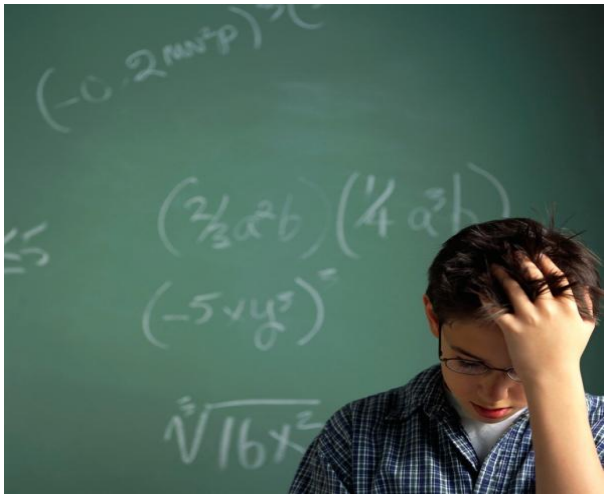
RTI in Math:
Evidence-Based Interventions for Struggling Students



NCTM 2014

New Orleans

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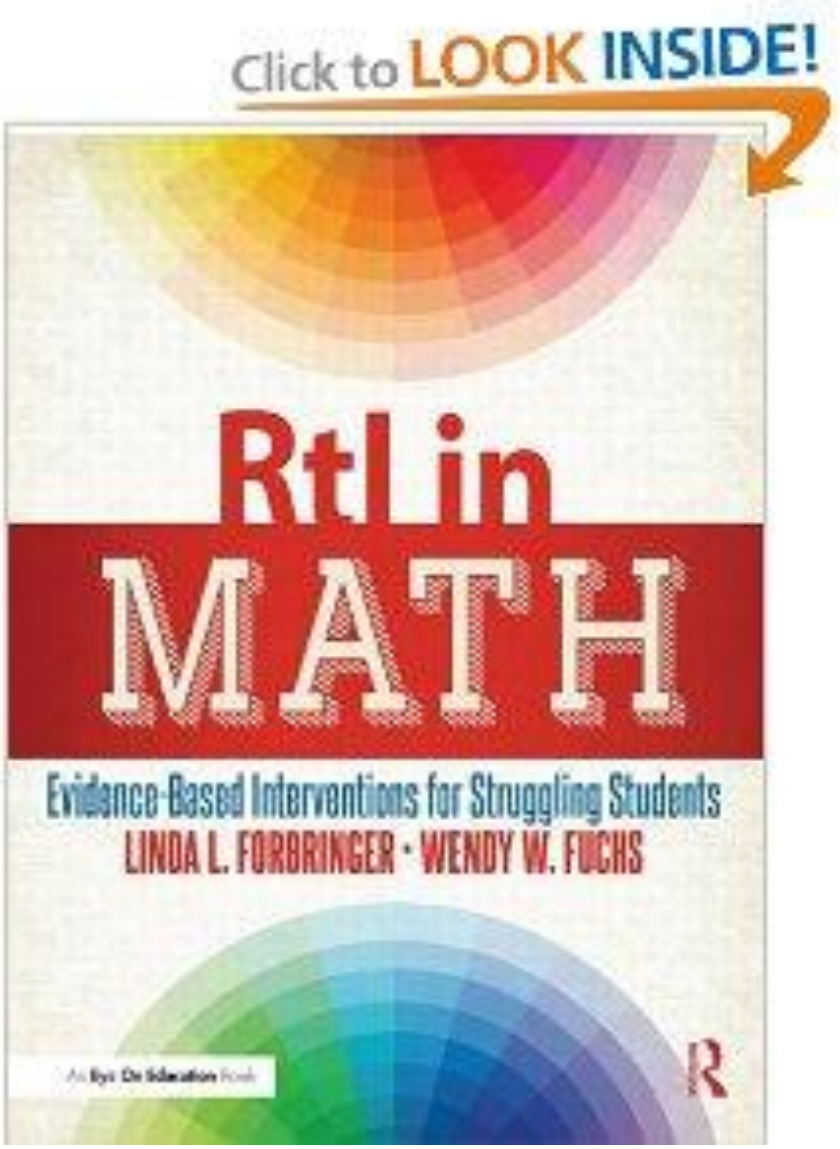
What evidence-based interventions support learners who struggle with mathematics?

Assisting Students Struggling with Mathematics: RtI for Elementary and Middle Schools (2009)

What Works Clearinghouse Practice Guide:

http://ies.ed.gov/ncee/wwc/publications/practiceguides/


Click to LOOK INSIDE!
RtI in
MATH
Evidence-Based Interventions for Struggling Students
LINDA L. FORBRINGER · WENDY W. FUCHS

WWC Recommendations

1. Universal Screening →
   Tiered, Targeted Interventions
2. Monitor Progress & Adjust Interventions
3. Include Motivational Support
4. Focus on Foundational Skills
   (whole numbers & rational numbers)

WWC Recommendations

5. Develop Fluency with Basic Facts
6. Use Visual Representations
7. Use Underlying Structures to Teach Problem Solving
8. Use Explicit Instruction During Interventions

5. What Works Clearinghouse Recommends

Develop Fluency with Basic Facts

Interventions at all grade levels should devote about

10 minutes each session

to building fluent retrieval of basic arithmetic facts.

To build fluency –

How many unfamiliar facts should students practice at one time?

Memorize the following numbers:

7 4 3 8 5 9 2

How many did you remember?

7 4 3 8 5 9 2

The number of items
that the average adult
can hold in working memory
is about

7 ± 2

Miller, 1956

Capacity increases with age:

Age	# of Items
15	7 ± 2
13	6 ± 2
11	5 ± 2
9	4 ± 2
7	3 ± 2
5	2 ± 2

Pascual-Leon, 1970

Research Findings

Students who struggle with mathematics typically have deficits in working memory.

What happens when the pace of instruction exceeds the learner's capacity?

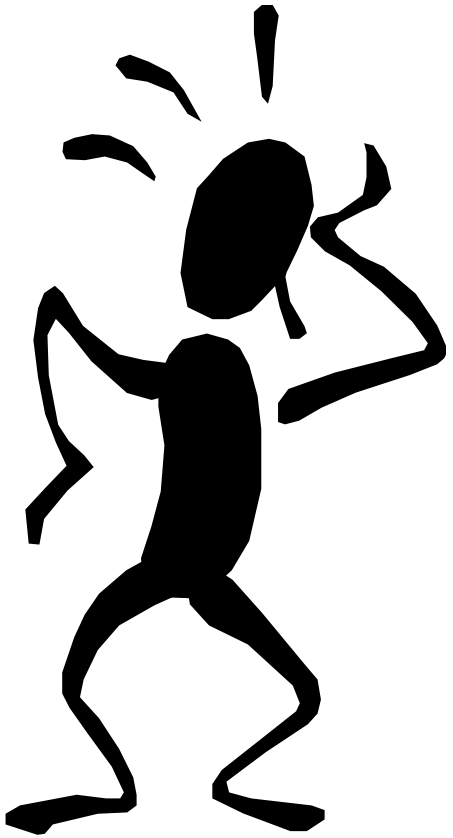
Memorize this list of numbers:

6 3 1 5 2 7 4 2 1 9 4

How well did you do?

6 3 1 5 2 7 4 2 1 9 4

When the task
exceeds the child's capacity,
little learning occurs!



Limit the number of new facts students practice at one time!

WWC suggests
2 new facts,
plus review

Potential Roadblock

- Commercial materials often practice too many facts at a time.

Differentiating Practice: Peer Tutoring

Examples:

- Classwide Peer Tutoring (CWP)
- Peer Assisted Learning Strategies (PALS)
- Etc.

