

Productive Struggle to Grow Stronger Mathematics Students in Grades K-12



April 10th, 2014

*National Council of Teachers Mathematics
Annual Meeting - New Orleans, LA*

#103 9:45-11:00am Grand Salon 4-7-10 (Hilton)

www.debbiewaggoner.com

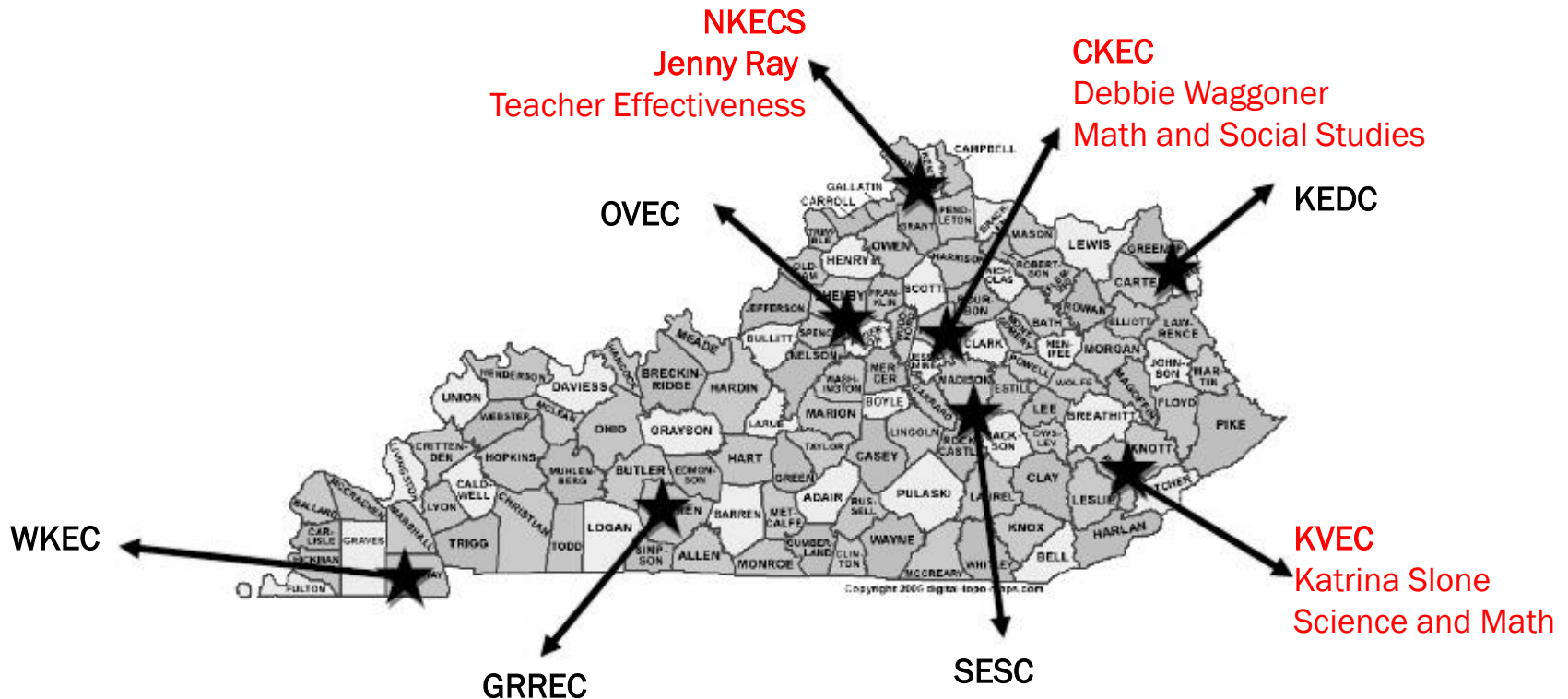
www.jennyray.net

Who are we?



**Kentucky Department of Education
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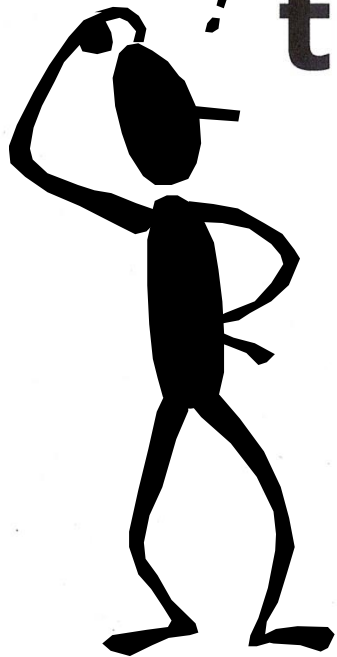
REAL



- Think of a time when you learned something outside of school.
- Why and how did you learn it?
- Was there a struggle?
- Do you still remember it?

