The Core of Number Sense

Adapted from Developing Essential Understanding of Number and Numeration & Teaching Student Centered Mathematics

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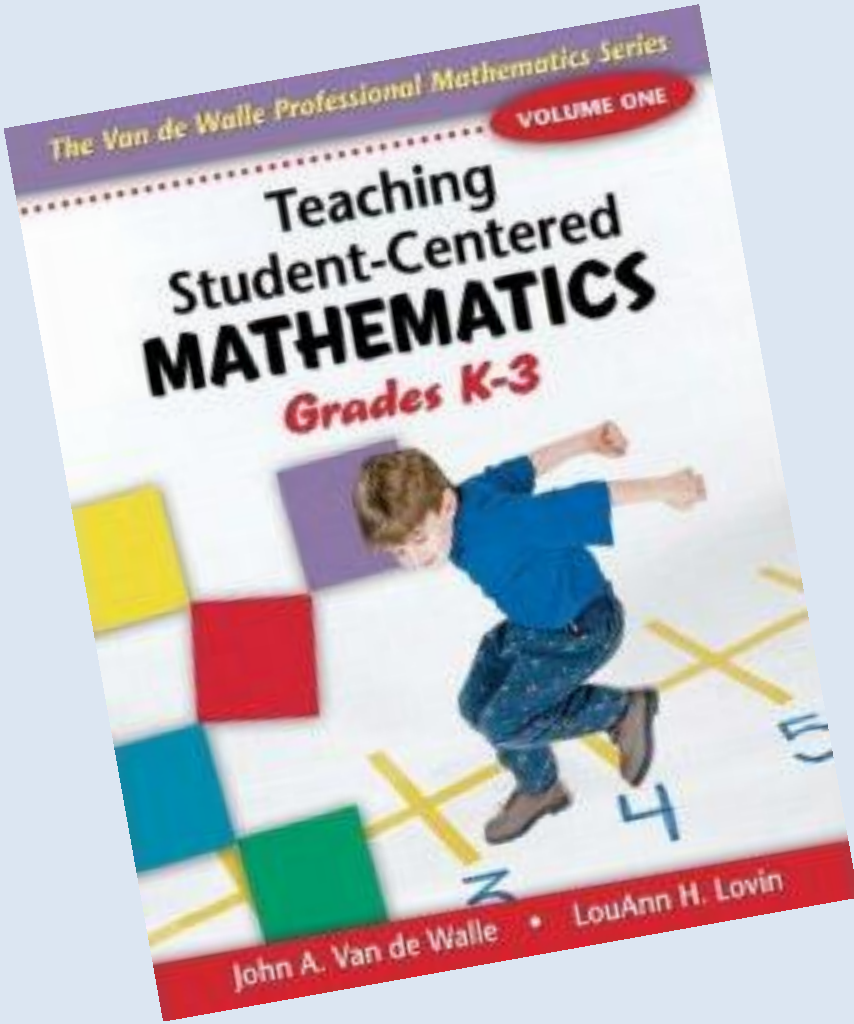
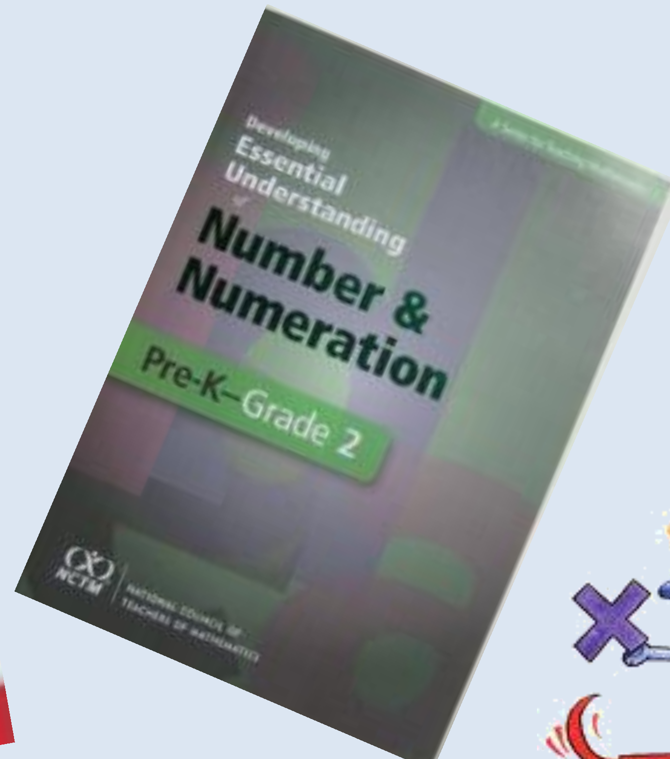
1:00 – 2:15

Information based on these books





Number Vocabulary

words you can define

words you have heard

words you don't know

- ☐ Discrete unit
- ☐ Continuous unit
- ☐ Conservation
- ☐ Subitizing
- ☐ Iteration
- ☐ Compensation
- ☐ Cardinality
- ☐ Ordinality
- ☐ Hierarchical Inclusion

Big Idea 1

1c. The relation between one quantity and another quantity can be an equality or inequality relation. (K.CC.6, 2.MD.4, MP7, MP8)

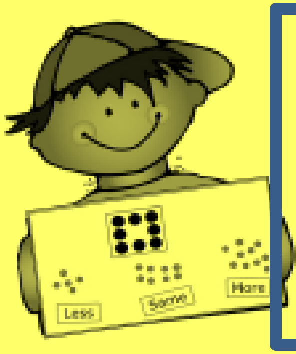
Four statements I can make about unequal sets:

1.

2.

3.

4.



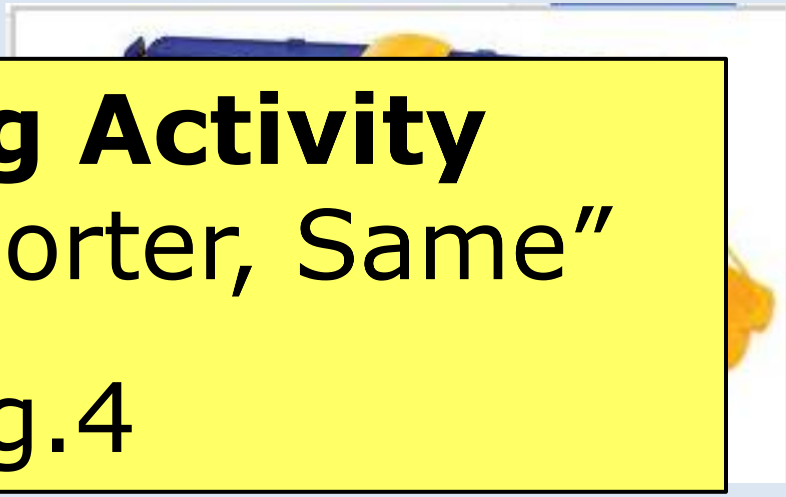
<u>VDW Activity 2.1: More/Less/ Same</u>, <u>p. 38 (105)</u>

At a workstation or table, provide about eight cards with sets of 4 to 12 objects, a set of small counters or blocks, and some word cards labeled *More*, *Less* and *Same*. Next to each card have students make three collections of counters: a set that is more, one that is less, and one that is the same. The appropriate labels are placed on the sets.

Big Idea #1

Number is an extension of more basic ideas about **relationships** between **quantities**.

1a. Quantities can be compared without assigning numerical value



Learning Activity

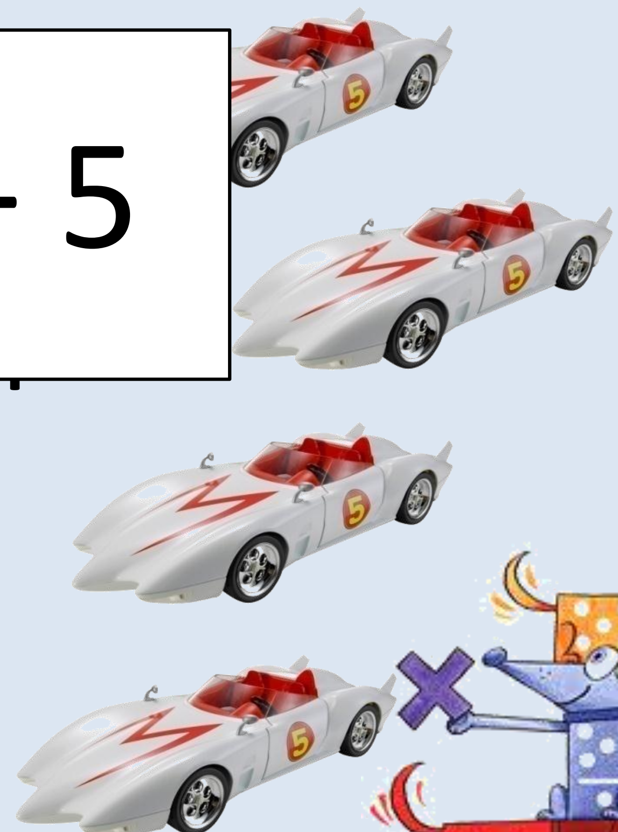
"Longer, Shorter, Same"

Pg.4

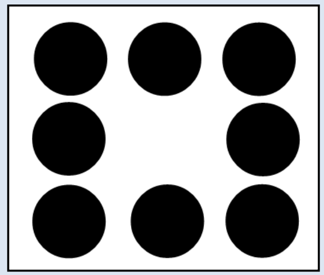
1b. All quantitative comparisons involve selecting particular attributes to compare.

