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Using Models To Explore Ratio and Proportional Relationships

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Ratio Tables

Students were asked to the find the price of 16 tickets if they knew that the price of 1 ticket was $5.50. Can you explain how each student created their ratio table to find the answer?

Maria

Tickets	1	10	2	4	6	16
Cost ($)	5.50	55.00	11.00	22.00	33.00	88.00

Steve

Tickets	1	2	4	8	16	
Cost ($)	5.50	11.00	22.00	44.00	88.00	

Andre

Tickets	1	10	5	15	16	
Cost ($)	5.50	55.00	27.50	82.50	88.00	

Unit Rate: How Did These Students Use the Ratio Table?

It took Sheena 2 hours to travel 81.2 mi. Sheena used a ratio table to calculate the average number of miles she drove per hour. Here is Sheena's scrap of paper.

Miles	81.2	812	406	40.6
Hours	2	20	10	1

Cindy remembers that she drove 50 mi on 2.5 gal of gasoline. She creates the following ratio table on a scrap of paper.

Miles	50	100	20
Gallons	2.5	5	1

Young says, "My car averages 24 mpg." Using this information and the ratio table below, calculate how far he can drive on 14 gallons of gas? Make sure to add labels.

Ratios and Rates, Section A: Single Unit Rate (Mathematics in Context, Britannica)

As students work with tables of quantities in equivalent ratios (also called ratio tables), they should practice using and understanding ratio and rate language.[6.RP.1, 6.RP.2] It is important for students to focus on the meaning of the terms "for every," "for each," "for each 1," and "per" because these equivalent ways of stating ratios and rates are at the heart of understanding the structure in these tables, providing a foundation for learning about proportional relationships in Grade 7.

Progressions for the Common Core State Standards in Mathematics: 6-7 Ratio and Proportional Relation, Commoncoretoools.wordpress.com

Double Number Line

Mr. Math's Supermarket sells Red Delicious Apples for $2.40 per pound. If Ms. Science wants to buy 1¾ lbs, how much will she have to pay? How many pounds can she buy for $6.00?

My strategy:

Another strategy:

7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction ½ / ¼ miles per hour, equivalently 2 miles per hour.*

A different way of using this model to help you assess your students' understanding is to ask them to make sense of a provided double number line. Students can label the intervals, analyze an incorrect solution, or simply answer the question.
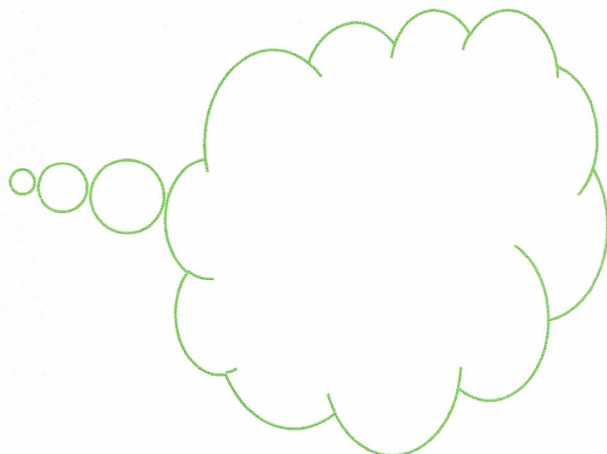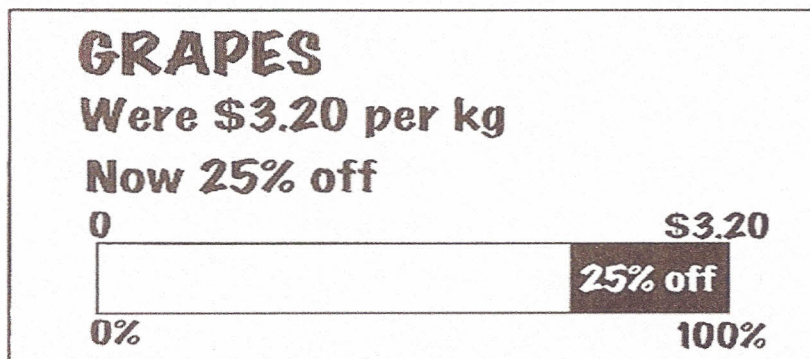
Georgia just read 16 pages in 4 minutes. She recorded her progress using the double number line shown below. How many pages did she read during the time represented in the circled section? How many minutes does this section represent?

16 Pages

4 minutes
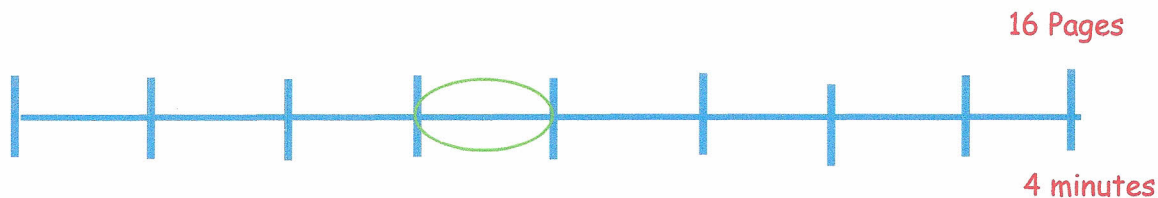
Adapted from: *Developing Essential Understanding, Ratios, Proportions, Proportional Reasoning, Grades 6 – 8*

Tape Diagram

Calculate the discounted price of grapes using information below.