Productive Struggle

"I have not failed. I've just found 10,000 ways that won't work."



Thomas A. Edison



Don't Prevent Students' Mistakes, Prepare for Them

*David Ginsberg
ASCD Smart Brief
January 2, 2012*

http://blogs.edweek.org/teachers/coach_gs_teaching_tips/2012/01/dont_prevent_students_mistakes_prepare_for_them.html



Today's Learning Targets



- o I can experience productive struggle and explain its purpose and benefits.
- o I can explain the purpose of Formative Assessment Lessons (FALs).
- o I can describe the process of implementing a FAL.
- o I can access and use the materials to plan and perform a FAL.

Formative Assessment???

Exit Slips

entrance slips

bell ringers

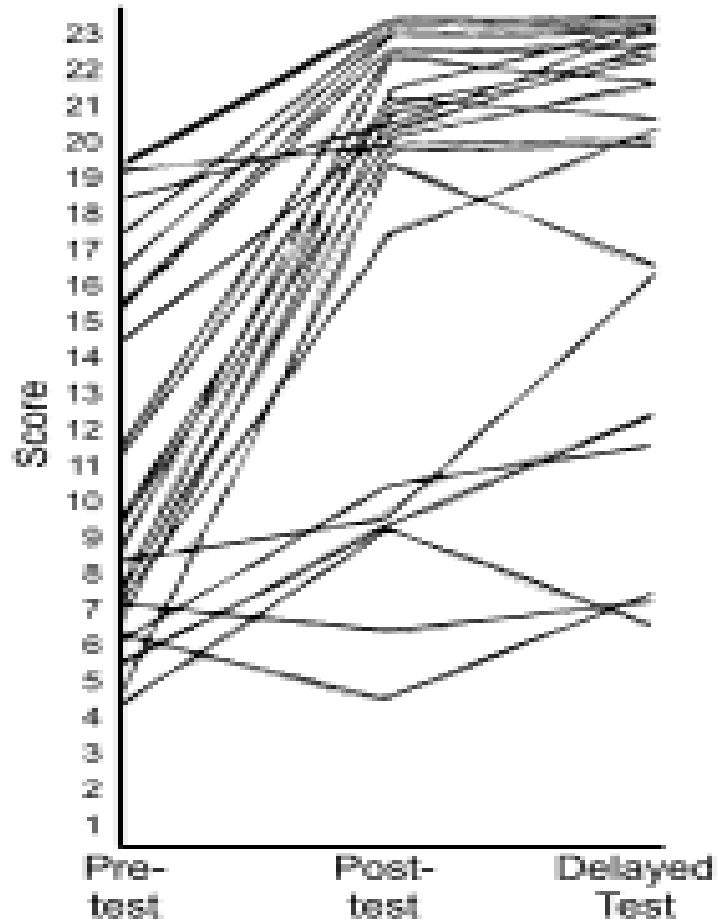
THUMBS UP/DOWN

Clickers

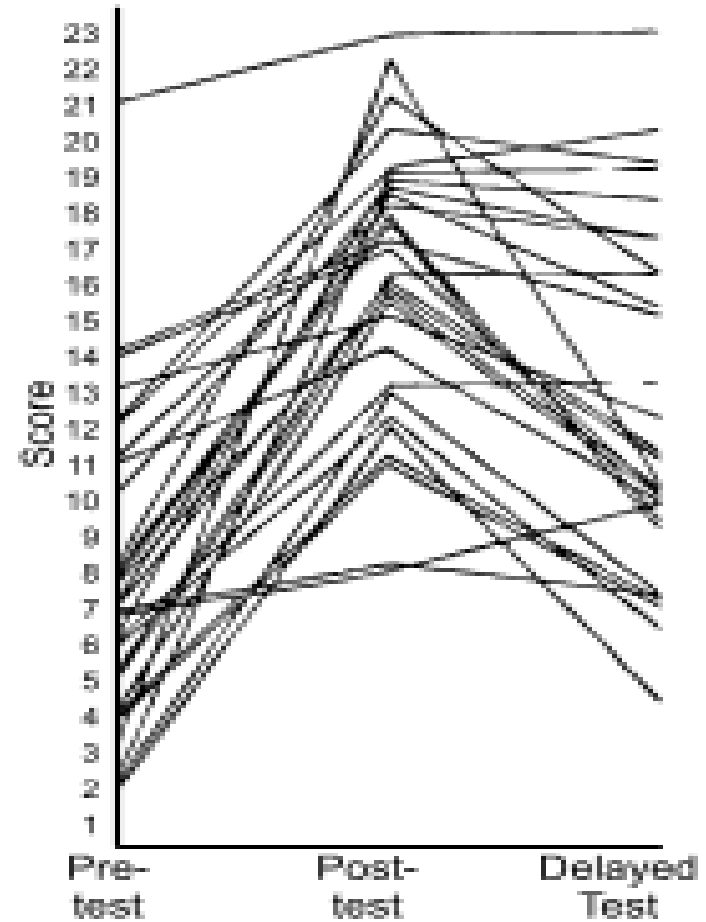
Common Assessments

WHITE BOARDS

Why Now?



Formative assessment techniques



Direct instruction

vs.

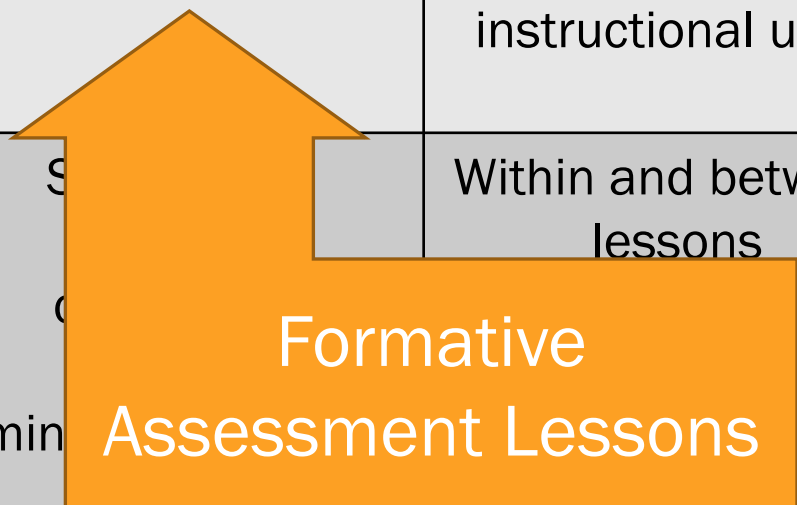


Five “Key Strategies” for Effective Formative Assessment

1. Clarifying, sharing, and understanding goals for learning and criteria for success with learners
2. Engineering effective classroom discussions, questions, activities, and tasks that elicit evidence of students’ learning.
3. Providing feedback that moves learning forward.
4. Activating students as owners of their own learning.
5. Activating students as learning resources for one another.

Typology of Kinds of Formative Assessment

Type	Focus	Length
Long-cycle	Across marking periods, quarters, semesters, years	4 weeks to 1 year
Medium-cycle	Within and between instructional units	1 to 4 weeks
Short-cycle	Within and between lessons	24-48 hours
Minute		5 seconds to 2 hours





A Snail in the Well

A snail is at the bottom of a well that is 10 feet deep. Each day he crawls up 3 feet and each night he slides back 2 feet. How many days will it take him to reach the top of the well? Show your work to defend your answer.

What's a Problem For?

- o What would students need to know to get started on this problem?
 - o Background knowledge, choose carefully based on where my students are...ZONE a little above
- o Is there more than one way to approach the problem?
 - o Make sure the task is OPEN and has multiple possible solution strategies...
- o How do I introduce the problem?
 - o Frame the task, relating to what we are learning...
- o What should I do after introducing the problem?
 - o Stay quiet as much as possible, answer questions with questions only to move the learning forward...



Sample Responses to Discuss

Here is some work on *A Snail in the Well* from students in another class.

For each piece of work:

1. Write the name of the student whose solution you are analyzing.
2. Describe the problem solving approach the student used.

For example, you might:

- Describe the way the student has organized the data.
 - Describe what the student did to calculate the day the snail reaches the top of the well.
3. Explain what the student needs to do to complete or correct his or her solution.

