The Mathematics of Angry Birds

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The Main Objectives

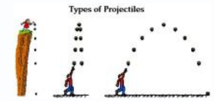
- Use data from the flight of an angry bird to develop models for the motion
- Explore the major variables of angle and initial velocity
- Explore the parametric relation

$x = (v_0\cos\theta)t + x_0$

$y = -0.5gt^2 + (v_0\sin\theta)t + y_0$

Begin with Simpler Examples

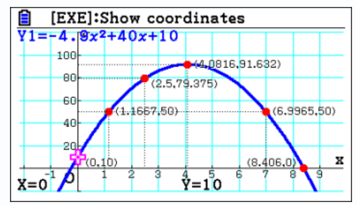
- Write a model for height as a function of time when an object is dropped from a height of 90 meters.
- Compute height for a specific time and time for a specific height
- Compute the time when the object strikes the ground



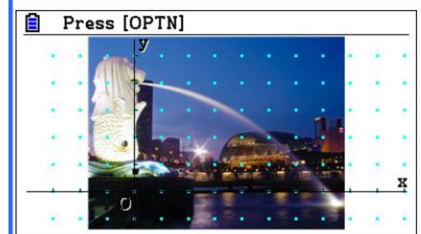
Example 2

- Write a model for height as a function of time when an object is thrown upwards with an initial velocity of 40 m/s from an height of 10 meters.
- Ask multiple questions
- Explore the effect of the initial velocity

Write a model for height as a function of time when an object is thrown upwards with an initial velocity of 40 m/s from an height of 10 meters.



A Parametric Model



Using Vertical and Horizontal Components

- The initial point is close to (0,4)
- The drop hits the surface at about (7,0)

$y=4 - 16t^2$

When $y=0$, $0=4 - 16t^2$

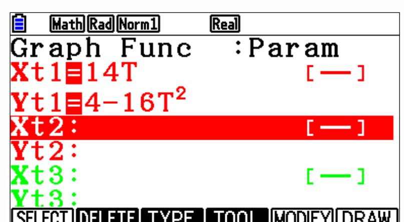
$t=1/2$

$x=vt$

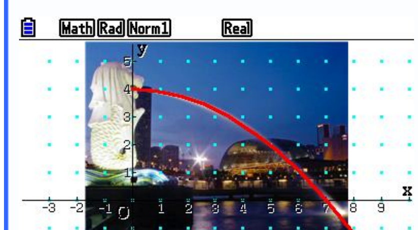
$7=v(1/2)$

$v=14$

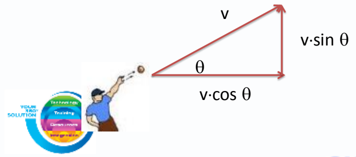
A Parametric Model



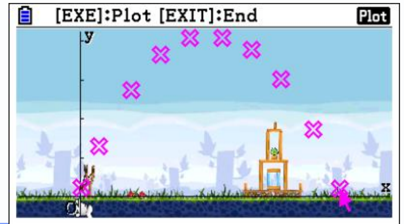
A Parametric Model



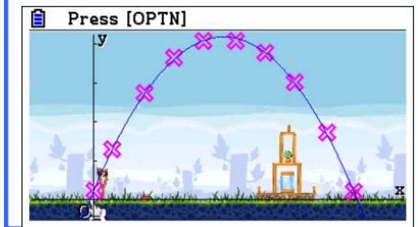
What if the initial velocity is not in the vertical or horizontal direction, but at an angle, θ?



A Demonstration of Data Collection



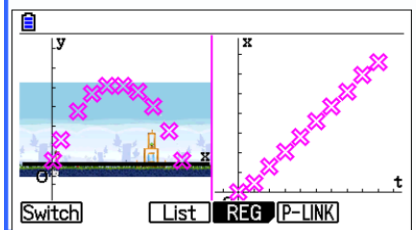
Create a Model by Regression



More About These Coefficients Later!

QuadReg
a =-0.0145123
b =2.39544602
c =5.98181537
r^2 =0.99938941
MSe=1.1348274
$y=ax^2+bx+c$
COPY DRAW

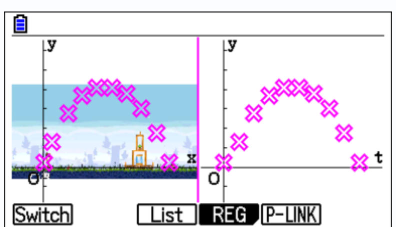
Let's Explore Horizontal Position vs. Time



Interpret Both Coefficients of the Model

LinearReg(a+bx)
a =-4.4002886
b =18.866378
r =0.99933381
r^2 =0.99866807
MSe=4.89551753
$x=a+bt$
COPY DRAW

Let's Explore Vertical Position vs. Time



Interpret the Coefficients of this Model

QuadReg
a =-4.9358911
b =45.1151026
c =-0.2461388
r^2 =0.99177329
MSe=15.2901893
$y=at^2+bt+c$
COPY DRAW