Process:

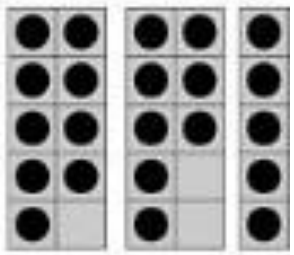
- Pre-assess. Create flashcards for facts that the student will learn first and a bank of facts for later practice.
- Partner Practice
  Partners take turns practicing the facts on 5 of their cards.
- Reward Improvement

www.promisingpractices.net/program.asp?programid=99

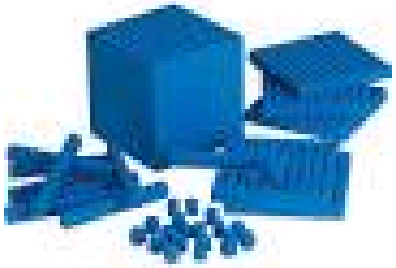
Differentiating Practice

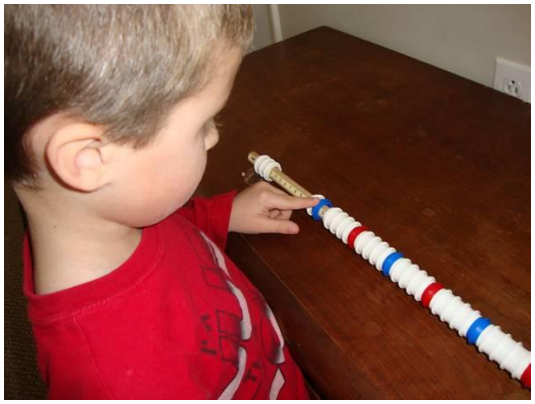
6. WWC Recommendation:

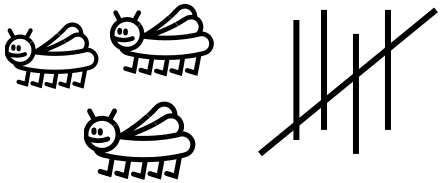
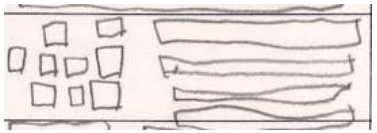
Use Visual Representations



- Ten Frames
- Mathline
- Base Ten Blocks
- DigiBlocks





Concrete	Representational	Abstract
• Manipulatives • Act it out	• Pictures • Drawings • Diagrams • Numberlines • Tally marks	• Words • Symbols
 	  	$2 + 3$ $a + b$ 5^2 one half

Concrete – Representational – Abstract

Explicitly link CRA!

II

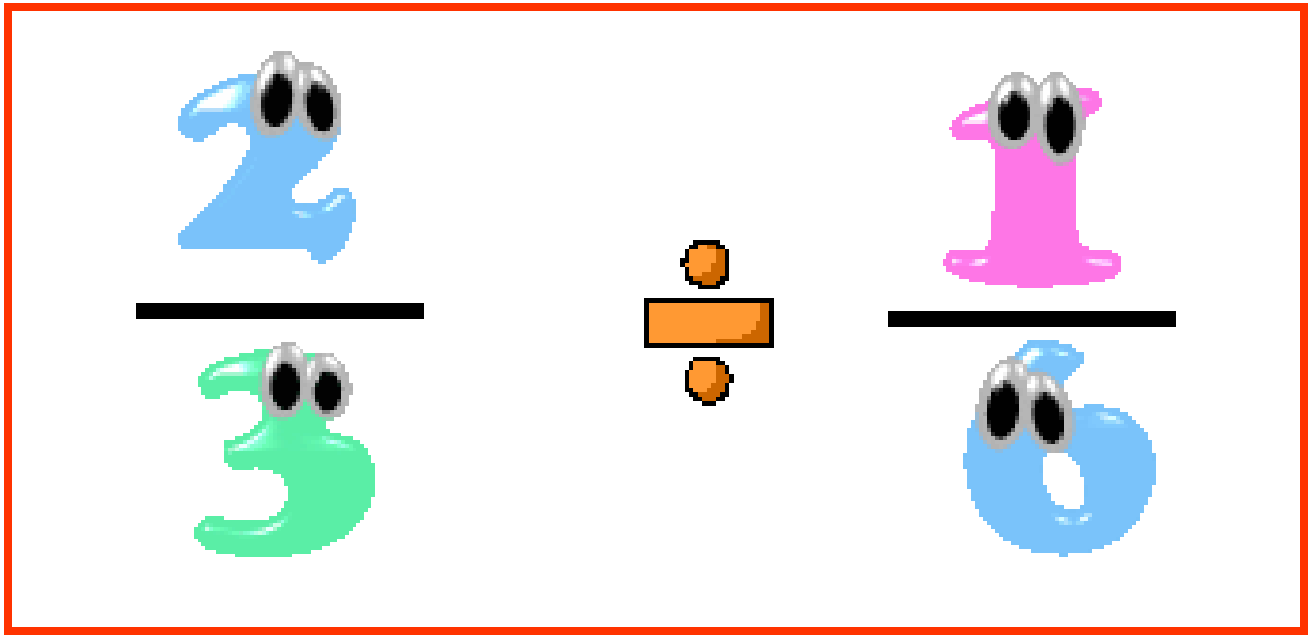
II

II

II

4 x 2 = 8

Dividing Fractions



5th grade:

$\frac{1}{2} \div 4$

5th grade:

$4 \div \frac{1}{2}$

6th grade:

$\frac{1}{2} \div \frac{1}{4}$

Dividing Fractions

"Division by fractions, the most complicated operation with the most complex numbers, can be considered as a topic at the summit of arithmetic."

Liping Ma, 1999

Solve: $1\frac{3}{4} \div \frac{1}{2}$

Represent $1\frac{3}{4} \div \frac{1}{2}$

U.S. Teachers: 39% could solve
4% could represent

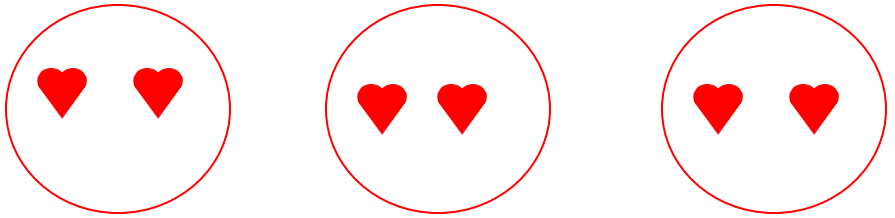
Chinese Teachers: 100% could solve
90% could represent

Modeling Division of Fractions

Review:

Dividing Whole Numbers- $6 \div 2 = 3$

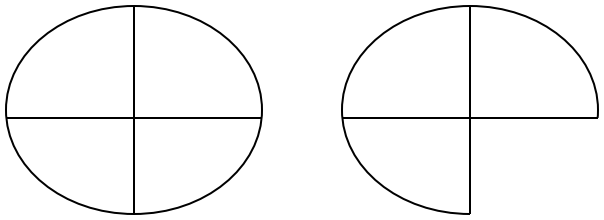
If I have 6 items, how many groups of 2 can I make?



Dividing Fractions:

If I have 1 ¾ items, how many groups of ½ can I make?

$1\frac{3}{4} \div \frac{1}{2} = 3\frac{1}{2}$

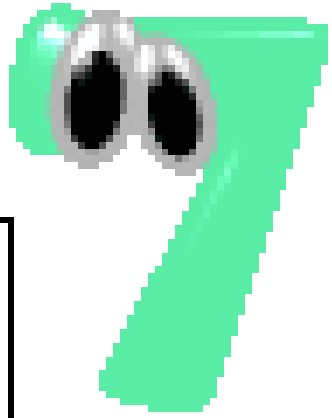


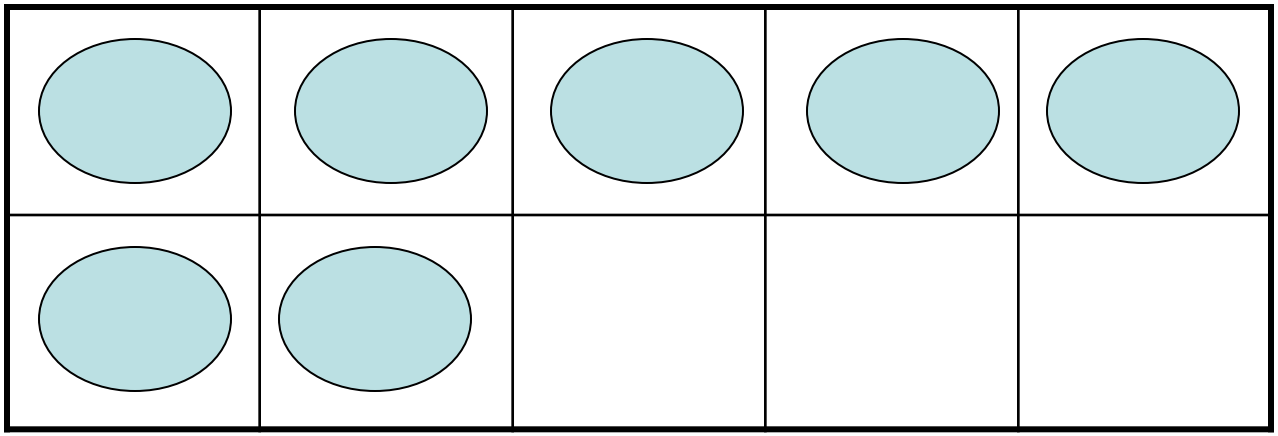
Fractions: Percentage of Problems Using Each Representation