_____’s Solution

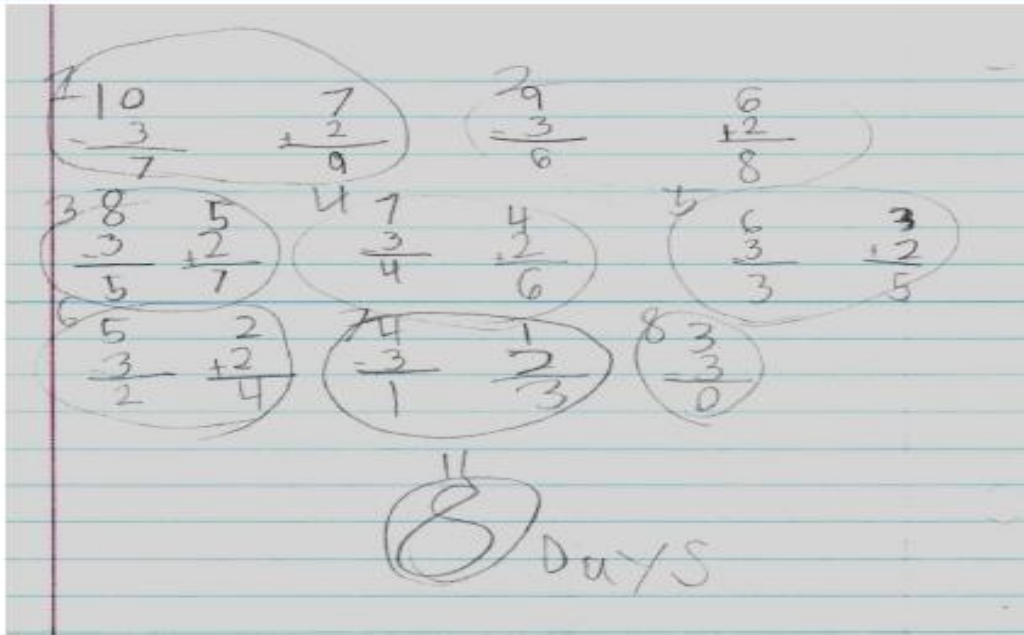
Will's Solution

Snail = 1 ft. a day, I know this because $3\text{ft.} - 2\text{ft.} = 1\text{ft.}$
If it takes 10 ft. to climb to climb
out of the well, then it takes
10 days.

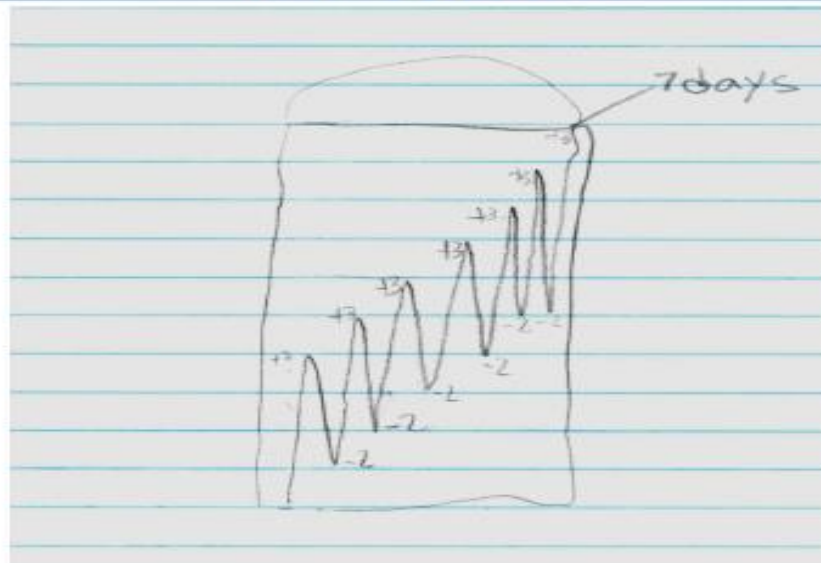
Whitney's Solution

Day One	+1
Day Two	+1
Day Three	+1
Day Four	+1
Day Five	+1
Day Six	+1
Day Seven	+1
Day Eight	+1
Day Nine	+1
Day Ten	+1
	10

