

thinking outside the box

A
CHOCOLATEY
OPTIMIZATION
PROBLEM

NCTM 2014 Annual
Meeting & Exposition
New Orleans
April 11, 2014



Taken from http://a.tgcdn.net/images/products/zoom/f158_chocolate_gaming_dice_set.jpg

3-8 Gallery Workshop

CCSSM Areas:	Measurement and Data Geometry Functions
Grade Band/Audience:	3 to 5; 6 to 8
Focus on Math:	This hands-on activity allows students to explore a real-world optimization problem. In order to perform the task, students need to identify patterns, determine relationships, use appropriate tools, and multiple problem solving strategies. They will have the opportunity to collaborate with classmates and construct viable arguments to support their mathematical reasoning. This student-centered task provides teachers with the opportunity to assess students' mathematical thinking and understanding.

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3rd & 4th Grade

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

CCSS for Mathematics

Measurement and Data (MD)

3.MD.8 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Measurement and Data (MD)

4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

Performance Task

It is almost Mother's Day in the United States. Godiva, a manufacturer of premium chocolates and related products, needs to add a new gift box to their Mother's Day Collection to launch their new kind of chocolate truffle, shaped like a cube.

Since you are the packaging engineer at the Research and Development department at Godiva, it is your task to design and construct a prototype of the gift box. You have to take into consideration the requirements from the Board of Directors.

- The base of each chocolate truffle is 1" x 1" square
- The gift box has to have a rectangular shape.
- Must hold 12 chocolate truffles.
- A ribbon must be wrapped around the box.
- You will have to use the least amount of ribbon.

Your task as the packaging engineer is to design a gift box, out of a rectangular sheet of material, that meets the requirements from the Board of Directors.

For this task, you will have to come up with different designs that hold 12 chocolate truffles. Think about all the different ways you could fit 12 chocolate truffles onto a rectangle.

You will record the dimensions of each design in a table. You can only use whole-number side lengths.

- Fill in the chart for the length and width of each of your designs.
- Find the area and perimeter of each of your rectangles.
- Decide how many chocolate truffles would fit in the box.
- Decide how long the ribbon needs to be.

Now, choose the design that uses the least amount of ribbon. Draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

4th Grade Extension

The ribbon that goes around the middle of the box is very expensive, so your board of directors has told you that you can only use 18" of ribbon for each box. Think about all the possible designs you could use to create this box. Fill in the table to show all the possible combinations for a rectangle with a perimeter of 18". Which design will hold the most candies? Choose a box that holds the most candies. This is the box you will construct as your prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

3rd Grade Rubric

Criteria	4	3	2	1
Problem Solving		<p>Uses the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Finds the area of a rectangle with whole-number sides by tiling it with unit squares and showing that the area is the same as would be found by multiplying the side lengths.</p> <p>Finds rectangles with the same perimeter and different areas and with the same area and different perimeters.</p>	<p>Uses some of the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Finds the area of a rectangle with whole-number sides by either tiling it with unit squares, or multiplying the side lengths, but not both.</p> <p>Finds rectangles with either the same perimeter or different areas or with the same area and different perimeters, but not both.</p>	<p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to find the area of a rectangle by tiling with unit squares or multiplying the lengths of the sides.</p> <p>Is not able to find rectangles with the same perimeter and different areas or with the same area and different perimeters.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

3rd Grade Rubric

Criteria	4	3	2	1
Data Collection		Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.	Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.	Does not collect sufficient data. Only 1 set of dimensions is represented in the table.
Gift Box Prototype		Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box. Draws the net and labels the dimensions on the 12in x 18in sheet of paper. Constructs the prototype of the gift box by using the net.	Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box. Draws the net but does not label the dimensions correctly. Constructs the prototype of the box.	Does not represent three-dimensional figures using nets made up of rectangles. Constructs the prototype of the box without using a net. Is not able to construct a prototype of the box.
Communication and Reasoning & Proof		Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.	Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.	Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

