have to fill each section of the caterpillar before they move on to the next. If, for instance, they have 5 triangles filled in on the first section of their caterpillar and they roll a 6 , they can take the 6 as a hexagon and fill the second section, waiting until their third or fourth turn to go back and fill in the remaining triangle on the first section. Some children, of course, will want to fill their caterpillar sections completely as they go, and may even opt to take all of their scores in triangles each time. You might want to spend some time observing at this Work Place or looking over children's record sheets to see whether they stick with triangles or move into taking equivalent amounts with larger blocks.

For challenge, students may write multiplication sentences like $(3 \times 2)+$ $(3 \times 6)=24$ or whatever grouping structures match the area of their shapes.

$\qquad$
Caterpillar Fill \& Add record sheet

$\qquad$
$\qquad$

NAME $\qquad$ DATE $\qquad$
Caterpillar Fill \& Add record sheet


## Activity 2

## WORK PLACE

## Build-4-Less

## Overview

Partners use pattern blocks to figure out how to build shapes shown on a problem sheet using the fewest blocks possible. They record their soultions by coloring the picture on a record sheet.

## This center will need

* pattern blocks-hexagons, trapezoids, triangles, and blue rhombuses only
* Build-4-Less sheets 1-6 (Teacher Masters 3-8, run 15 copies of each sheet)
ڤ red, green, yellow, and blue crayons


## Skills

Ł Recognize shapes having specified attributes (e.s., a certain number of angles or congruent faces) (2.G.1)
ڤ Draw shapes having specified attributes (e.s., a certain number of angles or congruent faces) (2.G.1)

* Identify triangles, quadrilaterals, and hexasons (2.G.1)
* Partition a hexason into 2 equal parts (2.G.3)
* Partition a hexagon into 3 equal parts (2.G.3)
* Use the terms halves and half of to talk about the 2 equal parts (2.G.3)
$\star$ Use the terms thirds and a third of to talk about the 3 equal parts (2.G.3)
* Describe a whole as 2, 3, 4 of two, three, four equal parts (2.G.3)
$\star$ Demonstrate an understanding that equal parts of identical wholes do not have to be the same shape (2.G.3)

Ł Fluently add with sums to 20 using mental strategies (2.OA.2)

* Recall from memory all sums of two 1-digit numbers (2.OA.2)


## Work Place Instructions

1. Get some pattern blocks and one of the problem sheets from the folder. Work with the blocks until you figure out how to make the shape(s) on the sheet with the fewest blocks possible. You will probably have to experiment for awhile and try several different arrangements until you find the one that uses the fewest blocks. You can build directly on the sheet itself or off to the side. Build this picture with the fewest possible number of blocks. Show yoursolution by coloring in the pattern blocks you used.

Activity 2 Build-4-Less (cont.)

2. When you think you have found a way to make the shape with the fewest blocks, record your solution by coloring in the picture on the sheet. If you have built directly on the shape, you may remove your blocks one at a time, coloring as you go, so you don't forget your own arrangement. You may also slide your blocks to the side and rearrange them so you can copy from the blocks to the sheet.

## Instructional Considerations

There are six sheets in this set, and you'll need to decide how many sheets you want to require for each visit. Some of the sheets are fairly challenging, and each requires a fair amount of work in building and rebuilding, and then coloring to show a solution. Perhaps you'll want to vary the requirement depending on the student. Here is the answer key for the six sheets. Please note that there are two solutions for one of the problems.

## ANSWER KEY FOR BUILD-4-LESS SHEETS

## Sheet 1

- The trapezoid: 4 (4 trapezoids)
- The star: 6 (6 rhombuses or 3 trapezoids and 3 triangles)

Sheet 2

- The large rhombus: 5 (4 trapezoids and 1 hexagon)


## Sheet 3

- The dos: 9 (5 rhombuses, 2 hexasons, 1 trapezoid, and 1 triangle)

Sheet 4

- The large trapezoid: 7 (2 hexasons and 5 trapezoids)

Sheet 5

- The large hexagon: 12 (6 hexagons and 6 trapezoids)

Sheet 6

- The large star: 13 ( 7 hexagons and 6 triangles)

Note You might want to post a chart that simply states the minimum number blocks for each shape. We find that some of our students are able to find the solutions above more easily if they know how many total blocks a figure requires, without knowing exactly how many triangles, hexagons, trapezoids, and/or rhombuses are needed for that figure.



DATE


Build this picture with the fewest possible blocks. Show your solution
by coloring in the pattern blocks you used.




[^0]:    * Pattern Blocks are the only manipulative required for this activity