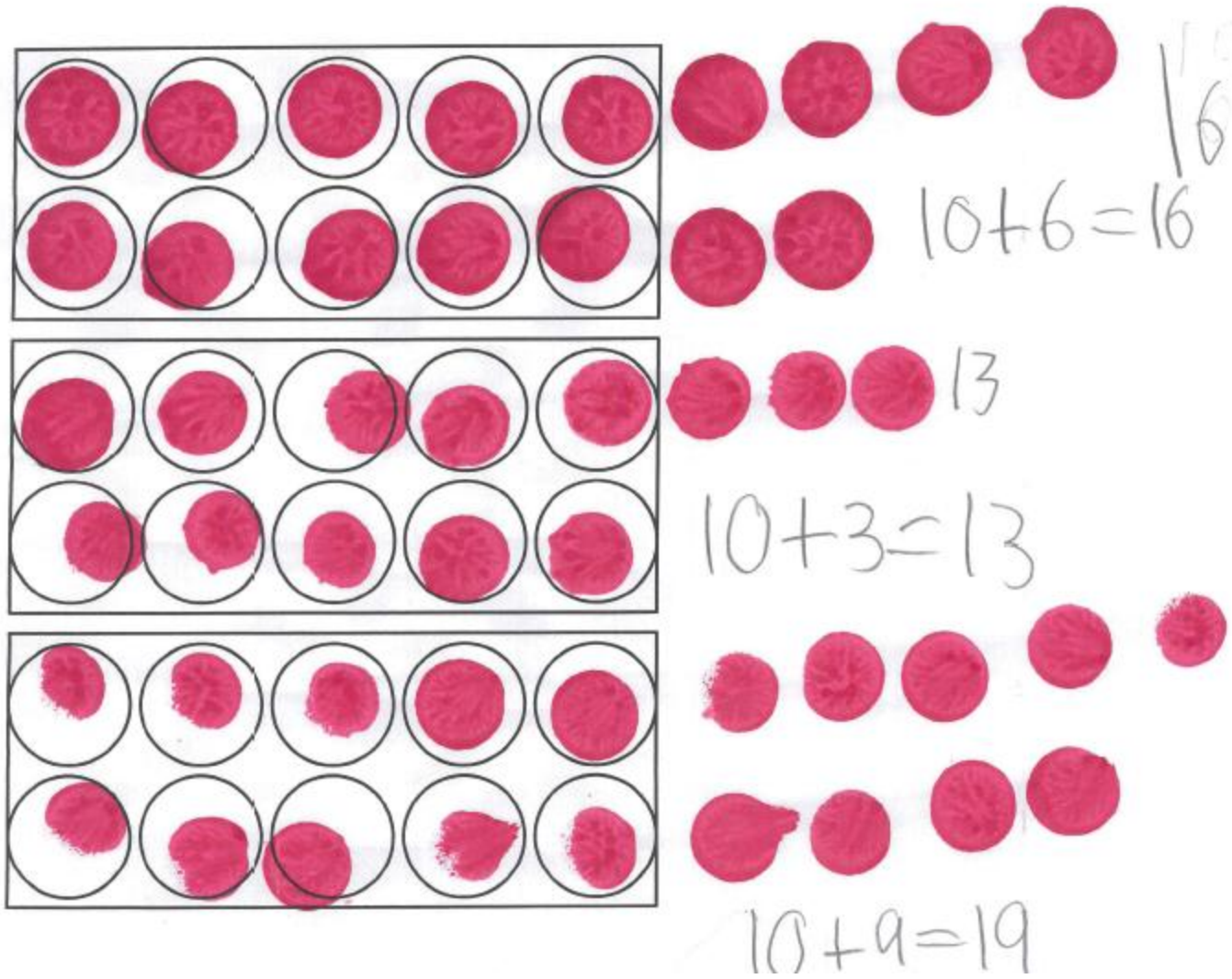
Representation	CMP	Thematics	Glencoe
Concrete: Manipulatives	2.86%	5.12%	0.25%
Representational: Pictures	27.31%	7.28%	7.36%
Abstract: Written language	31.94%	7.28%	6.38%
Abstract: Symbols	58.15%	88.95%	95.71%

Fraction Representation: The Not-So-Common Denominator among Textbooks *Mathematics Teaching in the Middle School 14(2), 78-84.* NCTM (2008)