3rd Grade

thinking outside the box

A CHOCOLATEY OPTIMIZATION PROBLEM

Box Design No.	Length	Width	Area of the Base	Number of Chocolate Truffles	Perimeter of the Base	Length of Ribbon Used

4th Grade Rubric

Criteria	4	3	2	1
Problem Solving		<p>Uses the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Applies the area and perimeter formulas for rectangles in real world and mathematical problems.</p> <p>Expresses measurements in a larger unit in a smaller unit.</p>	<p>Uses some of the attributes of shapes in order to determine areas and perimeters to design, develop, and produce packages.</p> <p>Applies the area and perimeter formulas for rectangles in real world and mathematical problems with some errors.</p> <p>Expresses measurements in a larger unit in a smaller unit with some errors.</p>	<p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the area and perimeter formulas for rectangles in real world and mathematical problems.</p> <p>Is not able to demonstrate how to express measurements in a larger unit in a smaller unit.</p>
Data Collection		<p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>	<p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>	<p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

4th Grade Rubric

Criteria	4	3	2	1
Gift Box Prototype		<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p>	<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p>	<p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p> <p>Is not able to construct a prototype of the box.</p>
Communication and Reasoning & Proof		<p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>	<p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>	<p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

Box Design No.	Length	Width	Area of the Base	Number of Chocolate Truffles	Perimeter of the Base	Length of Ribbon Used

Performance Task

It is almost Mother's Day in the United States. Godiva, a manufacturer of premium chocolates and related products, needs to add a new gift box to their Mother's Day Collection to launch their new kind of chocolate truffle shaped like a cube.

Since you are the packaging engineer at the Research and Development department at Godiva, it is your task to design and construct a prototype of the gift box. You have to take into consideration the requirements from the Board of Directors.

- The dimensions of each cubed chocolate truffle are 1 in x 1 in x 1 in.
- The gift box has to have a rectangular shape.
- The gift box must be constructed from a rectangular sheet of material with dimensions 12 in x 18 in.
- The net of the gift box must be drawn on the rectangular sheet of material. The new machines will stamp out the net of the box and production line workers will assemble the box and carefully place the truffles inside it.

Your task as the packaging engineer is to design and construct a packaging box made out of the rectangular sheet of material that will hold the most truffles.

As part of the engineering process, you will have to come up with different designs before you construct the final one. You will record the dimensions of each design in the following table. You can only use whole-number side lengths.

After you analyze the data of the different dimensions and choose the final design, you will have to draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

6th Grade Extension

As part of the engineering process, you will have to come up with lots of different designs before you construct the final one. You will record the dimensions of each design in the following table. You will have to use whole-number side lengths, as well as fractional side lengths.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

5th & 6th Grade

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

CCSS for Mathematics

Measurement and Data (MD)

5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- Apply the formulas $V = lwxh$ and $V = bwh$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

Geometry (G)

6.G.2 Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real world and mathematical problems.

5th Grade Rubric

Criteria	4	3	2	1
Problem Solving		<p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths by: packing it with unit cubes and applying the formulas $V = l \times w \times h$ and $V = b \times h$.</p> <p>Decomposes three-dimensional shapes and finds volumes of right rectangular prisms by viewing them as decomposed into layers of array of cubes.</p>	<p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms by either packing it with unit cubes or applying the formulas $V = l \times w \times h$ and $V = b \times h$, but not both.</p>	<p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly and does not determine the volume of a right rectangular prism by packing it with unit cubes.</p>
Data Collection		<p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>	<p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>	<p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

5th Grade Rubric

Criteria	4	3	2	1
Gift Box Prototype		<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p>	<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p>	<p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p>
Communication and Reasoning & Proof		<p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>	<p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>	<p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

Box Design No.	Length	Width	Height	Volume	Number of Chocolate Truffles

6th Grade Rubric

Criteria	4	3	2	1
Problem Solving		<p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths and fractional side lengths by applying the formulas $V = l \times w \times h$ and $V = b \times h$.</p>	<p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms only using whole-number side lengths by applying the formulas $V = l \times w \times h$ and $V = b \times h$.</p>	<p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly.</p>
Data Collection		<p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>	<p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>	<p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

6th Grade Rubric

Criteria	4	3	2	1
Gift Box Prototype		<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p>	<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p>	<p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p>
Communication and Reasoning & Proof		<p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>	<p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>	<p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

Box Design No.	Length	Width	Height	Volume	Number of Chocolate Truffles

7th & 8th Grade

How do packaging engineers use what they know about the properties of a product and the needs of the consumer to design packages?