GRAPES
Were $3.20 per kg
Now 25% off

```
0                        $3.20
┌────────────────┬──────────┐
│                │ 25% off  │
└────────────────┴──────────┘
0%                        100%
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More or Less, Section C: Many Changes (Mathematics in Context, Britannica)

This is Craig's receipt from a restaurant. He decided to leave a 20% tip. How much did he spend altogether? Use the bar below to show your thinking.

GUEST CHECK

80918

#7   19.00

#12   21.00

Total $40.00

THANKS!

- Use of the tape diagram helps students who only have the "key words" approach. "Altogether" may not be useful in this situation.

- The visual aspect of the model will help students develop the understanding that the total payment is 120% (bill and tax).

- From this understanding, the one step multiplication algorithm is developed.
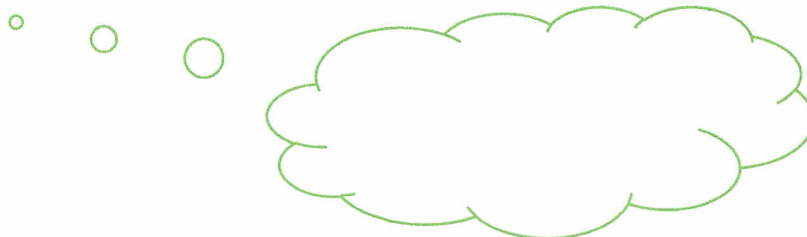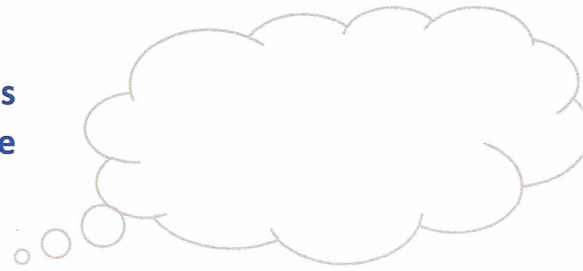
Adapted from *Models You Can Count On*, Section B: The Bar Model, p. 20 (Mathematics in Context, Britannica)

Percent increase and decrease problems require students to pay close attention to 'the whole'.

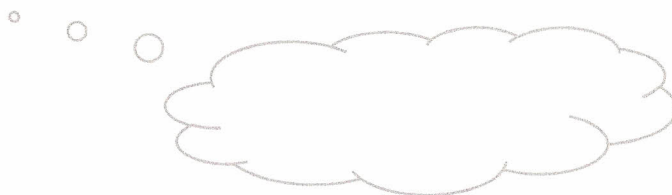After a 25% discount the price of this item was $96.00. What was the price of this item before the discount?

25% Discount
Sale price
$96.00

A digital player costs $96.00 now, but its price will go up by 25%. What will the price of this item be after the increase?

$96.00
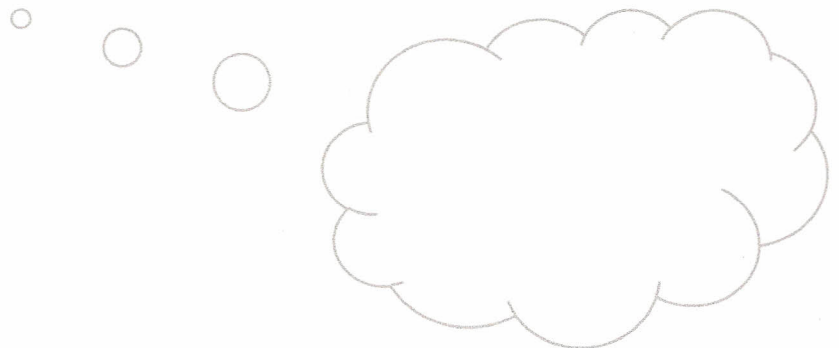
SALE

Making Sense

A new flat-screen TV is on sale for $220. If the regular price was $400, what is the percent of the discount?

SUMMARY: How are these models alike? How are they different?

Type of Model	Information	Instructional Stategies
Table of Equivalent Values (Ratio Table)		
Double Number Line		
Tape Diagram		

CONCLUSION:

- *Be aware of overestimating students' understanding on the basis of their ability to use an algorithm correctly.*

- *A better way to gauge students' proportional reasoning is to **provide problems that represent a variety of types and go beyond the typical missing-value problems.***

- ***By posing problems of assorted types and forms,** you will be able to determine with greater accuracy how your students are reasoning.*

Developing Essential Understanding of Ratios, Proportions & Proportional Reasoning, NCTM

Final Thoughts and References

Strategies for Solving Problems

Although it is traditional to move students quickly to solving proportions by setting up an equation, the Standards do not require this in Grade 6. There are a number of strategies for solving problems that involve ratios. As students become familiar with relationships among equivalent ratios, their strategies become increasingly abbreviated and efficient.

Progressions for the Common Core State Standards in Mathematics: 6-7 Ratio and Proportional Relation, *Commoncoretoools.wordpress.com*

REFERENCES:

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