Effective Visuals: Ten Frames

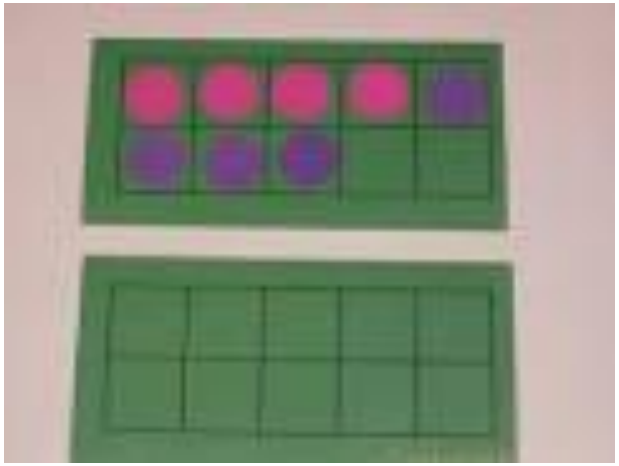





16
10+6=16
13
10+3=13
10+9=19

Ten Frames

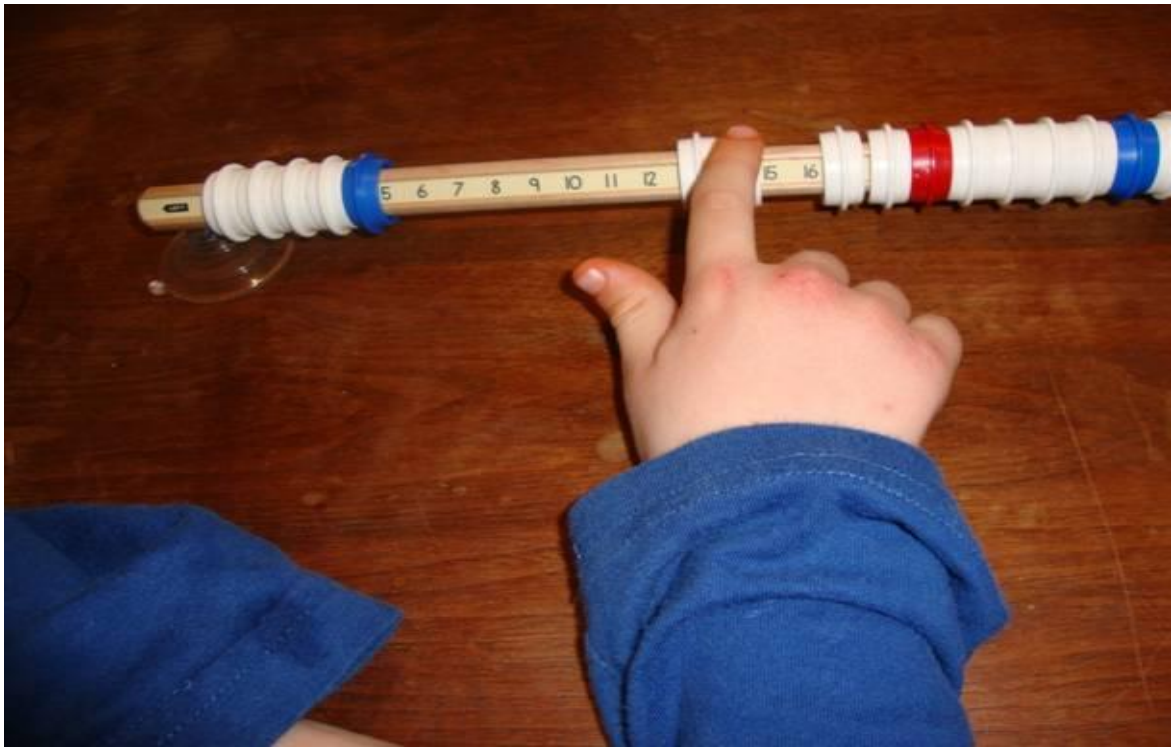
dot_card_and_ten_frame_package2005.pdf



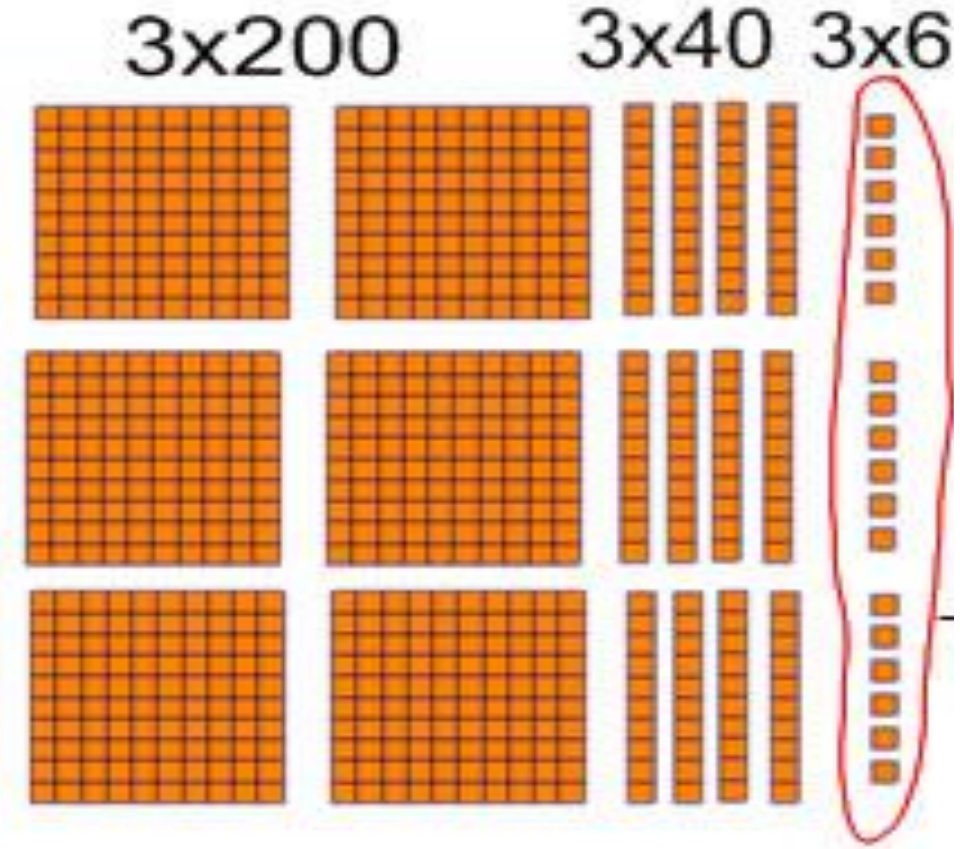
Concrete → Visual → Abstract Representation: Number lines

Mathline

www.howbrite.com




ACTIVITIES FOR
BASE TEN BLOCKS

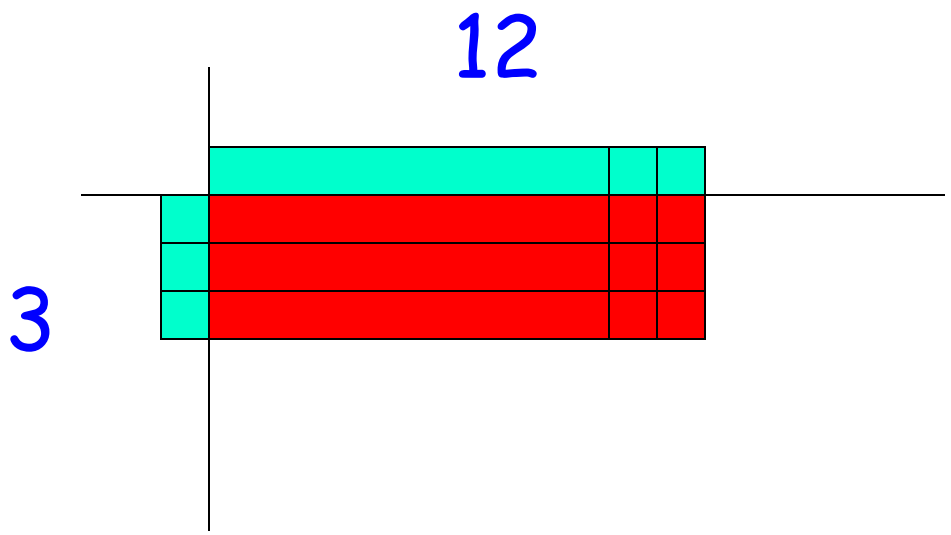


3 x 246 = 738

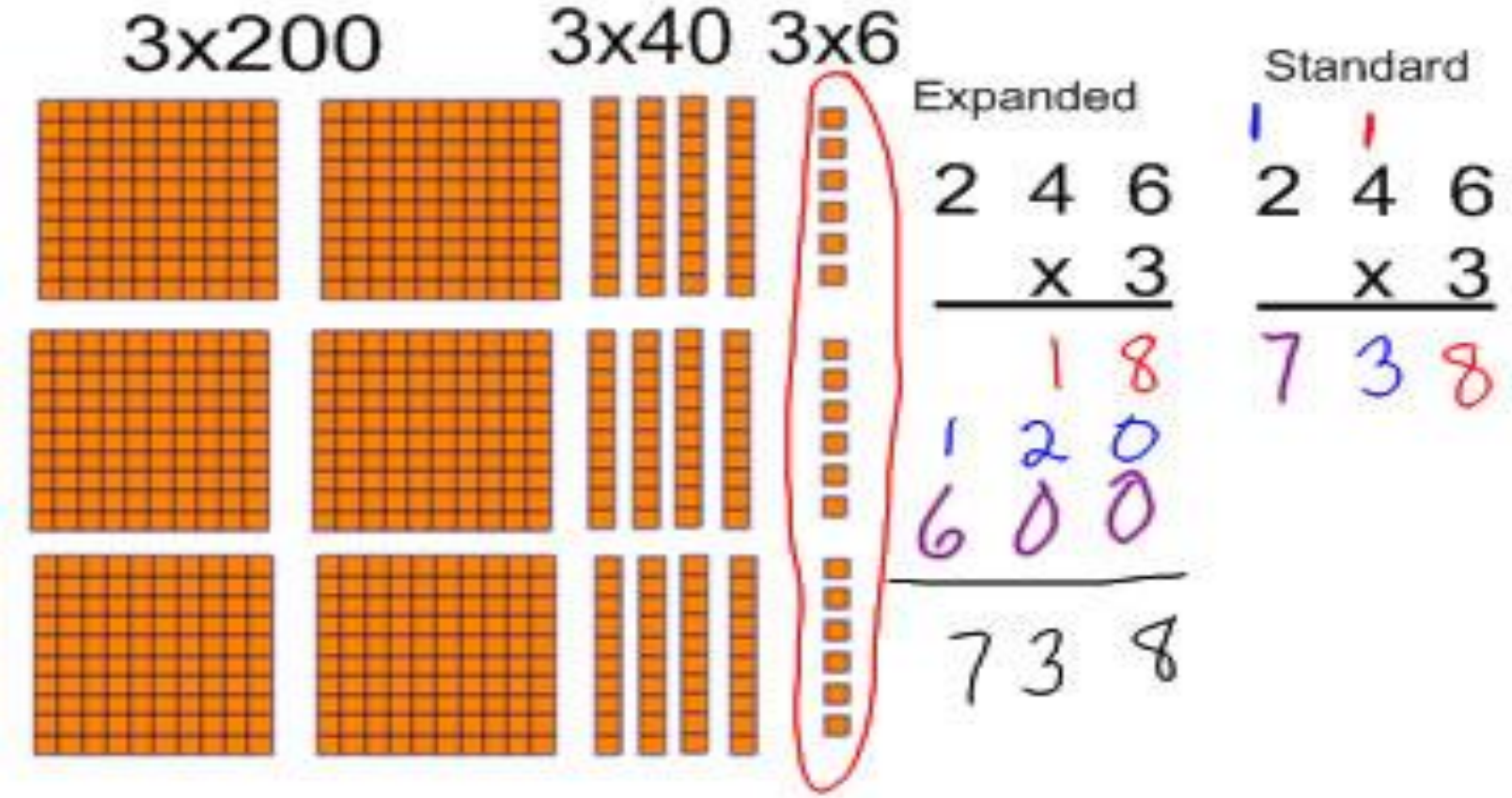
The Low Stress or Partial Products Algorithms: Representing Multiplication with Arrays and Area Models