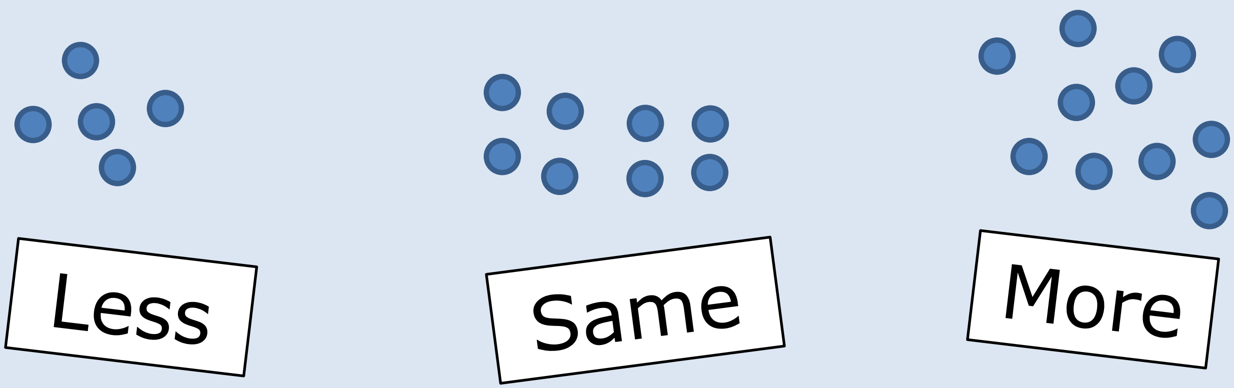
1c. The relation between one quantity and another quantity can be an equality or inequality relation.

$$8 + 4 = \square + 5$$

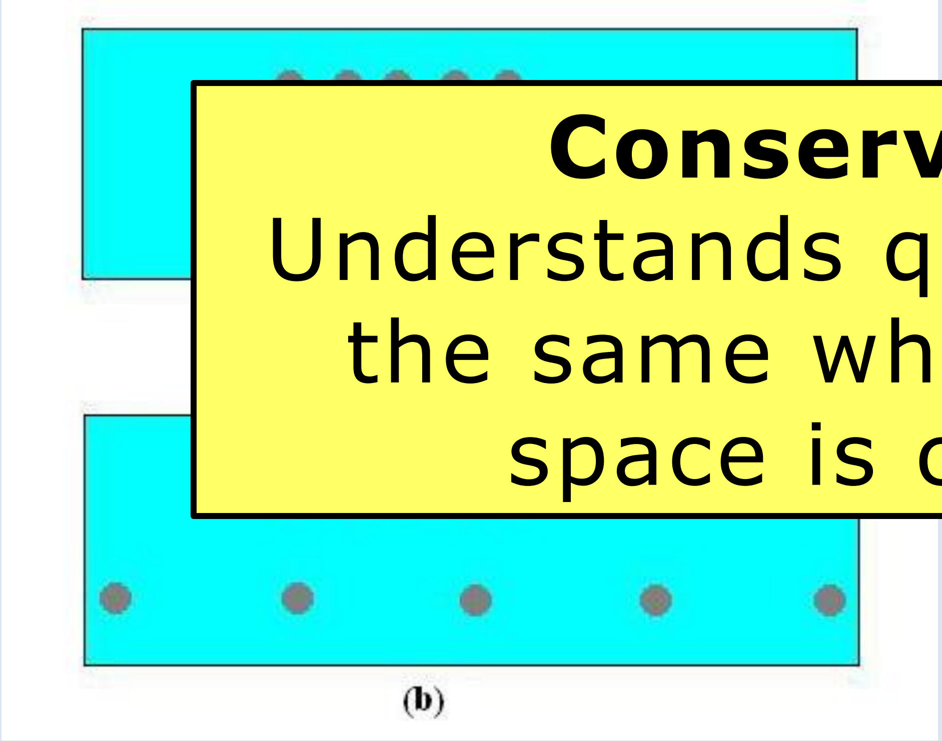


More/Less/Same Activity
(with counters)



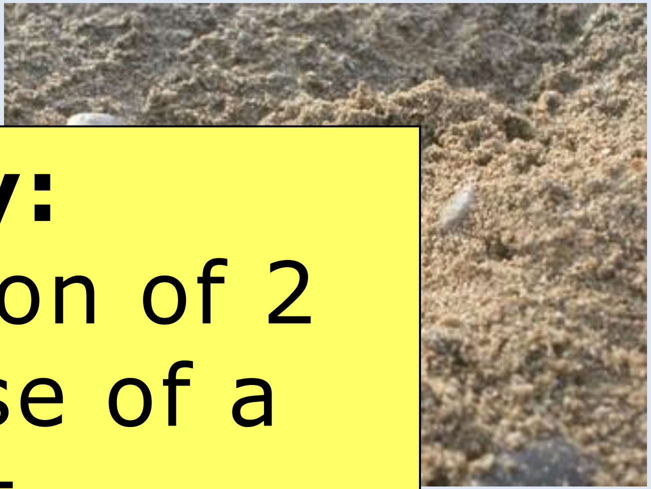


1d. Two important properties of equality and order relations are **conservation** and **transitivity**!



Conservation:
Understands quantity stays the same when physical space is changed

Transitivity



Transitivity:
Indirect comparison of 2 objects by the use of a third object

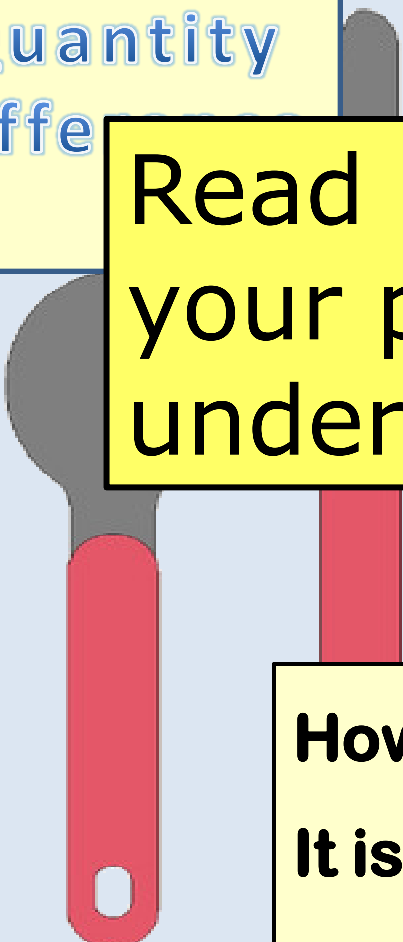
Connections:
Looking Back and Ahead in Learning

Early Algebraic Thinking: **Transitive Principle -** relates to what students experienced using the stick measuring the two different holes.

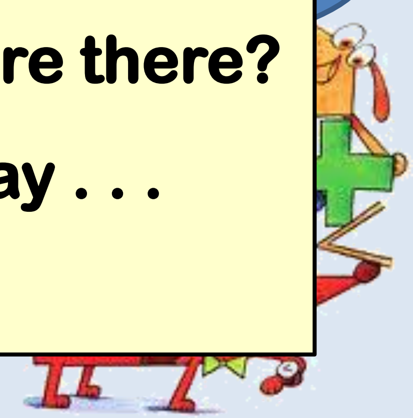
For any numbers a, b, c, if a = b and b = c, then a = c.

1e Composing and Decomposing Quantities pg. 9

Quantity
diffe



Read 1e on page 9. Talk to your partner about your understanding of this concept.