CCSS for Mathematics

Geometry (G)

7.G.6 Solve real world and mathematical problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons and right prisms.

Functions (F)

8.F.5 Describe quantitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear).

Performance Task

It is almost Mother's Day in the United States. Godiva, a manufacturer of premium chocolates and related products, needs to add a new gift box to their Mother's Day Collection to launch their new kind of chocolate truffle shaped like a cube.

Since you are the packaging engineer at the Research and Development department at Godiva, it is your task to design and construct a prototype of the gift box. You have to take into consideration the requirements from the Board of Directors.

- The dimensions of each cubed chocolate truffle are 1in x 1in x 1in.
- The gift box has to have a rectangular shape.
- The gift box must be constructed from a rectangular sheet of material with dimensions 12in x 18in.
- The net of the gift box must be drawn on the rectangular sheet of material. The new machines will stamp out the net of the box and production line workers will assemble the box and carefully place the truffles inside it.
- The cost of a squared inch of packaging material is \$0.21.
- The cost of producing 1 chocolate truffle is \$0.68.
- The sale price of 1 chocolate truffle is \$1.42.

Your task as the packaging engineer is to design and construct a packaging box made out of the rectangular sheet of material that will generate the most profit.

As part of the engineering process, you will have to come up with lots of different designs before you construct the final one. You will record your data in the following table.

After you analyze the data of the different dimensions and choose the final design, you will have to draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

8th Grade Extension

Analyze the data by analyzing the relationships between different quantities. Determine if the relation is a function. If it is a function, determine if it is increasing or decreasing, and linear or nonlinear.

- Height vs Volume
- Volume vs Profit
- Surface Area vs Profit
- Height vs Profit

After you analyze the data of the different dimensions and choose the final design, you will have to draw the net of the box and construct the prototype gift box. You will be able to use tape to bind the edges.

You will have to present your prototype and a proposal to the Board of Directors. Make sure that the proposal includes the advantages of the design and why they should adopt and start producing it.

7th Grade Rubric

Criteria	4	3	2	1
Problem Solving		<p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths and fractional side lengths by applying the formulas $V = l \times w \times h$ and $V = b \times h$.</p> <p>Finds surface areas of prisms by decomposing them into pieces whose area they can determine or by using the net.</p>	<p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms only using whole-number side lengths by applying the formulas $V = l \times w \times h$ and $V = b \times h$.</p> <p>Does not find correctly the surface area of the prism.</p>	<p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly.</p> <p>Is not able to find the surface area of the prism.</p>
Data Collection		<p>Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.</p>	<p>Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.</p>	<p>Does not collect sufficient data. Only 1 set of dimensions is represented in the table.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

7th Grade Rubric

Criteria	4	3	2	1
Gift Box Prototype		<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to find the surface area and construct the prototype box.</p> <p>Draws the net and labels the dimensions on the 12in x 18in sheet of paper.</p> <p>Constructs the prototype of the gift box by using the net.</p>	<p>Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box.</p> <p>Draws the net but does not label the dimensions correctly.</p> <p>Constructs the prototype of the box.</p>	<p>Does not represent three-dimensional figures using nets made up of rectangles.</p> <p>Constructs the prototype of the box without using a net.</p>
Communication and Reasoning & Proof		<p>Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.</p>	<p>Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.</p>	<p>Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