$12 \times 3 = 36$

$$\begin{array}{r} 12 \\ \times\ 3 \\ \hline 30 \\ 6 \\ \hline 36 \end{array}$$



The representation should match the abstract process.

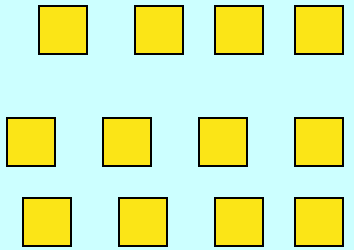


3 x 246 = 738

Modeling Multiplication
The Standard Algorithm

14

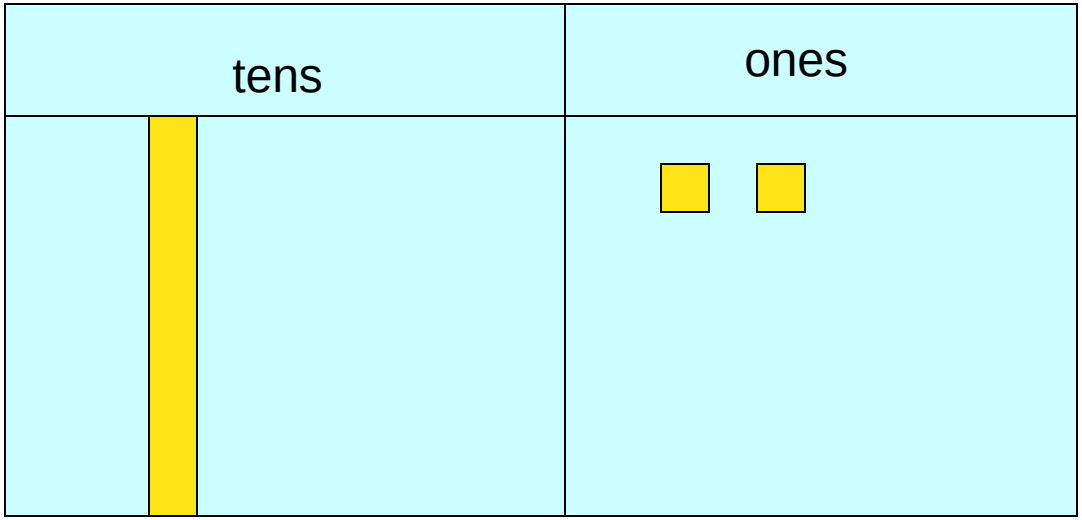
X 3

tens	ones
	

✓ **Multiply ones.**
Regroup and Record.
Multiply tens.
Add regrouped tens.
Record.

Modeling Multiplication
The Standard Algorithm

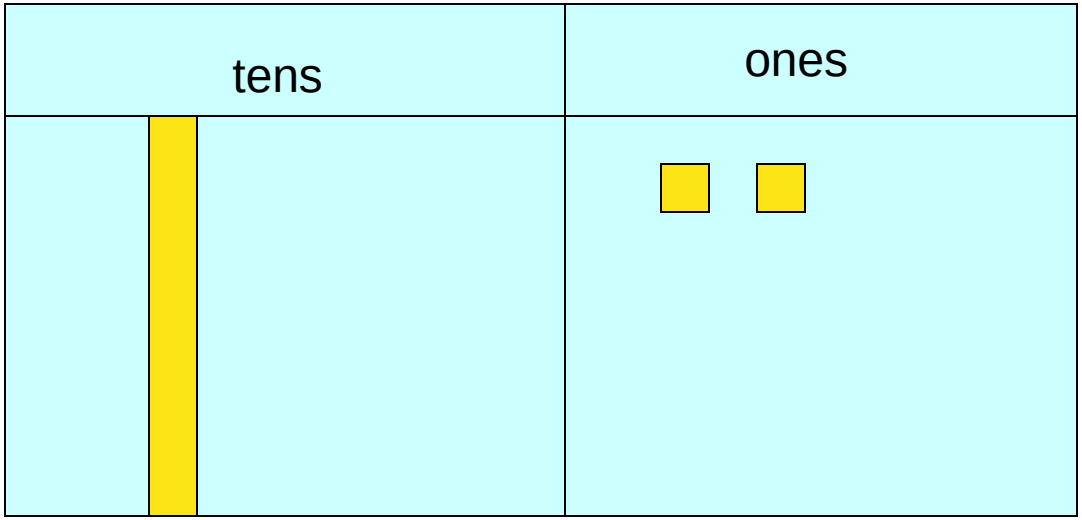
$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline \end{array}$$



✓ **Multiply ones.**
Regroup and Record.
Multiply tens.
Add regrouped tens.
Record.

Modeling Multiplication
The Standard Algorithm

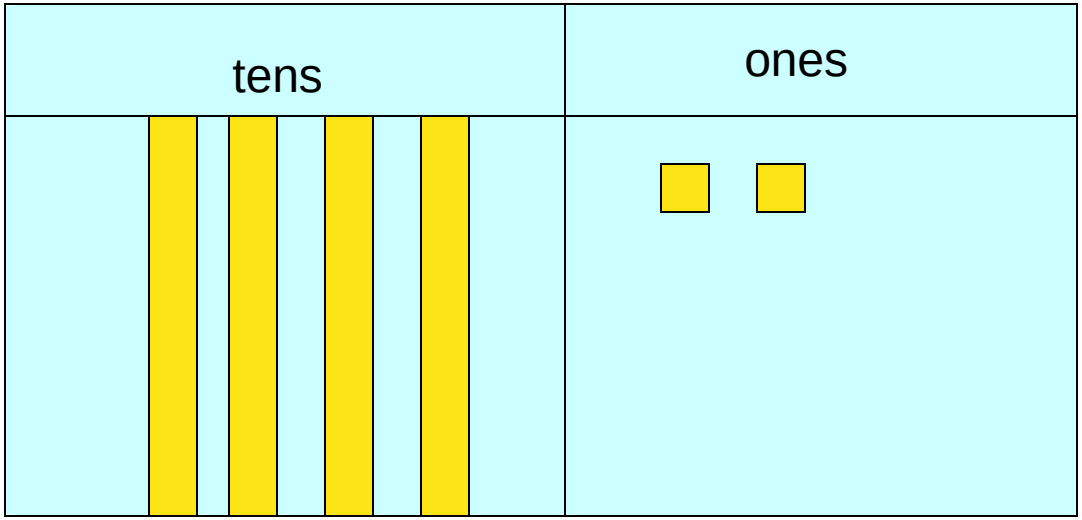
$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline 2 \end{array}$$



Multiply ones.
Regroup and Record.
Multiply tens.
Add regrouped tens.
Record.