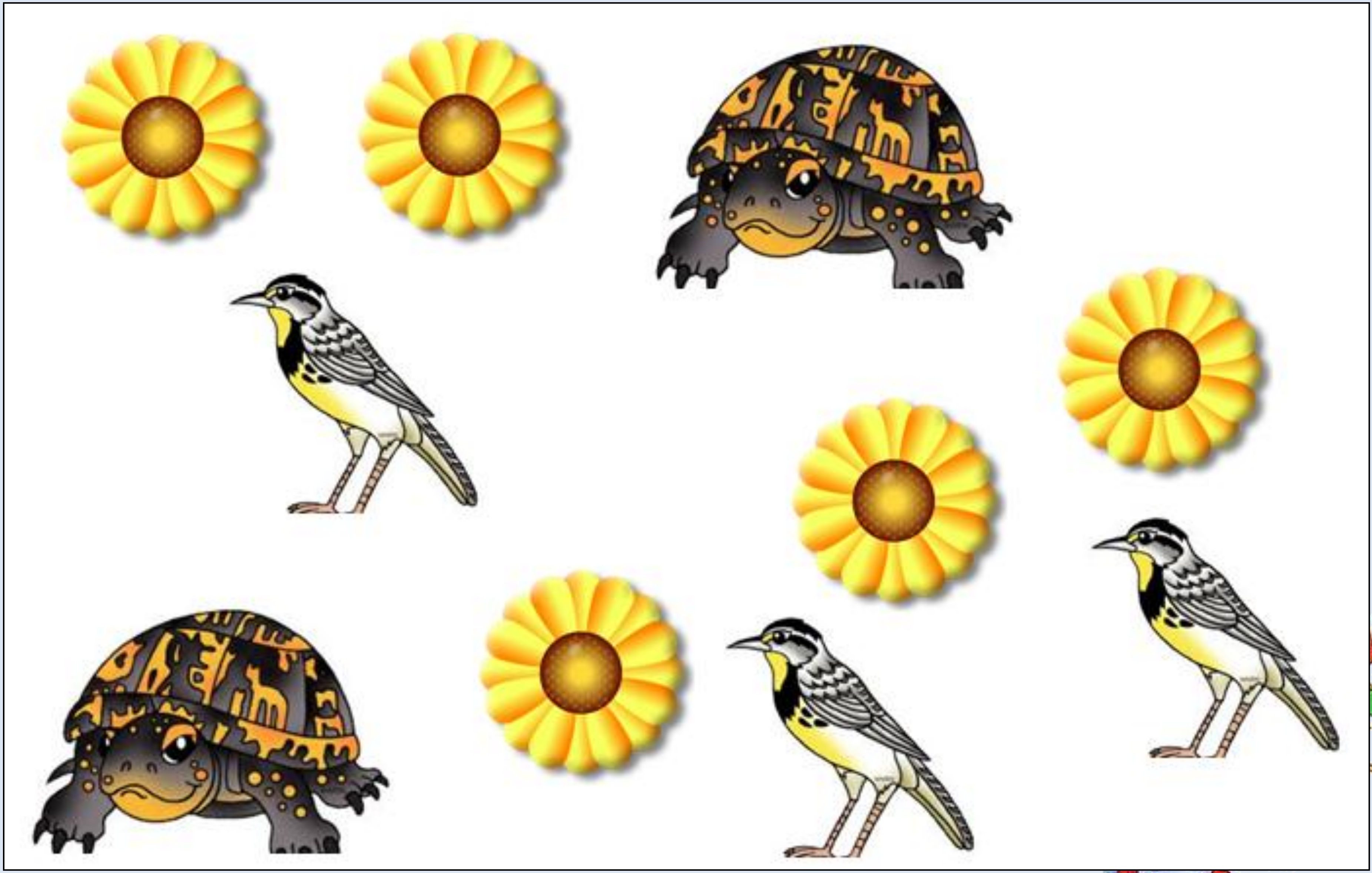


How many more birds than worms are there?

It is less confusing for students to say . . .

How many birds won’t get a worm?

Let’s Do a Number Chat!



Big Idea #2

The selection of a unit makes it possible to use numbers in comparing quantities.

2a. Using numbers to describe relationships between or among quantities depends on identifying a unit.

MARBLES are a clear unit of one

M is greater than **P**

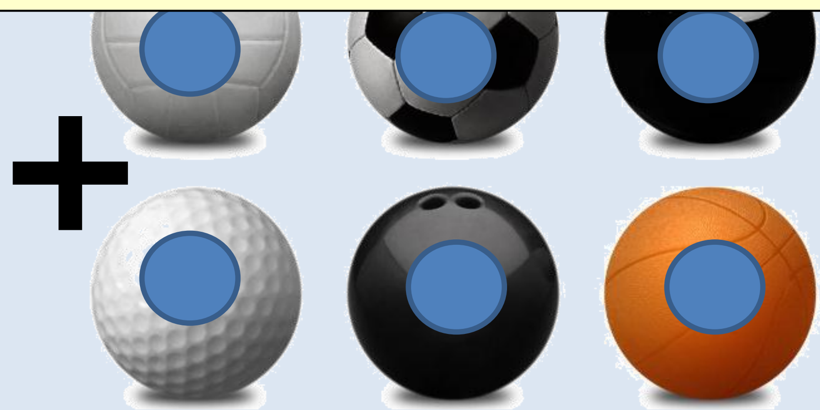
P is less than **M**

What are the units when counting this set?

Make a note on pg. 11 ~ Counting with meaning (NOT rote) involves understanding and identifying units!

You can have units that are **discrete** or **continuous**.

Using DIFFERENT units allows us to measure quantity in Different ways!!



Exactly 6

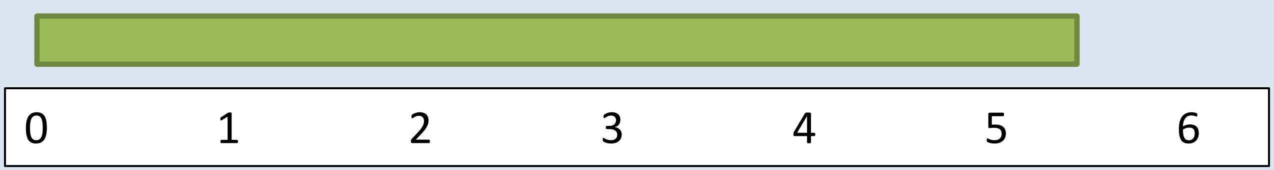
Discrete Unit:
Exact count of separate things

Continuous Unit:
Units that can be divided into smaller amounts (measurement)

Connections:
Looking Back and Ahead in Learning

Working with Rational Numbers is directly linked to these big ideas in number. Continuous quantities tie in with measurement and fractions.

Once students understand the implications of determining units and unit sizes, then they can use the number line in a more meaningful way including the development of rational number.



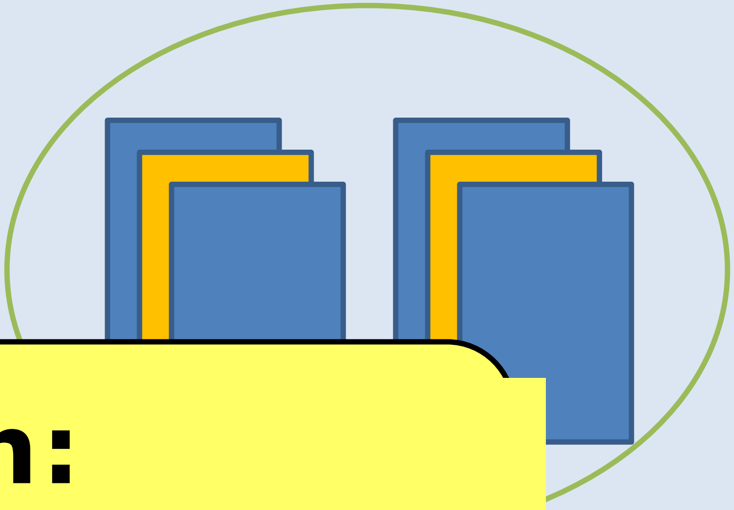
Measuring a length using ½ units (any type of unit) can be started at an early age. Students understand ½ and the “fairness” necessary when using ½.

2b. The size of a unit determines the number of times that it must be **iterated** to count or measure a quantity.

E+E+E+E+E+E = R

Iteration of unit to create new

(Co



Iteration:

Repeating the same unit

K= A + A + A or K= 3A

Unit A iterated 3 times

(Decomposing)

Number lines help establish the **magnitude** of numbers.

Magnitude:

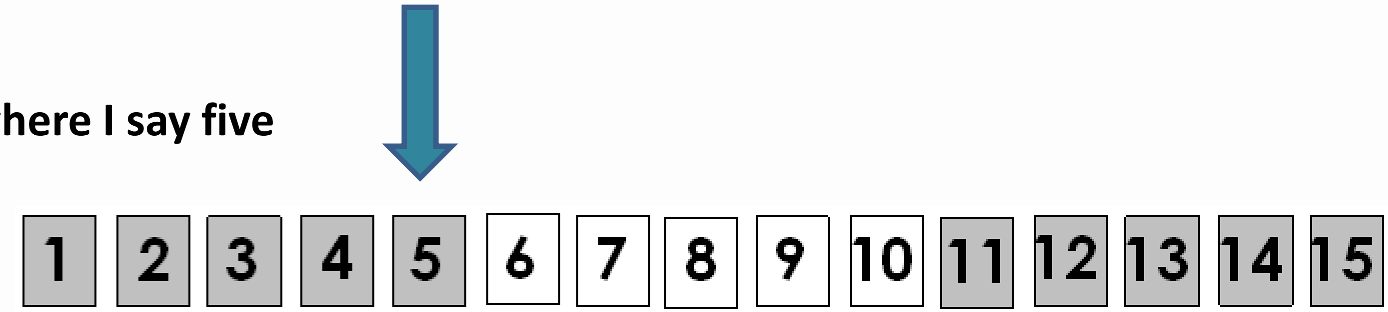
Distance of a number from zero

2 and 6 as distances from 0.