7th Grade

thinking outside the box

A CHOCOLATEY OPTIMIZATION PROBLEM

Box Design No.	Length	Width	Height	Volume	Area of Faces						Surface Area
					F1	F2	F3	F4	F5	F6	

7th Grade

thinking outside the box

A CHOCOLATEY OPTIMIZATION PROBLEM

Box Design No.	Volume	Surface Area	Cost of Packaging Material	Cost of Producing Chocolate Truffles	Sales	Profit

8th Grade Rubric

Criteria	4	3	2	1
Problem Solving		<p>Uses the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms with whole-number side lengths and fractional side lengths by applying the formulas $V = l \times w \times h$ and $V = b \times h$.</p> <p>Finds surface areas of prisms by decomposing them into pieces whose area they can determine.</p> <p>Analyzes the data by analyzing the relationships between different quantities. Determines if the relation is a function. If it is a function, determines if it is increasing or decreasing, and linear or nonlinear.</p>	<p>Uses some of the attributes of shapes in order to determine volumes to design, develop, and produce packages.</p> <p>Finds the volume of right rectangular prisms only using whole-number side lengths by applying the formulas $V = l \times w \times h$ and $V = b \times h$.</p> <p>Does not find correctly the surface area of the prism.</p> <p>Analyzes some of the relationships between quantities.</p>	<p>Is not able to use the attributes of shapes to design, develop, and produce packages.</p> <p>Is not able to apply the volume formulas correctly.</p> <p>Is not able to find the surface area of the prism.</p> <p>Is not able to determine the relationship between quantities.</p>

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

8th Grade Rubric

Criteria	4	3	2	1
Data Collection		Collects data and organizes it in a table. More than 4 sets of dimensions are represented in the table.	Collects data and organizes it in a table. Between 2-3 sets of dimensions are represented in the table.	Does not collect sufficient data. Only 1 set of dimensions is represented in the table.
Gift Box Prototype		Represents three-dimensional figures using nets made up of rectangles and uses the nets to find the surface area and construct the prototype box. Draws the net and labels the dimensions on the 12in x 18in sheet of paper. Constructs the prototype of the gift box by using the net.	Represents three-dimensional figures using nets made up of rectangles and uses the nets to construct the prototype box. Draws the net but does not label the dimensions correctly. Constructs the prototype of the box.	Does not represent three-dimensional figures using nets made up of rectangles. Constructs the prototype of the box without using a net.
Communication and Reasoning & Proof		Drafts a proposal in persuasive language that is addressed to the Board of Directors. The proposal includes: the advantages of the design, why it should be adopted and produced.	Drafts a proposal. The proposal includes one of the following: the advantages of the design, why it should be adopted and produced.	Does not draft a proposal or the proposal drafted includes none of the following: the advantages of the design, why it should be adopted and produced.

4=Exceeds Standards 3=Meets Standards 2=Almost Meets Standards 1=Beginning to Meet Standards

8th Grade

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Box Design No.	Length	Width	Height	Volume	Area of Faces						Surface Area
					F1	F2	F3	F4	F5	F6	

8th Grade

thinking outside the box

A CHOCOLATEY OPTIMIZATION PROBLEM

Box Design No.	Volume	Surface Area	Cost of Packaging Material	Cost of Producing Chocolate Truffles	Sales	Profit

measurement

LINEAR MEASURE

AREA, PERIMETER, AND VOLUME



The following big ideas and reasons for using non-standard and standard units were taken from Van de Walle & Lovin (2006), *Teaching Student-Centered Mathematics Grades K-3* (pp. 252-257).

Big Ideas

1. Measurement involves a comparison of an attribute of an item or situation with a unit that has the same attribute. Lengths are compared to units of length, areas to unit of area, volumes to unit of volume, and so on. Before anything can be measured meaningfully, it is necessary to understand the attribute to be measured.
2. Meaningful measurement and estimation of measurements depend on a personal familiarity with the unit of measure being used.
3. Estimation of measures and the development of personal benchmarks for frequently used units of measure help students increase their familiarity with units, prevent errors in measurements, and aid in the meaningful use of measurement.
4. Measurement instruments are devices that replace the need for actual measurement units. It is important to understand how measurement instruments work so that they can be used correctly and meaningfully.
5. Area and volume formulas provide a method of measuring these attributes by using only measures of length.
6. Area, perimeter, and volume are related to each other, although not precisely or by formula. For example, as the shapes of regions or three-dimensional objects change but maintain the same areas or volumes, there is a predictable effect on the perimeters and surface areas.