Modeling Multiplication
The Standard Algorithm

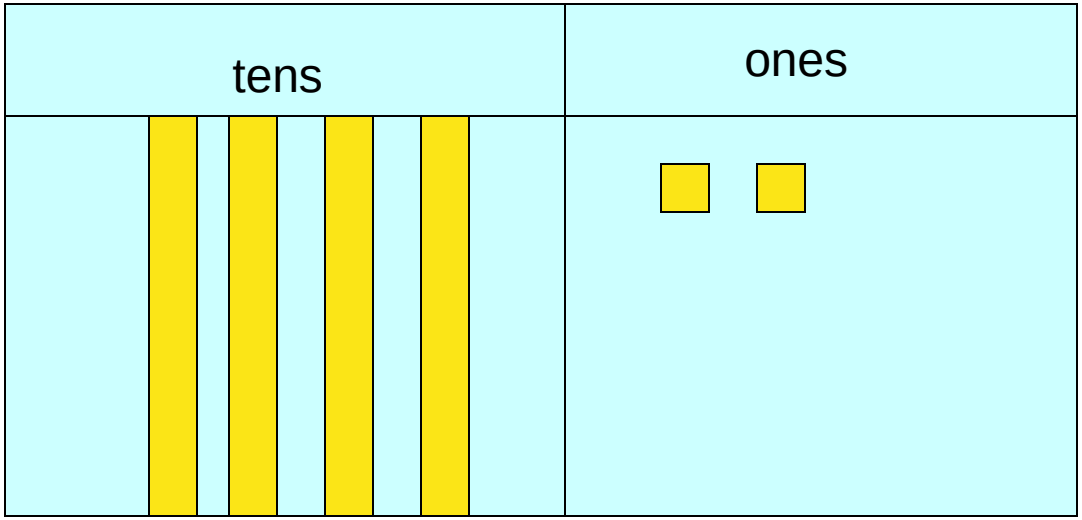
$$\begin{array}{r} {}^{1}4 \\ \times\ 3 \\ \hline 2 \end{array}$$



- ✓ **Multiply ones.**
- ✓ **Regroup and Record.**
- ✓ **Multiply tens.**
- **Add regrouped tens.**
- **Record.**

Modeling Multiplication
The Standard Algorithm

$$\begin{array}{r} {}^{1}4 \\ \times\ 3 \\ \hline 4\ 2 \end{array}$$



- **Multiply ones.**
- **Regroup and Record.**
- **Multiply tens.**
- **Add regrouped tens.**
- **Record.**

Effective Visuals: Digi-blocks

- http://www.digi-block.com/

Research Findings

Concrete: 3 lessons

Representational: 3 lessons

Abstract

Hudson, Peterson, Mercer & McLeod, 1988; Peterson, Mercer & O'Shea, 1988; Butler, Miller, Crehan, Babbitt & Pierce, 2003; Harris, Miller & Mercer, 1995; Mercer & Miller, 1992; Miller, Harris, Strawser, Jones & Mercer, 1998; MMiller, Mercer & Dillon, 1992

Concrete – Representational – Abstract

3 concrete → 3 pictorial → abstract

II

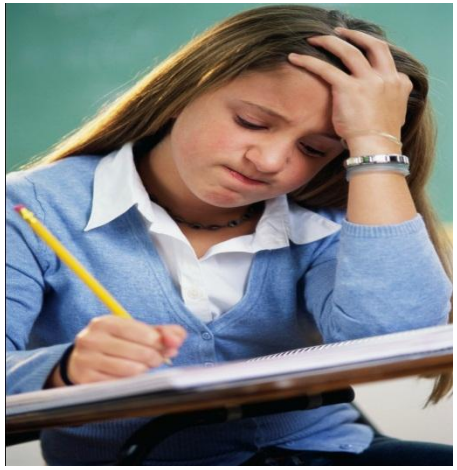
II

II

3 x 2 = 6

Potential Roadblock

- Commercial materials
  - Often fail to provide any concrete or visual representation
  - Move too quickly to abstract words and symbols
  - Do not provide a clear link from concrete to representational to abstract presentations



7. WWC Recommendation: Problem Solving

- Interventions should include instruction on solving word problems that is based on common underlying structures.