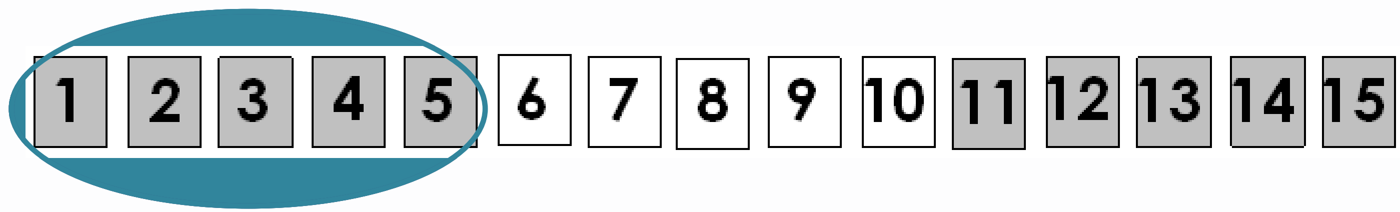
0	1	2	3	4	5	6	7	8	9	10	11	12
0	1	2	3	4	5	6	7	8	9	10	11	0
0	1	2	3	4	5	6	7	8	9	10	0	1
0	1	2	3	4	5	6	7	8	9	0	1	2
0	1	2	3	4	5	6	7	8	0	1	2	3
0	1	2	3	4	5	6	7	0	1	2	3	4
0	1	2	3	4	5	6	0	1	2	3	4	5
0	1	2	3	4	5	0	1	2	3	4	5	6
0	1	2	3	4	0	1	2	3	4	5	6	7
0	1	2	3	0	1	2	3	4	5	6	7	8
0	1	2	0	1	2	3	4	5	6	7	8	9
0	1	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10	11
0	1	2	3	4	5	6	7	8	9	10	11	12

Number Path

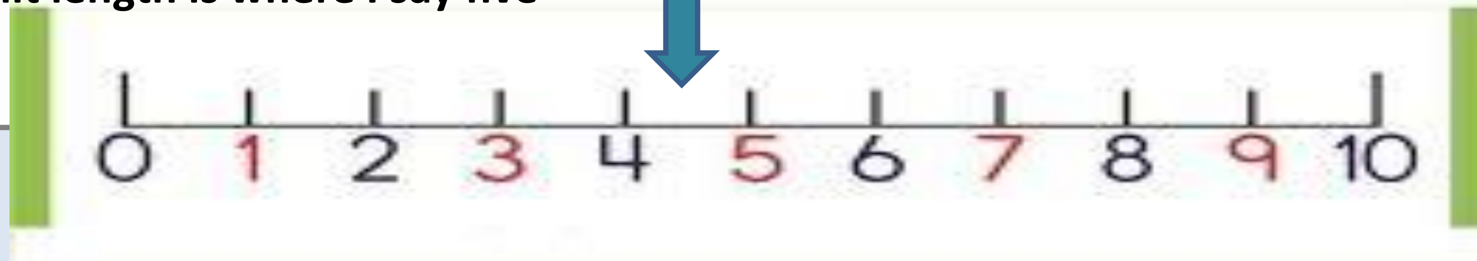
This square is where I say five



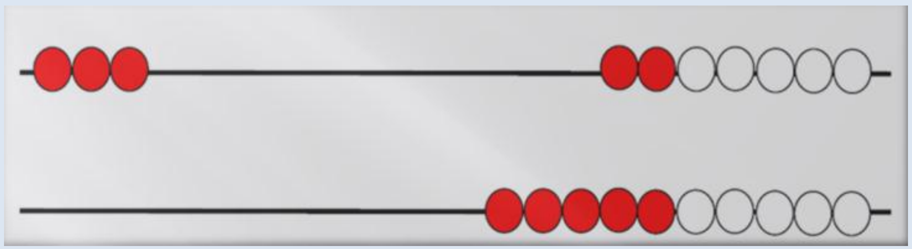
These are five squares (cardinality)

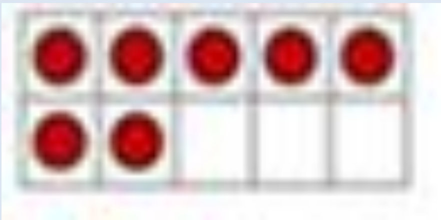


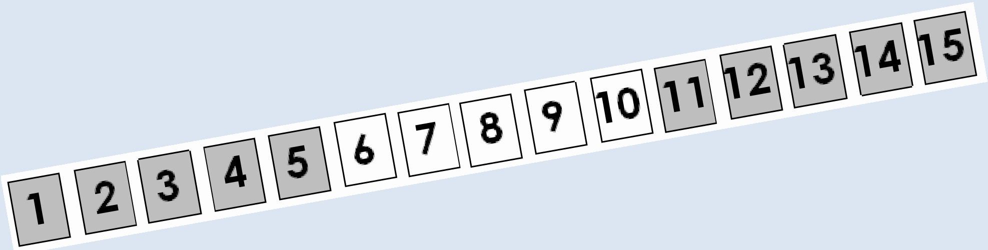
Misconception:
This unit length is where I say five



2c Quantities represented by numbers can be decomposed /composed into part-whole relationships.







Composing & Decomposing Numbers

Part – Part- Whole

Find the partners. Then discuss patterns you see. [Answers are filled in]

A. Write the partner equation.
[Write the partner expression in an equation.]

6 = 5 + 1

6 = 4 + 2

6 = 3 + 3

6 = 2 + 4

6 = 1 + 5

B. Write equations for the partner switches.

6 = 5 + 1

6 = 1 + 5

6 = 3 + 3

6 = 4 + 2

6 = 2 + 4

6= 4 + 2

Compensation

Compensation:
Understanding that decreasing from one part and increasing it to another leaves the quantity unchanged

A ten-frame representation

Connections:
Looking Back and Ahead in Learning

The context of the problem, rather than the considerations of efficiency, often suggests to children the methods that they use to solve it.

Example:
Jon had 42 pieces of gum. Kyle had 31 pieces. How many more pieces does Jon have than Kyle?

42 – 31 = ?   OR
31 + ? = 42
The second problem could be solved using decomposition. 42 decomposed into 31 for one part and then ? for the next part. Or adding onto 31 by skip counting – 10 and then 1 more.

Big Idea #3

Meaningful counting integrates different aspects of number and sets, such as sequence, order, one-to-one correspondence, ordinality and cardinality.

3a. Number-word sequence, combined with the order inherent in the natural numbers, can be used as foundation for counting.

If a child can correctly say the first five counting numbers,

“one, two, three, four, five,”

will the child necessarily be able to determine how many blocks there are in this collection?

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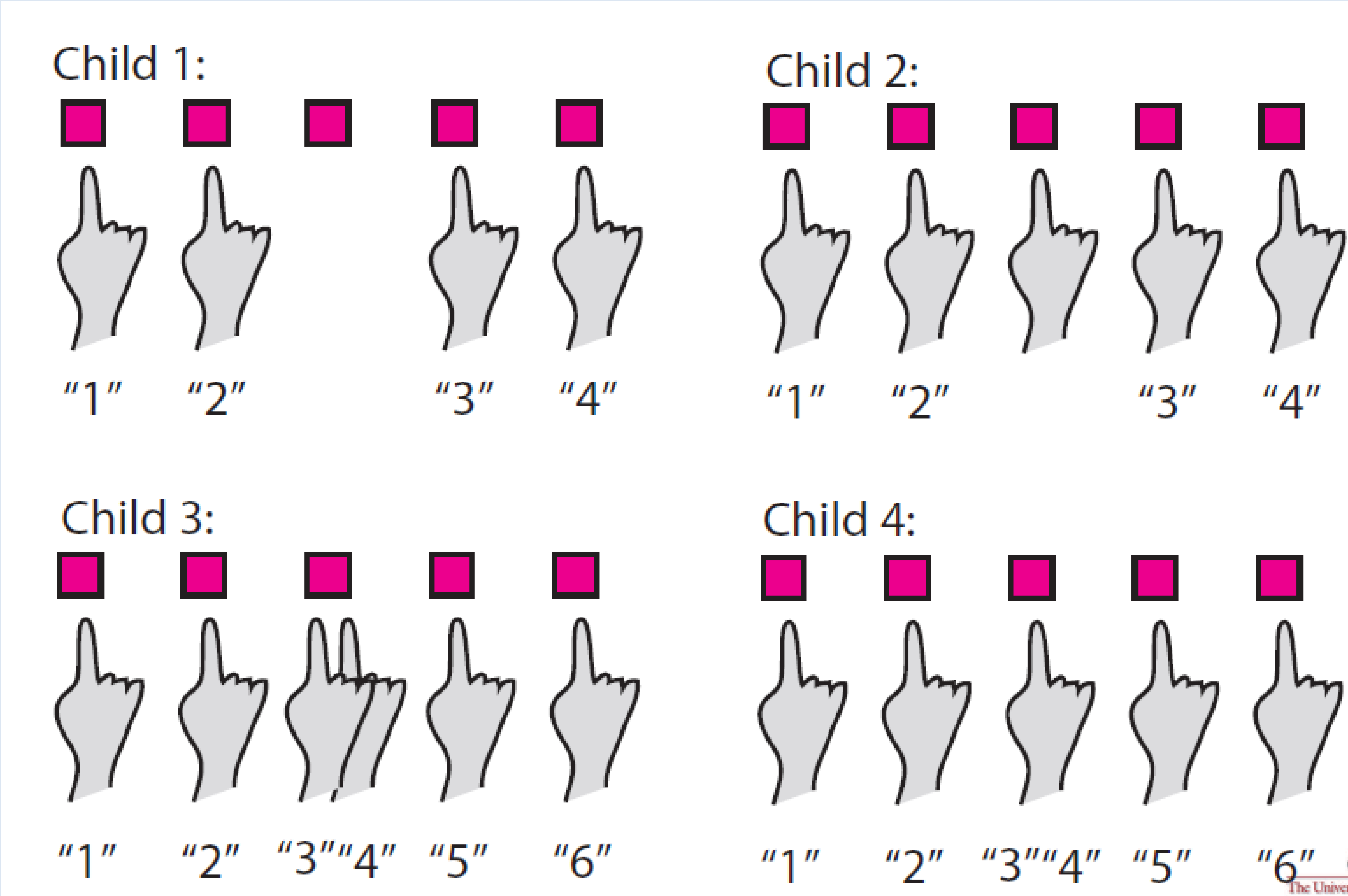