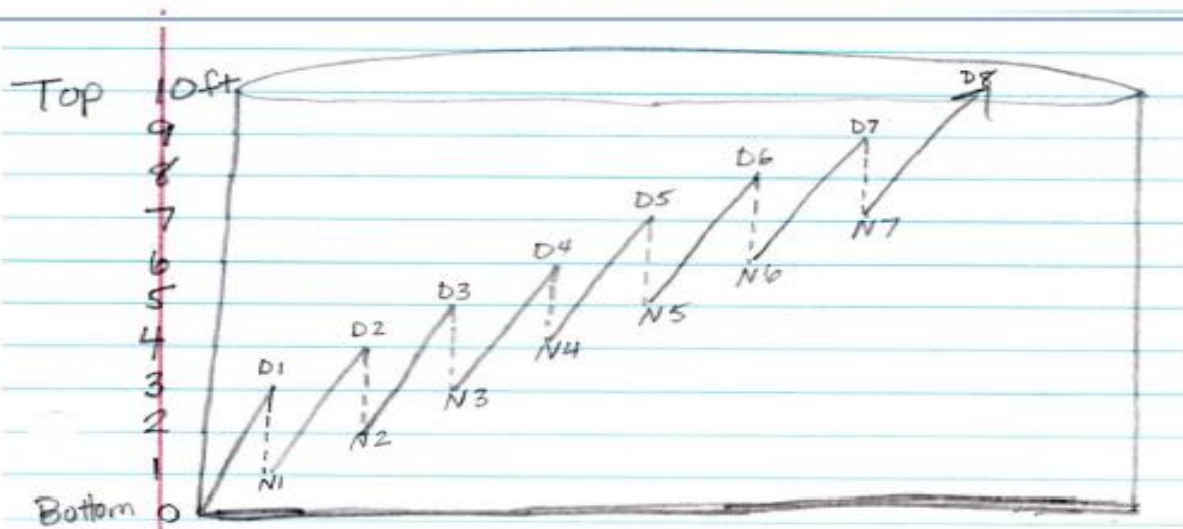
Chuck's Solution



Tim's Solution



Denise's Solution



Bill's Solution

"Well 10' deep

up - 3, ngn + down 2 foot

day	d	N	Total
1	7	9	9
2	6	8	8
3	5	7	7
4	4	6	6
5	3	5	5
6	2	4	4
7	1	3	3
<u>8</u>	<u>0</u>		
9			

Collaborative work

- (1) Share your method with your partner(s) and your ideas for improving your individual solution.
- (2) Together in your group, agree on the best method for completing the problem.
- (3) Produce a poster, showing a joint solution to the problem.
- (4) Make sure that everyone in the group can explain the reasons for your chosen method, and describe any assumptions you have made.
- (5) Check your work.

How Did You Work?

Post-Task Reflection: A Snail in a Well

Tick the boxes and complete the sentences that apply to your work.

1.) Check one, then complete the sentence below:

Our group work was better than my own individual work.

OR

My own individual work is better than our group work.

I prefer (*circle one*) **our method** / **my method** because:

2.) Check one, then complete the sentence below:

Our method is similar to: _____
(*add name of sample response*)

OR

Our method is different from **all** of the sample responses.

I prefer (*circle one*) **our method** / **the sample response** method because:

3.) Check one and complete the sentence:

We checked our method by: _____

OR

We could check our method by: _____

This problem solving Formative Assessment Lesson is designed to be part of an instructional unit. The results of this task should be used to inform the instruction that will take place for the remainder of the unit.

Mathematical goals

This problem solving lesson is intended to help you assess how well students are able to use addition and subtraction in a problem solving situation. In particular, this lesson aims to identify and help students who have difficulties with:

- Choosing an appropriate, systematic way to collect and organize data.
- Examining the data and looking for patterns
- Describing and explaining findings clearly and effectively.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

1. Make sense of problems and persevere in solving them.
4. Model with mathematics.
8. Look for and make use of repeated reasoning.

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards*:

Operations and Algebraic Thinking

- 1-OA: Represent and solve problems involving addition and subtraction.
- 2-OA: Represent and solve problems involving addition and subtraction.
- 3-OA: Solve problems involving the four operations, and identify and explain patterns in arithmetic.
- 4-OA: Use the four operations with whole numbers to solve problems.