  - What Works Clearinghouse 2009

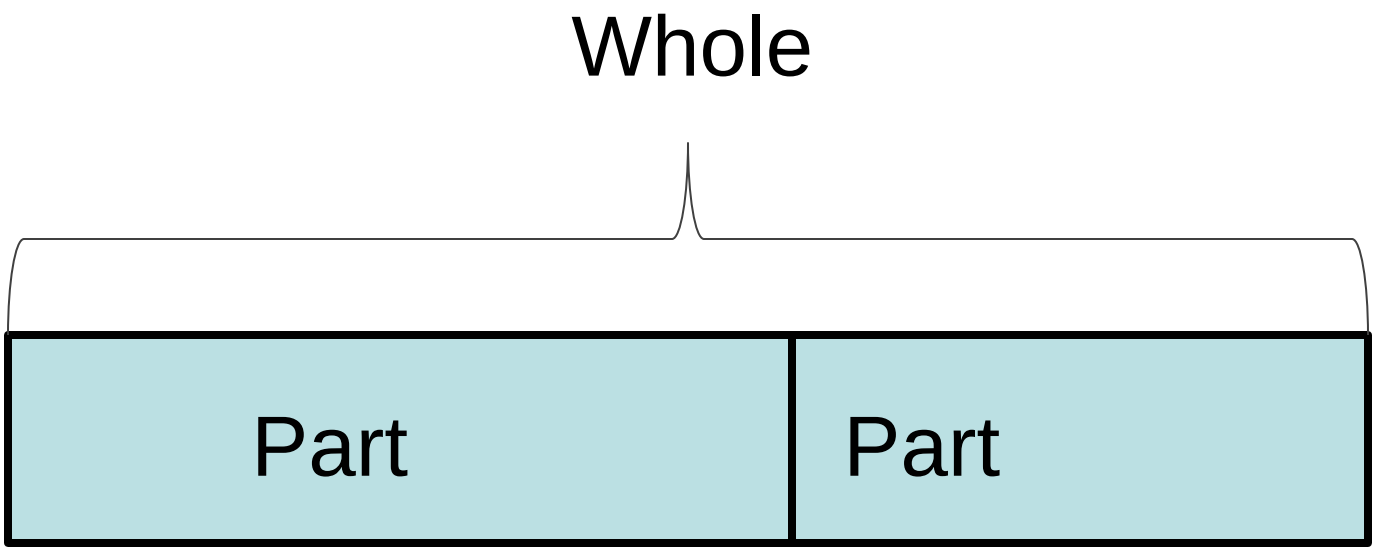
Schema-based Instruction

Addition & Subtraction

3 Basic Structures:

- Group (Part-Whole)
- Change
- Compare

Group Problem

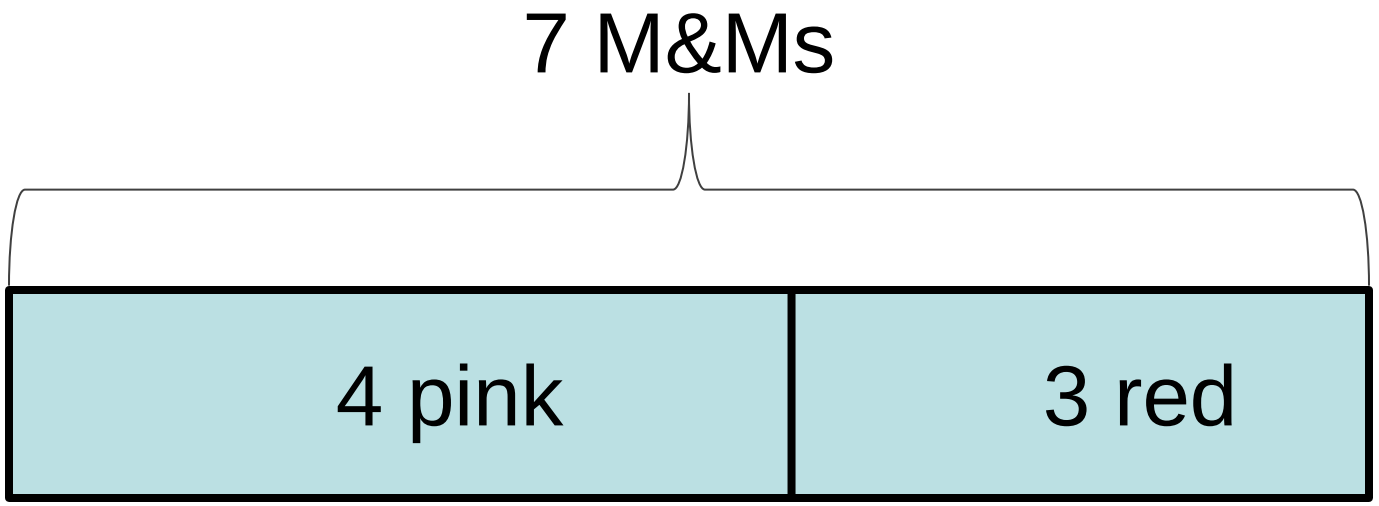


Group Problem

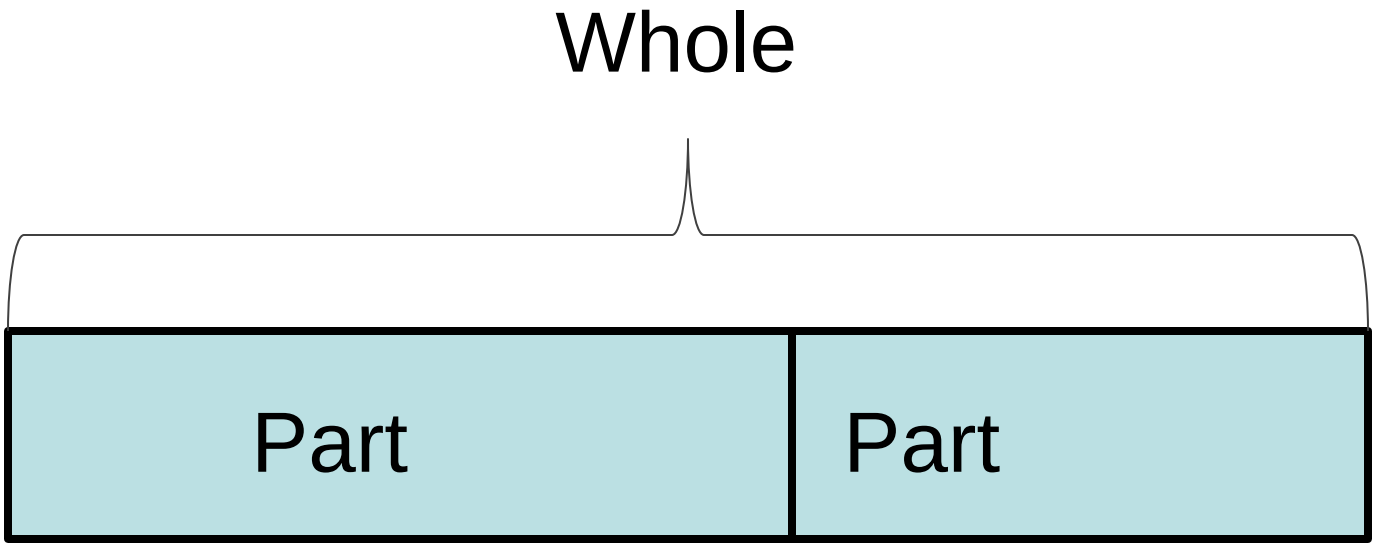
I have 7 M&Ms.

4 of them are pink.

3 of them are red.