Let's Explore the Angle and Initial Velocity

$v_0 sin(t) = 45.115$

$v_0 cos(t) = 18.866$

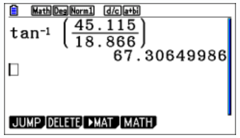
- There are several ways to compute the values

Method 1 - Division

$v_0 sin(t) = 45.115$

$v_0 cos(t) = 18.866$

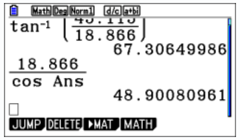
$tan(t) = 45.115/18.866$



Method 2 - Substitution

$v_0 cos(t) = 18.866$

$v_0 = 18.866 / cos(t)$

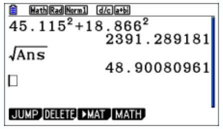


Method 3 – Trig Identity

$[v_0 sin(t)]^2 + [v_0 cos(t)]^2 = v_0^2$

$v_0 sin(t) = 45.115$

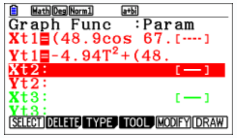
$v_0 cos(t) = 18.866$



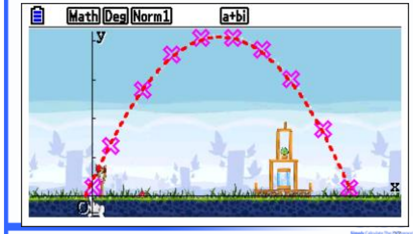
Checking the Parametric Model

$X = 48.90 \cos(67.31)T – 4.40$

$Y = -4.94T^2 + 48.90 \sin(67.31)T – 0.25$



Checking the Parametric Model



Revisiting the (x, y) Model

$$x = (v_0 \cos\theta)t$$

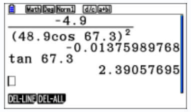
$$\frac{x}{v_0 \cos\theta} = t$$

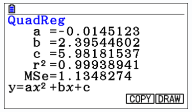
$$y = \frac{-1}{2}gt^2 + (v_0 \sin\theta)t + y_0$$

$$y = \frac{-1}{2}g\left(\frac{x}{v_0 \cos\theta}\right)^2 + (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) + y_0$$

$$y = \frac{-g}{2(v_0 \cos\theta)^2}x^2 + (\tan\theta)x + y_0$$

Revisiting the (x, y) Model





Explorations

Use the graphs of (t,x), (t,y), and (x,y) to compute the results:

Q1 What is the bird's position at time t=2.5 seconds?

Q2 How long is the bird in flight?

Q3 What is the time when the bird is at maximum height?

Q4 What is the maximum height?

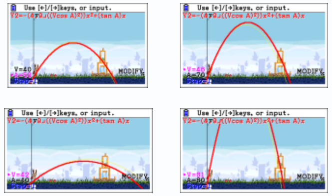
Q5 At what time(s) is the bird at height 60 meters?

Q6 How far did the bird fly horizontally?

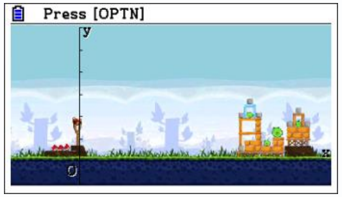
Q7 What is the height when the horizontal position is 150 meters?

Q8 What is the horizontal position when the height is 60 meters?

Extension – find combinations of velocity and angle that will hit the pig



Extension – find the unique combination of velocity and angle that pass through three selected points

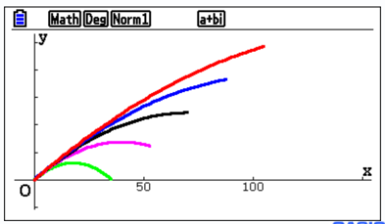


Extension – find the unique combination of velocity and angle that pass through three selected points

One more extension

- Suppose several birds are fired at the same angle, say 60°, but at different velocities {20,30,40,50,60}, and stop at time t =3.5.
- x = ({20,30,40,50,60} cos(60))t
  y = -4.9t² + ({20,30,40,50,60} sin(60))t

Birds are fired at the same angle, 60°, but at different velocities, and stop at time t =3.5.



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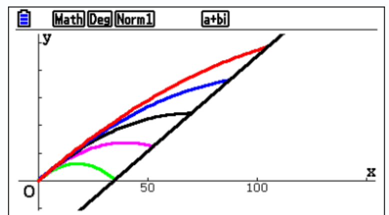

Solution

- Suppose several birds are fired at the same angle, say 60°, but at different velocities {20,30,40,50,60}, and stop at time t =3.5.
- The birds (points) are collinear with slope tan (60°) and y-intercept -4.9(3.5²).

Your Challenge

- Suppose several birds are fired at the same velocity, say 40 m/s, but at different angles and stop at time t =2.5.
- The birds (points) are not collinear , but there is a relation.
- Here's a hint