Mathematical Practices

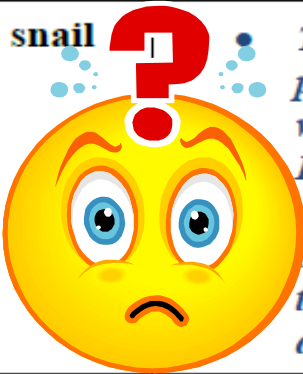
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

FAL Outline:

1. Pre-Assessment – Individual student work
← Determine FEEDBACK QUESTIONS
2. Intro Lesson
3. Collaborative Activity
← Oral FEEDBACK QUESTIONS
4. Whole Class Discussion
5. Post-Assessment
← Written FEEDBACK QUESTIONS

Formative Assessment Lesson

Common Issues – Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student forgets to consider what the snail does each day and each night.	<ul style="list-style-type: none">• <i>How could you make this task easier?</i>• <i>What sort of picture could you draw that might be helpful?</i>• <i>How can you show the path the snail follows until he gets out of the well?</i>
Student work is unsystematic.	<ul style="list-style-type: none">• <i>What pattern do you notice?</i>• <i>What is the same and what is different about how the snail moves during the day and at night?</i>• <i>How can you organize your work?</i>
Student assumes that the initial pattern continues indefinitely and over-generalizes.	<ul style="list-style-type: none">• <i>What do you think about how far the snail travels each day?</i>• <i>Does the snail always fall back?</i>
Student writes answer without explanation.	<ul style="list-style-type: none">• <i>How could you explain/show how you got your answer so that someone in another class understands?</i>• <i>How can you use numbers, words, or pictures to describe the path of the snail?</i>
Student correctly identifies when the snail gets out of the well.	 <ul style="list-style-type: none">• <i>Think of another way of solving the problem. Is this method better or worse than your original one?</i>• <i>Explain your answer.</i>• <i>Can you make a new problem with a different size well and/or a snail that travels different amounts each day and night?</i>

Beads Under the Cloud
I can generate and analyze patterns.

How many beads are hidden under the cloud?

NOTE - the cloud is not drawn to scale, and
HINT - remember planes often fly through clouds...

Name _____

0	
1	● ● ●
2	●
3	● ● ● ● ●
4	
5	● ● ●
6	
7	
8	● ● ●
9	● ●

I can identify numbers 0-9 by counting and matching.

Students	Misconceptions	What will you do next to help clear up misconceptions?
All	The size of the cloud - they thought the size of the cloud related to the # of beads under it.	Explain, more in depth, what "drawn to scale" means.
Majority	All sample responses are correct answers.	Discussion about why you give sample responses - to see strategies. Their job is to analyze the responses.
AD KF CB	MR MC JP	Some of the beads from the pattern "under" the cloud are "out" of the cloud.
		Look at ways to subtract those beads AFTER solving problem to avoid confusion

Students	Misconceptions	What will you do next to help clear up misconceptions?
Number ID	Don't know ⁶⁹ the numeral	• number structure with fingers • flashing numeral cards
Random dots	Mis-counting	Modeling organized counting
Ten Frame	Filling the ten frame	Model counting aloud when dotting

What do we really want our students to know and be able to do?

- o “And I’m calling on our nation’s governors and state education chiefs to develop standards and assessments that don’t simply measure whether students can fill in a bubble on a test, but whether they possess 21st Century skills like **problem solving and critical thinking and entrepreneurship and creativity.**”

o President Obama, 1 March 2009

Mathematics Assessment Project

- o Designed and developed well-engineered assessment tools (FALs) to support US schools in implementing the Common Core State Standards for Mathematics (CCSS).
- o Funding is provided by the Bill and Melinda Gates Foundation through the University of California, Berkeley.
- o <http://map.mathshell.org/materials/lessons.php>
- o KDE mathematics specialists are developing FALs for grades K-5.
- o www.debbiewaggoner.com
- o www.jennyray.net

Re-teaching vs. Re-engagement



Re-teaching

Teach unit again

Address missing basic skills

On the same or similar problems

Practice more

Cognition lower

Re-engagement

Revisit student thinking

Address conceptual understanding

Examine task from different perspective

Critique student approaches

Cognition higher

Two Kinds of FALs

Concept Focused

- Specific content is central to the activity
- Generally one correct answer, but may be a variety of ways to get that answer
- Usually includes a small group/pairs activity that requires manipulation of mathematical information (often in the form of card sorts, etc.)

Problem Solving Focused

- Activity draws on knowledge about a variety of content
- Sometimes a number of answers are plausible but must be defended
- Usually includes a small group/pairs activity that requires analyzing sample student work in order to look at different strategies for solving the problem at hand.