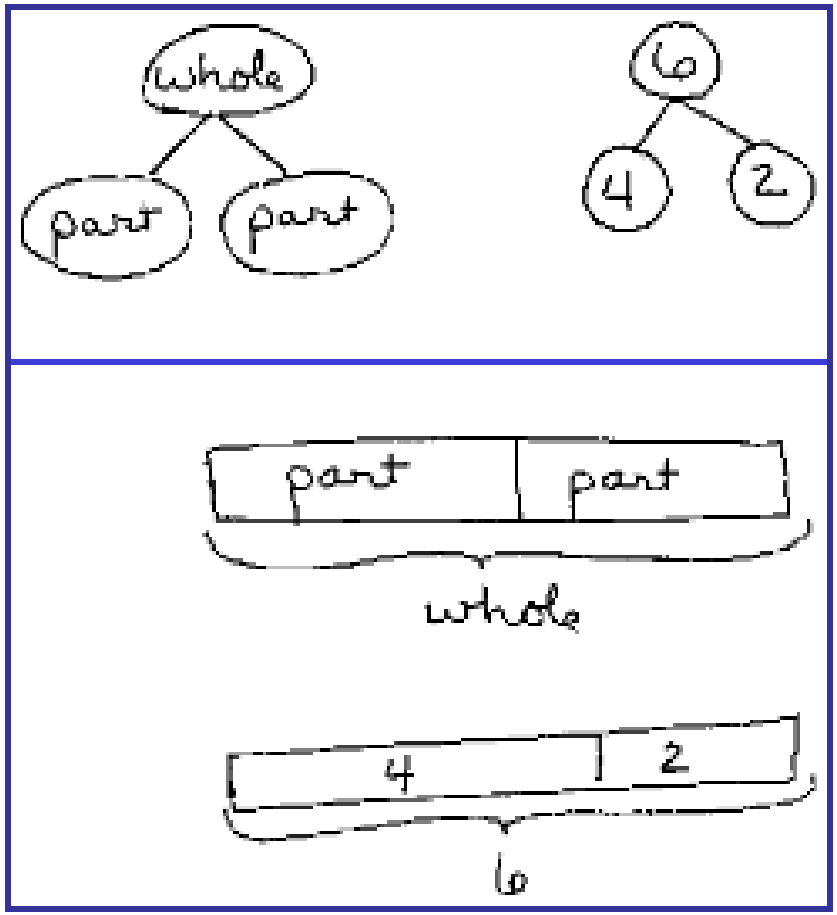


Group Problem

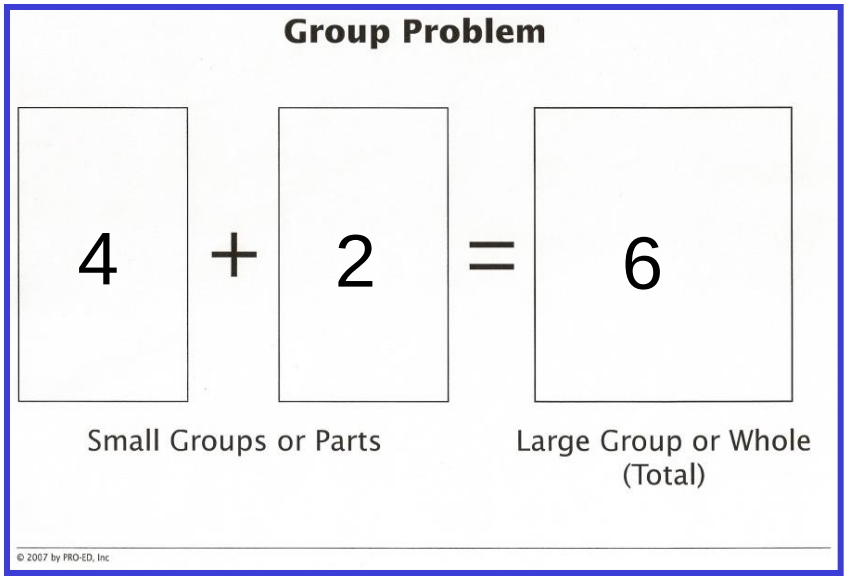


Part + Part = Whole
Whole – Part = Part

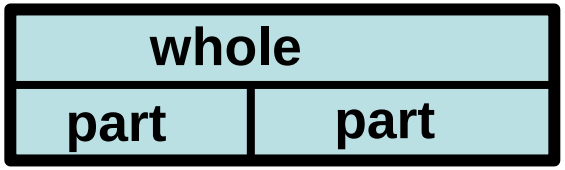
Representing Group Problems



Singapore Math

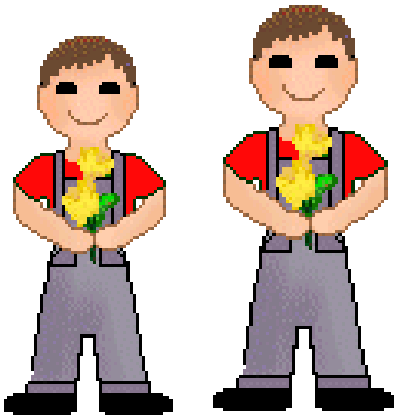


Jitendra



Everyday Math

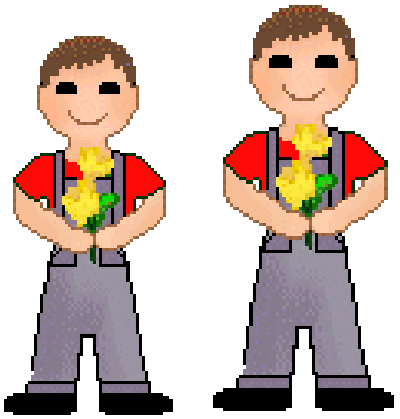
‘Change’ Problems

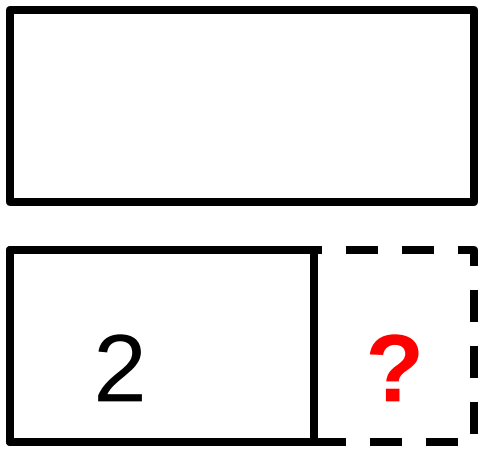


3	2

5

‘Compare’ Problems

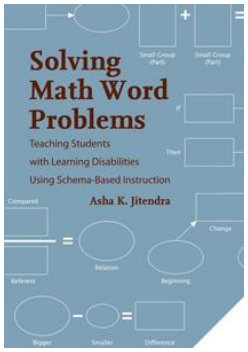




Potential Roadblock

Commercial materials seldom organize problems by underlying structures.

Resources



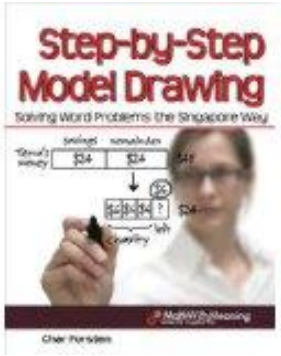
1. Solving Math Word Problems:

http://www.proedinc.com/customer/productView.aspx?ID=4145

2. Go Solve Computer Program

http://www.tomsnyder.com/products/product.asp?SKU=GOSGOS

Resources

3. 

Step-by Step Model Drawing
Char Forsten
(Singapore Math)

4. Thinking Blocks
Model Your Math Problems

http://www.mathplayground.com/wordproblems.html

Resources

5. Pirate Math

Fuchs – Vanderbilt University

www.kc.vanderbilt.edu/pals

8. WWC Recommendation:

Students requiring math interventions learn best with explicit instruction.

Characteristics of Learners Requiring Math Interventions

Frequently have deficits in:

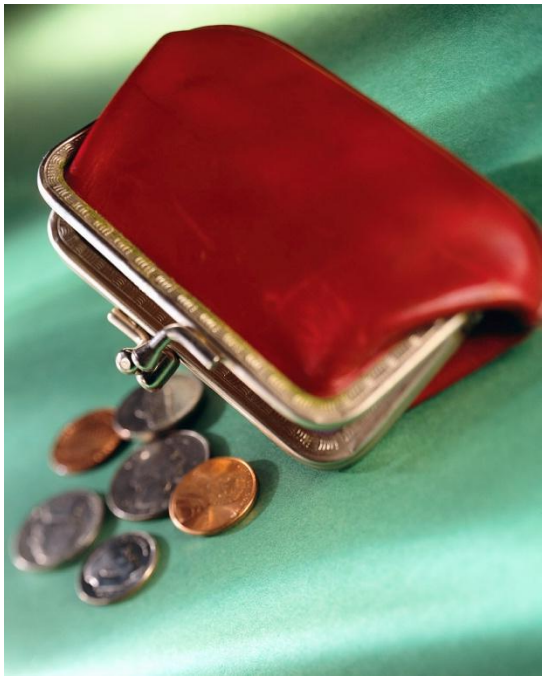
- Language
- Memory
- Executive Functioning
- Motivation & Attention

Allsop et al., 2010; Geary, 2004; Hallahan et al., 2005; Mabbott & Bisanz, 2008; Morzzacco, 2007; Swanson, Jerman & Zheng, 2009

How does explicit instruction differ from the inquiry model?

Example:

Counting Coins



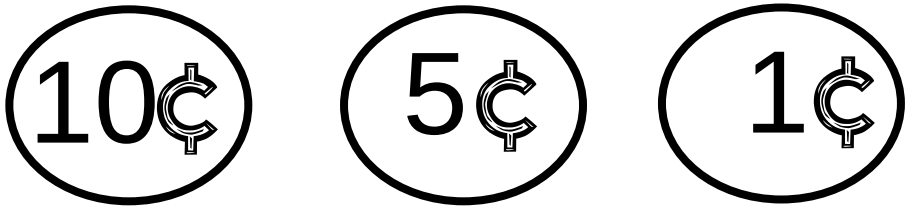
1. Review Prerequisite Skills

- Identify coins and coin values
- Count by 1’s, 5’s, 10’s
- Count on

2. Model Explicit Strategies

Counting Coins

1. Group like coins.
2. Order from largest value to smallest value.
3. Total the largest coins.
4. Move to the next largest coins. Count on (by skip counting) to find the new total.
5. Repeat step 3 until you have counted all the coins.



3. Guided Practice

- Students demonstrate understanding before working independently.
- Teachers provide scaffolded support and feedback, monitor understanding, and gradually fade support.

4. Independent Practice

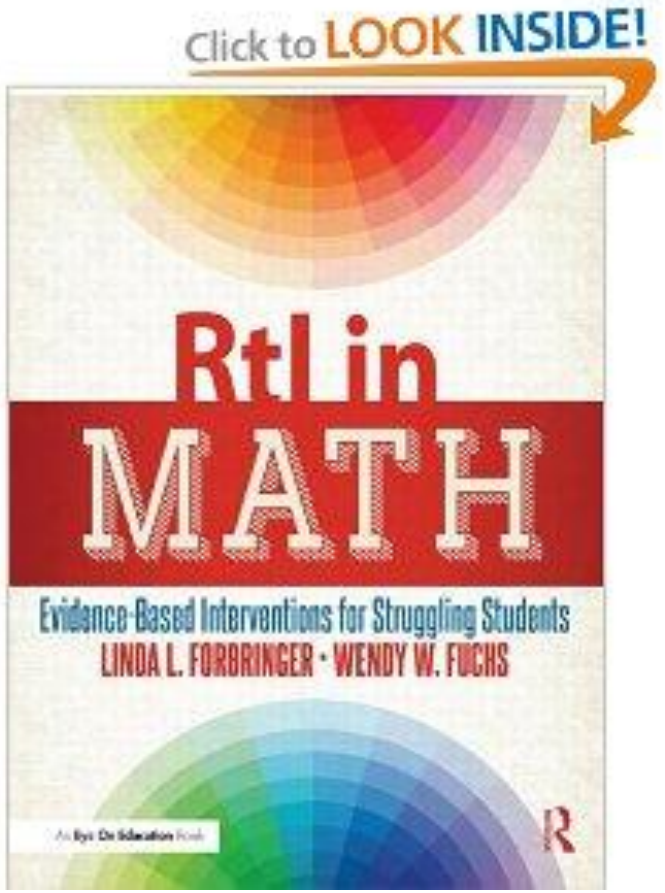
Only when students can practice successfully!

Successful practice = **90-100% accuracy**

Summary

1. Universal Screening → Tiered, Targeted Interventions
2. Monitor Progress & Adjust Interventions
3. Include Motivational Support
4. Focus on Foundational Skills (whole numbers & rational numbers)
5. Develop Fluency with Basic Facts
6. Use Visual Representations
7. Use Underlying Structures for Problem Solving
8. Use Explicit Instruction During Interventions

Thank you!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RtI in
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