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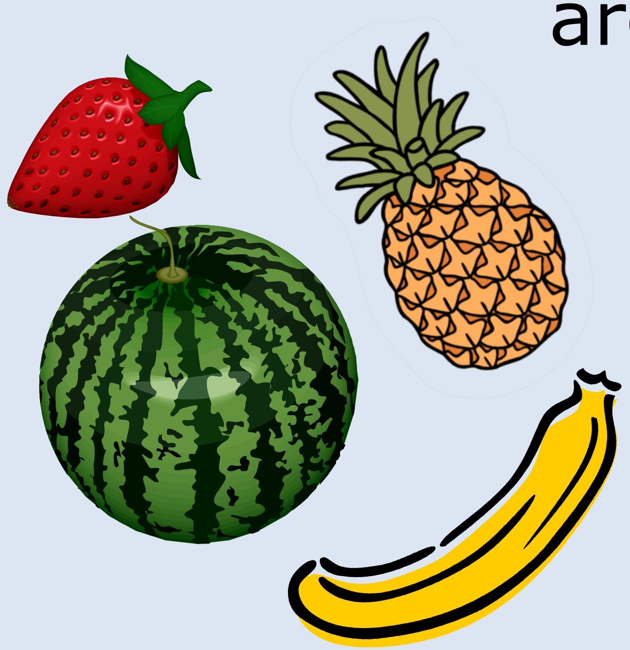
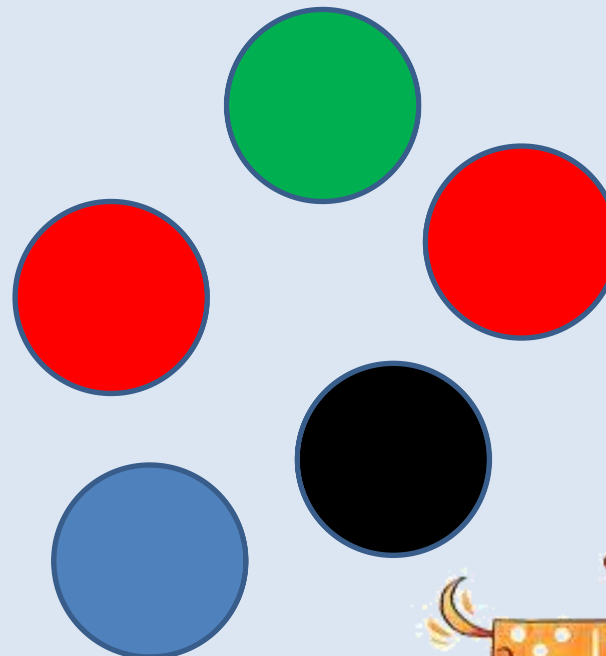
3b. Counting includes one-to-one correspondence, regardless of the kind of objects in the set and the order in which they are counted.

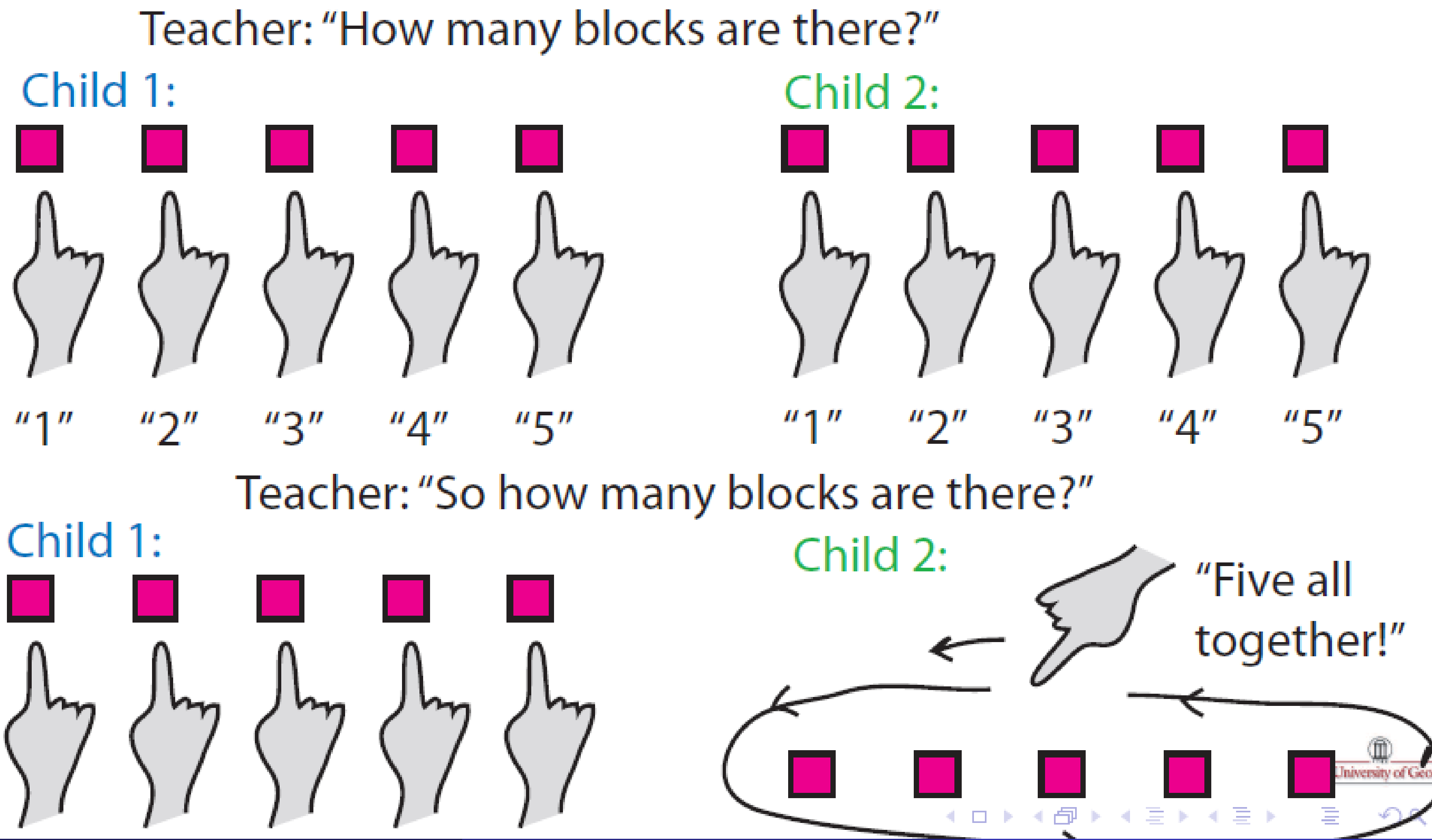




As long as we maintain a 1 to 1 matching, we can count the objects in the collection in a different order.

Counting includes cardinality and ordinality of sets of objects.