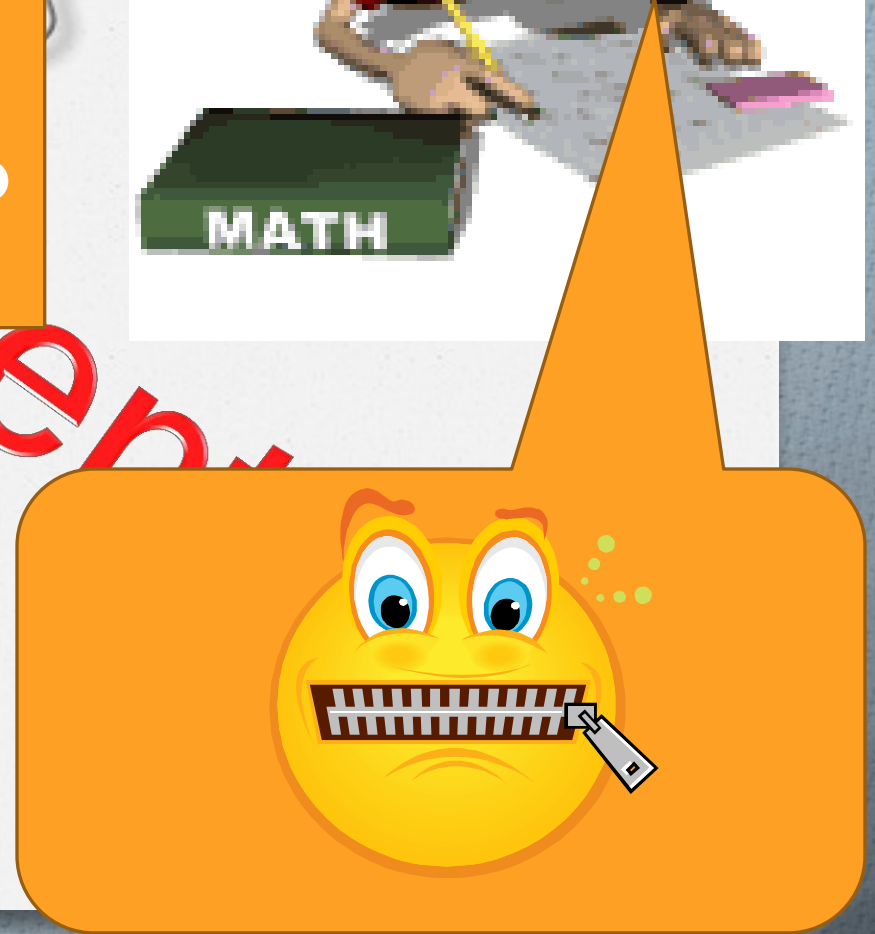
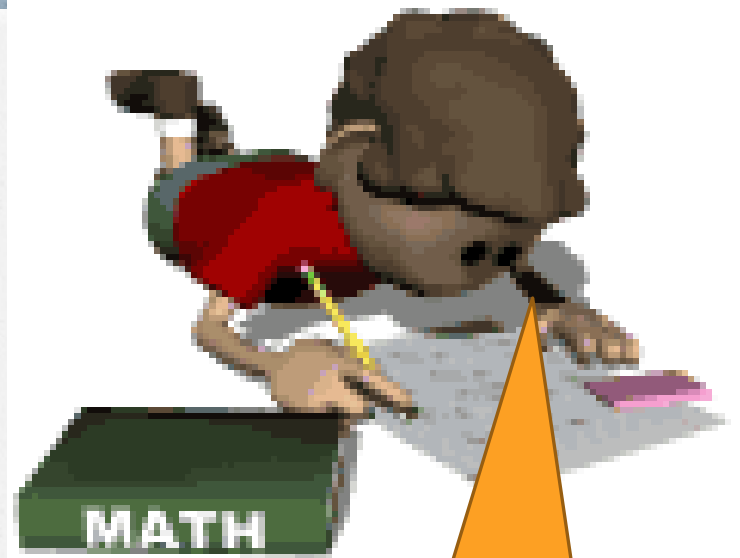
Both types include both concepts and problem solving, but each puts more emphasis on one than the other.



Learning by Doing!



What does
teacher do
during this time?



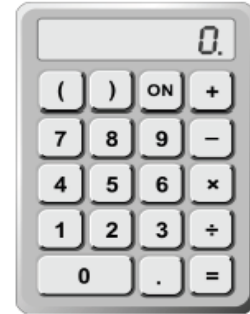
Percent Changes

One month Rob spent \$8.02 on his phone. The next month he spent \$6.00. To work out the average amount Rob spends over the two months, you could press the calculator keys:

(8 . 0 2 + 6) ÷ 2 =

1. Tom usually earns \$40.85 per hour. He has just heard that he has had a 6% pay raise. He wants to work out his new pay on this calculator. It does not have a percent button.