(Van de Walle & Lovin, 2006, p. 252)

Informal Units and Standard Units: Reasons for Using Each

(Van de Walle & Lovin, 2006, p. 256)

Use of Informal Units:

- ☑ Informal units make it easier to focus directly on the attribute being measured. For example, in a discussion of how to measure the area of an irregular shape, units such as lima beans, square tiles, or circular counters may be suggested. Each unit covers area and each will give a different result. The discussion focuses on what it means to measure area.
- ☑ By selecting units carefully, the size of the numbers in early measurement can be kept reasonable. The measures of length for 1st grade students can be kept less than 20 even when measuring long distances simply by using longer units.
- ☑ Informal units provide a good rationale for standard units. A discussion of the need for a standard unit can have more meaning after groups in your class have measured the same objects with their own units and arrived at different answers.
- ☑ Using informal units can be fun.

Use of Standard Units:

- ☑ Knowledge of standard units is a valid objective of a measurement program and must be addressed. Students must not only develop a familiarity with standard units but must also learn appropriate relationships between them.
- ☑ Once a measuring concept is fairly well developed, it is frequently just as easy to use standard units.

To measure something, one must perform three steps:

1. **Decide on the attribute to be measured.**
2. **Select a unit that has the attribute.**
3. **Compare the units, by filling, covering, matching, or some other method, with the attribute of the object being measured.**

(Van de Walle & Lovin, 2006, p. 253)

References:

Van de Walle & Lovin (2006) *Teaching Student-Centered MATHEMATICS Grades K-3*. Pearson Education

Ryan, J., & Williams, J. (2007). *Children's mathematics 4-15: Learning from errors and misconceptions*. Open University Press.

Common Errors and Misconceptions in Mathematics

Focus on Measurement

The following errors and misconceptions were derived from the National Mathematics Assessment for Learning and Teaching (MaLT), administered to sample students aged 5 to 14 years. The reporting has been adapted from *Children's Mathematics 4-15* (2007) by Julie Ryan and Julian Williams (pp. 174 – 221).

Children Aged 5 - 7

- Misconception: “Longer” is “More.” (i.e., if there are 10 gummy worms in a row, and only 3 cars in a row, but the 3 cars are longer, children may identify the cars as “more”).
- Common Error: When measuring in non-standard units, children will often not match the end-points of the objects.
- Misconception: A ruler must be longer than the item measured (i.e., children believe that a ruler could not measure a door).
- Common Error: Two-step complexity when reading clocks. (i.e., would write 2 hours later than 11 o'clock is 2 o'clock).

Children Aged 8 - 10

- Misconception: There are 100 minutes in an hour.
- Common Error: Children ignore scale (i.e., inches vs. centimeters)
- Common Error: Converting units of measurement.
- Common Error: Children confuse the hour and minute hand on a clock.
- Common Error: Confuses clockwise and anti-clockwise; Only turns in half-turn prototypes (i.e., cannot identify 90 degrees as a quarter-turn).

Children Aged 11 - 12

- Common Error: Confuse area and perimeter.
- Misconception: Diagonal and side of square are equal (i.e., counts diagonal of a unit square as 1).
- Misconception: There are 10 or 1000 centimeters in a meter (i.e., converts 1 meter, 20 centimeters to 12 or 1020 meters).
- Common Error: Counts each interval on scale as a unit (i.e., speedometer dial reads 20, 40, 60....200 – child reads each interval as one unit)
- Common Error: Direction of turn (i.e., confuse clockwise and anti-clockwise).