Cardinality and Ordinality



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Cardinality:

Understands last number word said when counting tells “how many”

Ordinality:

A number indicating a series or specific order (1^{st}, 2^{nd}, 3^{rd})

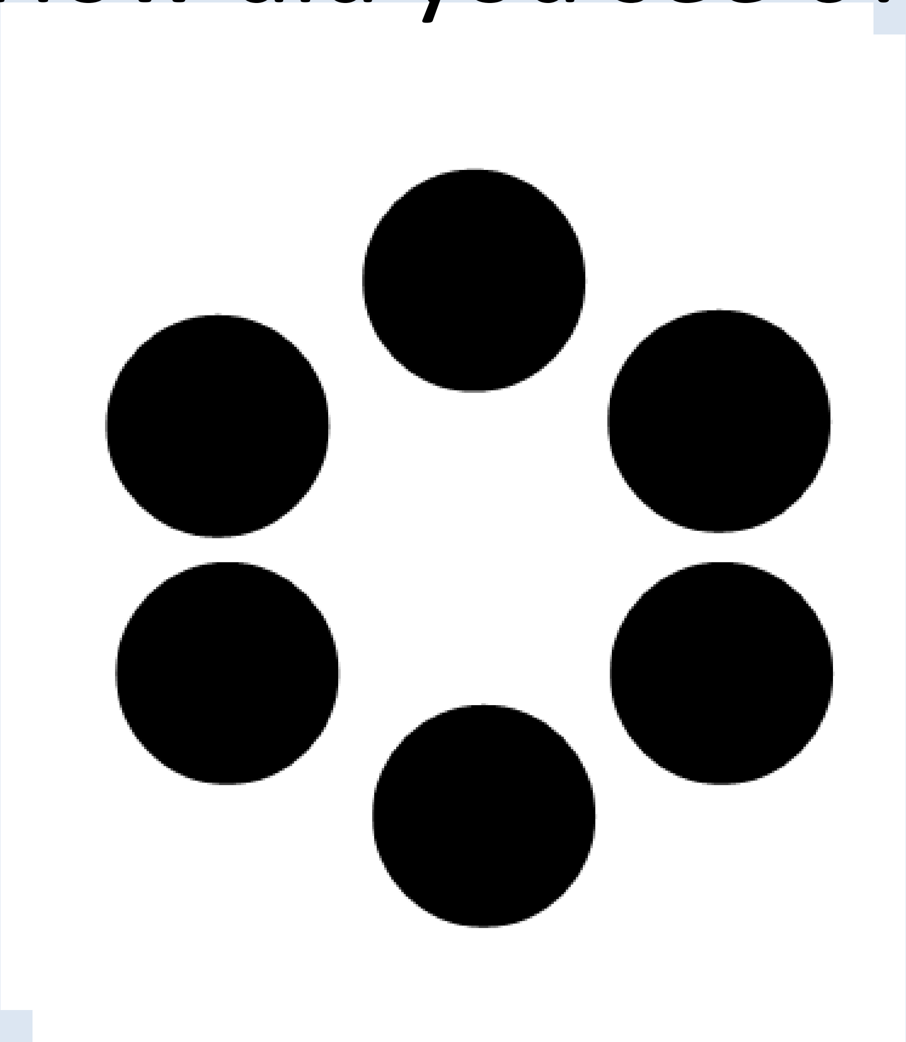
Subitizing

Subitizing is "instantly

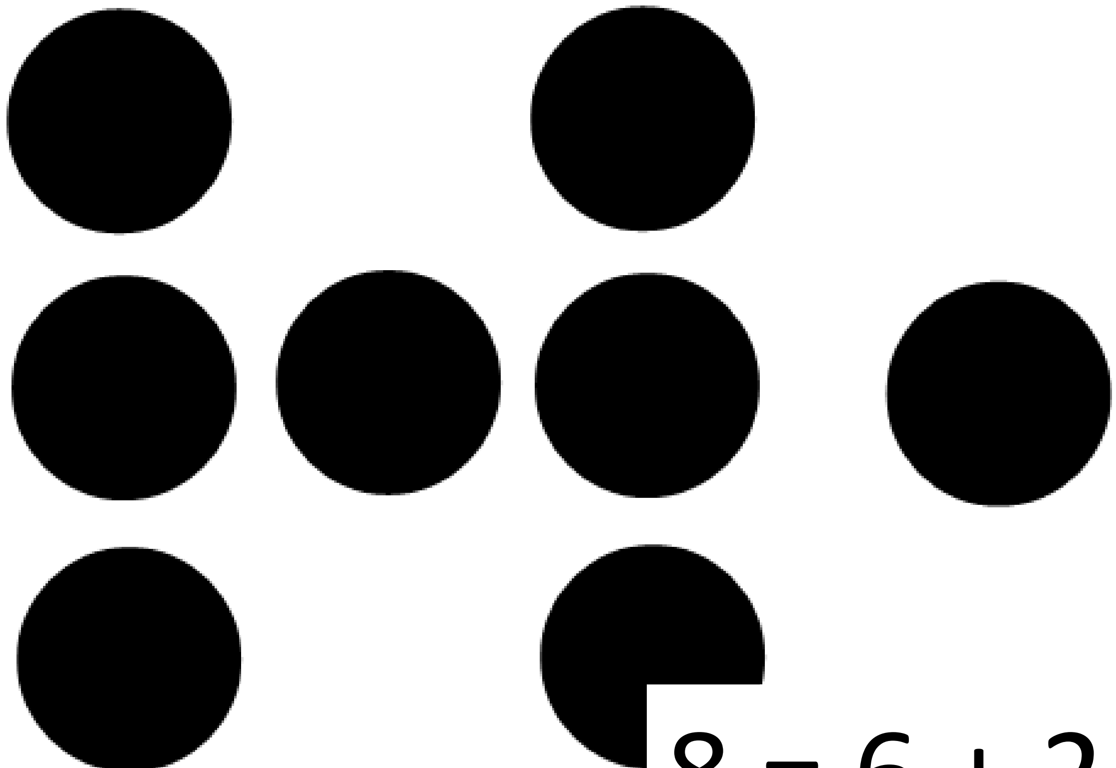
Let’s try it!

numerosity of a group.

How did you see 6?

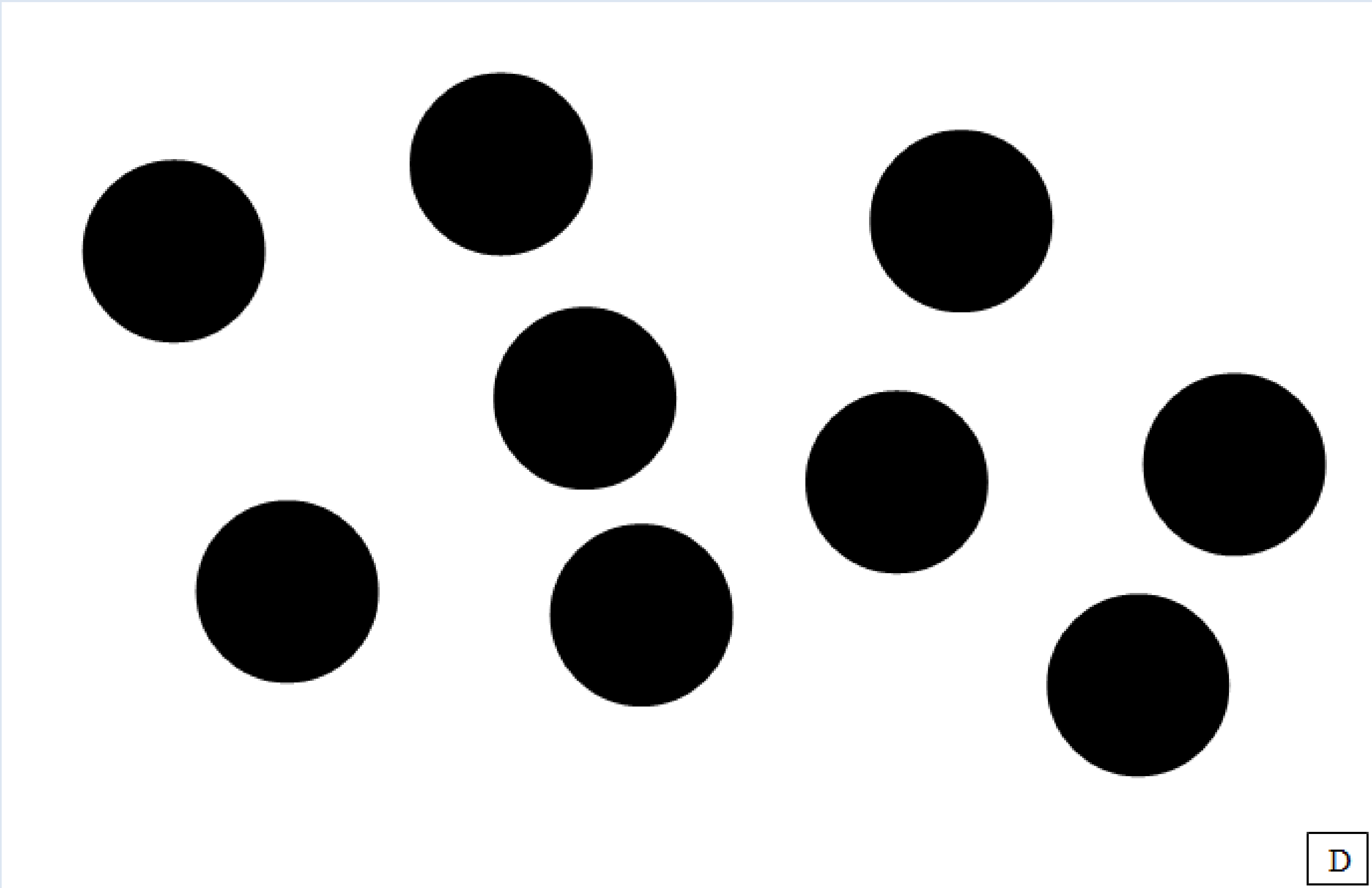


$8 = 6 + 1 + 1$



$8 = 6 + 2$

$8 = 3 + 3 + 2$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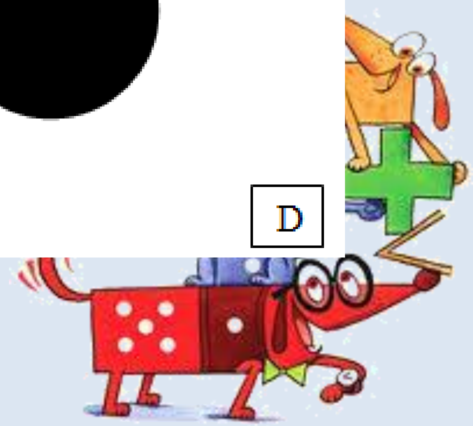


Two Types of Subitizing

Perceptual

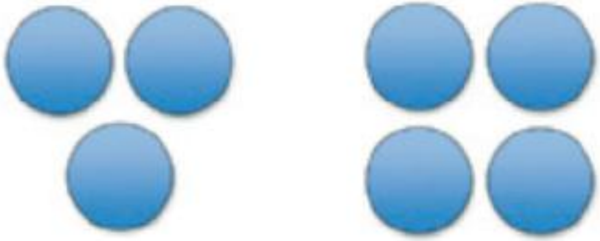
Conceptual

How does this impact future mathematics?

Subitizing ~ Connections to Arithmetic

Games and practice with subitizing develop students' thinking about number so they may enter the world that sees number as composed of other numbers. We read 7, for example, in stages: ones world, subitizing, and comprehending number at the symbolic level.

Presenting 7 as a collection of objects strung out one by one is appropriate initially while students are working on correspondence and counting skills.

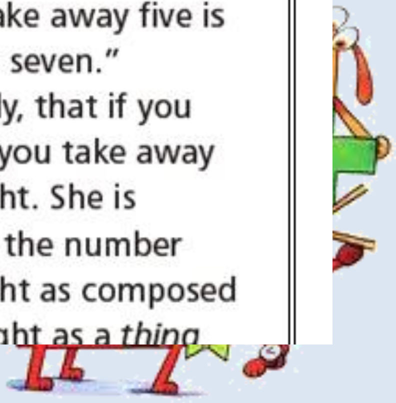


This example of a student's work confounds number symbols with concrete objects:

8 – 5 = 8   7 – 4 = 7

Student explanation: "Eight take away five is eight. Seven take away four is seven."

This student means, literally, that if you have an eight and a four and you take away the four, you still have the eight. She is applying concrete qualities to the number symbols. She does not see eight as composed of other numbers; she sees eight as a *thing*

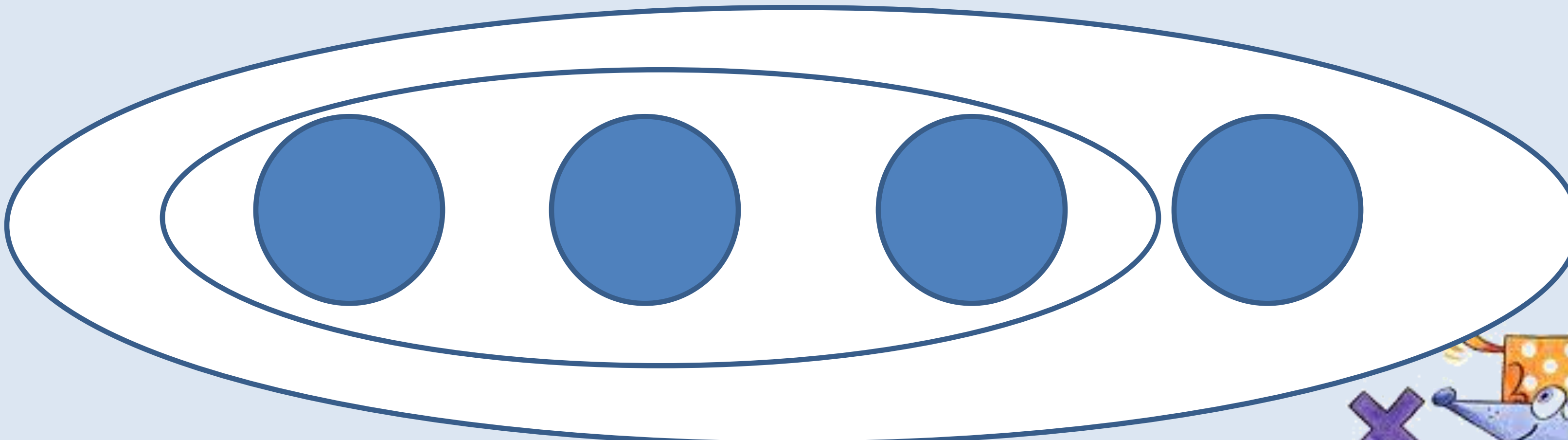


Subitizing:

Quickly recognizes and names how many without counting

3d Counting strategies are based on order and **Hierarchical Inclusion** of numbers.

- A quantity of 3 is part of a quantity of 4.
- A set is made up of subsets.



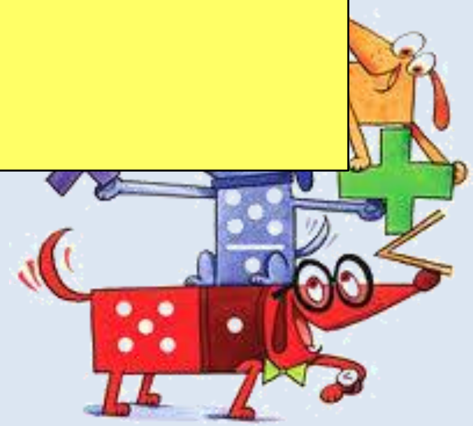
Inclusion:

Understands quantities of previous numbers make up last number named

(3 is “nested” in 4)

or

(3 is included in 4)



Abstract Understanding of Numbers

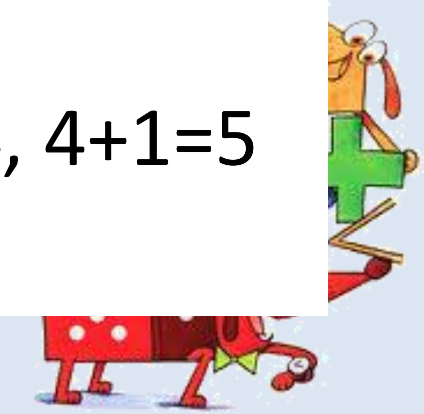
Counting Up

Counting Down

Sequencing

Counting On

2+1=3, 3+1=4, 4+1=5



Setting up the Counting On strategy

Hide them

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How many bugs are there altogether?

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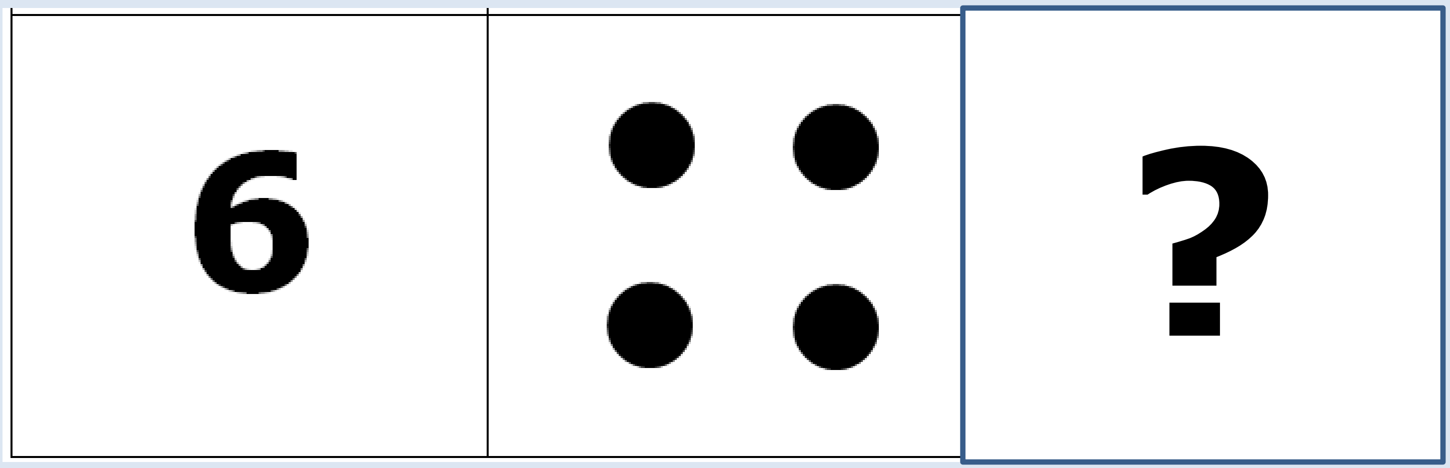
Missing Part cards



Turn to your neighbor and explain one of the Big Ideas that gave you something to think about and to try something different in your classroom.

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Big Idea #4

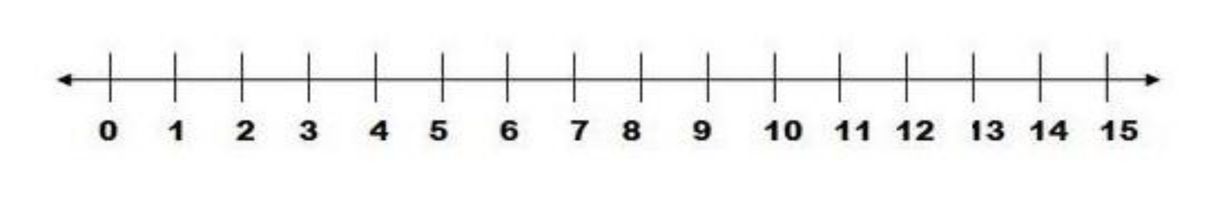
Numbers are abstract concepts.

Patterns in the number-word sequence play an important role in the development of an abstract concept of number.

What about the "teen" numbers? What happens when kids get to those numbers?

Teen Numbers & Crossing the Decade game

Different Tools



Hundreds Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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Number Sequence is INFINITE

If we can count or write up to a certain number, we can write the next one.

There is always one more!

Number Symbols are representations of abstract mental objects!!!

or 7 oranges

Are these the same thing?

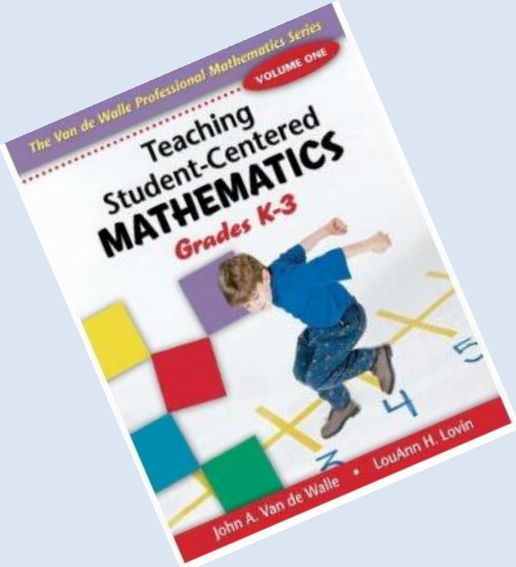
CAT

Big Idea #5

Our base-ten positional number system is a very efficient way to represent numbers in writing.

Our number system uses only 10 different digits. They are recycled to make an infinite series of numbers.

Why would it be important to use the number symbols as soon as possible?



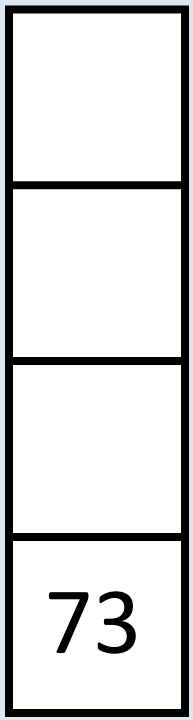
Activity 2.30

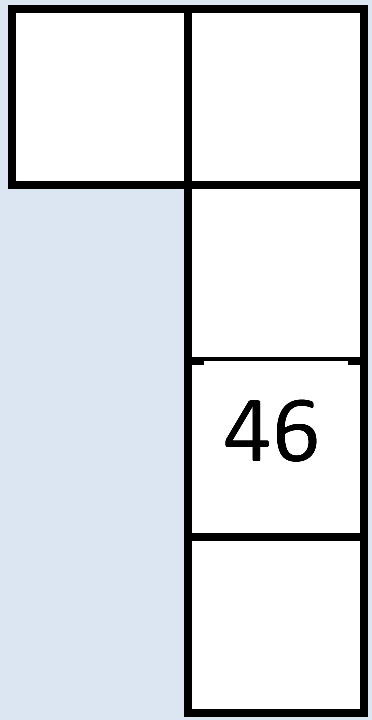
Missing Numbers, p. 58

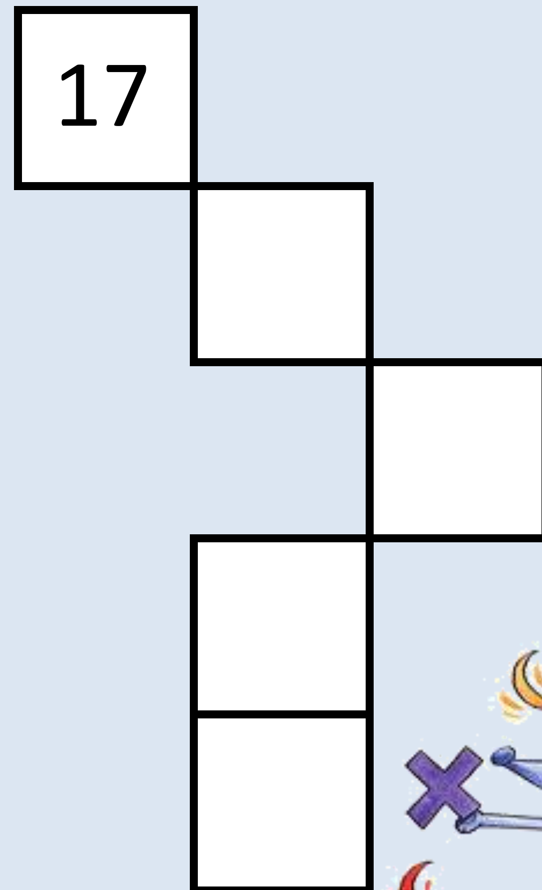
	202		204		206		208		210
221		223		225		227		229	
291				295					300

Number Puzzles

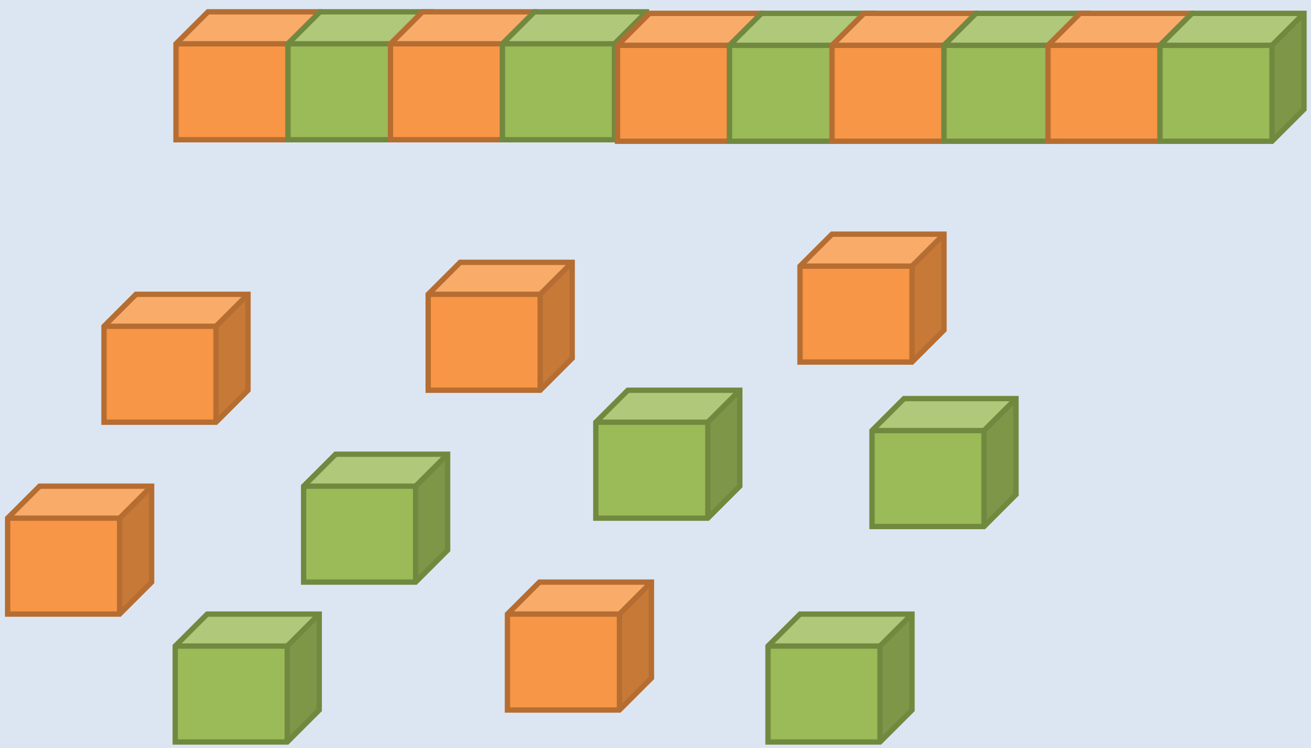
Write the missing numbers. Use after work with hundred grids.







Grouping ten is important. But ungrouping ten is equally as important. (Composing and decomposing number)



The value of a digit in a written numeral depends on its place, or position, in a number. So units are of different sizes.

We can count like units, or groups. So 10 tens is the same as 100 ones.

Turn to your neighbor and explain one of the Big Ideas that gave you something to think about and to try something different in your classroom.