Which keys must he press on his calculator?
Write down the keys in the correct order.
(You do not have to do the calculation.)



-
2. Maria sees a dress in a sale. The dress is normally priced at \$56.99. The ticket says that there is 45% off. She wants to use her calculator to work out how much the dress will cost. It does not have a percent button.

Which keys must she press on her calculator?
Write down the keys in the correct order.
(You do not have to do the calculation.)

-
3. Last year, the price of an item was \$350. This year it is \$450. Lena wants to know what the percentage change is. Write down the calculation she will need to do to get the correct answer. (You do not have to do the calculation.)

-
4. In a sale, the prices in a shop were all decreased by 20%. After the sale they were all increased by 25%. What was the overall effect on the shop prices? Explain how you know.

Pre- Assessment



Misconceptions & Feedback Questions



- What *misconceptions* might your students have with this FAL or an activity like this one?
- What possible *feedback questions* could you ask to *move their learning forward*?



Provide feedback that moves students forward.

Common issues:

Suggested questions and prompts:

Student makes the incorrect assumption that a percentage increase means the calculation must include an addition

For example: $40.85 + 0.6$ or $40.85 + 1.6$. (Q1.)

A single multiplication by 1.06 is enough.

- Does your answer make sense? Can you check that it is correct?
- “Compared to last year 50% more people attended the festival.” What does this mean? Describe in words how you can work out how many people attended the festival this year. Give me an example.
- Can you express the increase as a single multiplication?

Student makes the incorrect assumption that a percentage decrease means the calculation must include a subtraction

For example: $56.99 - 0.45$ or $56.99 - 1.45$. (Q2.)

A single multiplication by 0.55 is enough.

- Does your answer make sense? Can you check that it is correct?
- In a sale, an item is marked “50% off.” What does this mean? Describe in words how you calculate the price of an item in the sale. Give me an example.
- Can you express the decrease as a single multiplication?

Student converts the percentage to a decimal incorrectly

For example: 40.85×0.6 . (Q1.)

- How can you write 50% as a decimal? How can you write 5% as a decimal?

Student uses inefficient method

For example: First the student calculates 1%, then multiplies by 6 to find 6%, and then adds this answer on:

$(40.85 \div 100) \times 6 + 40.85$. (Q1.)

Or: $56.99 \times 0.45 = \text{ANS}$, then $56.99 - \text{ANS}$ (Q2.)

A single multiplication is enough.

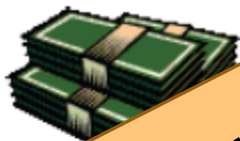
- Can you think of a method that reduces the number of calculator key presses?
- How can you show your calculation with just one step?



Provide feedback that moves students forward.

<p>Student is unable to calculate percentage change</p> <p>For example: $450 - 350 = 100\%$ (Q3.)</p> <p>Or: The difference is calculated, then the student does not know how to proceed or he/she divides by 450. (Q3.)</p> <p><i>The calculation $(450 - 350) \div 350 \times 100$ is correct.</i></p>	<ul style="list-style-type: none">• Are you calculating the percentage change to the amount \$350 or to the amount \$450?• If the price of a t-shirt increased by \$6, describe in words how you could calculate the percentage change. Give me an example. Use the same method in Q3.
<p>Student subtracts percentages</p> <p>For example: $25 - 20 = 5\%$. (Q4.)</p> <p><i>Because we are combining multipliers: $0.8 \times 1.25 = 1$, there is no overall change in prices.</i></p>	<ul style="list-style-type: none">• Make up the price of an item and check to see if your answer is correct.
<p>Student fails to use brackets in the calculation</p> <p>For example: $450 - 350 \div 350 \times 100$. (Q4.)</p>	<ul style="list-style-type: none">• In your problem, what operation will the calculator carry out first?
<p>Student misinterprets what needs to be included in the answer</p> <p>For example: The answer is just operator symbols.</p>	<ul style="list-style-type: none">• If you just entered these symbols into your calculator would you get the correct answer?

Card Set Money Cards (1)



\$200

\$160

As I walk around, take note of the questions I ask you and the comments I make to you. Are they questions or comments that encourage you to engage in productive struggle, or do I GPS you to an answer?

Percents, Decimals, and Fractions (1)

What's the teacher doing now?



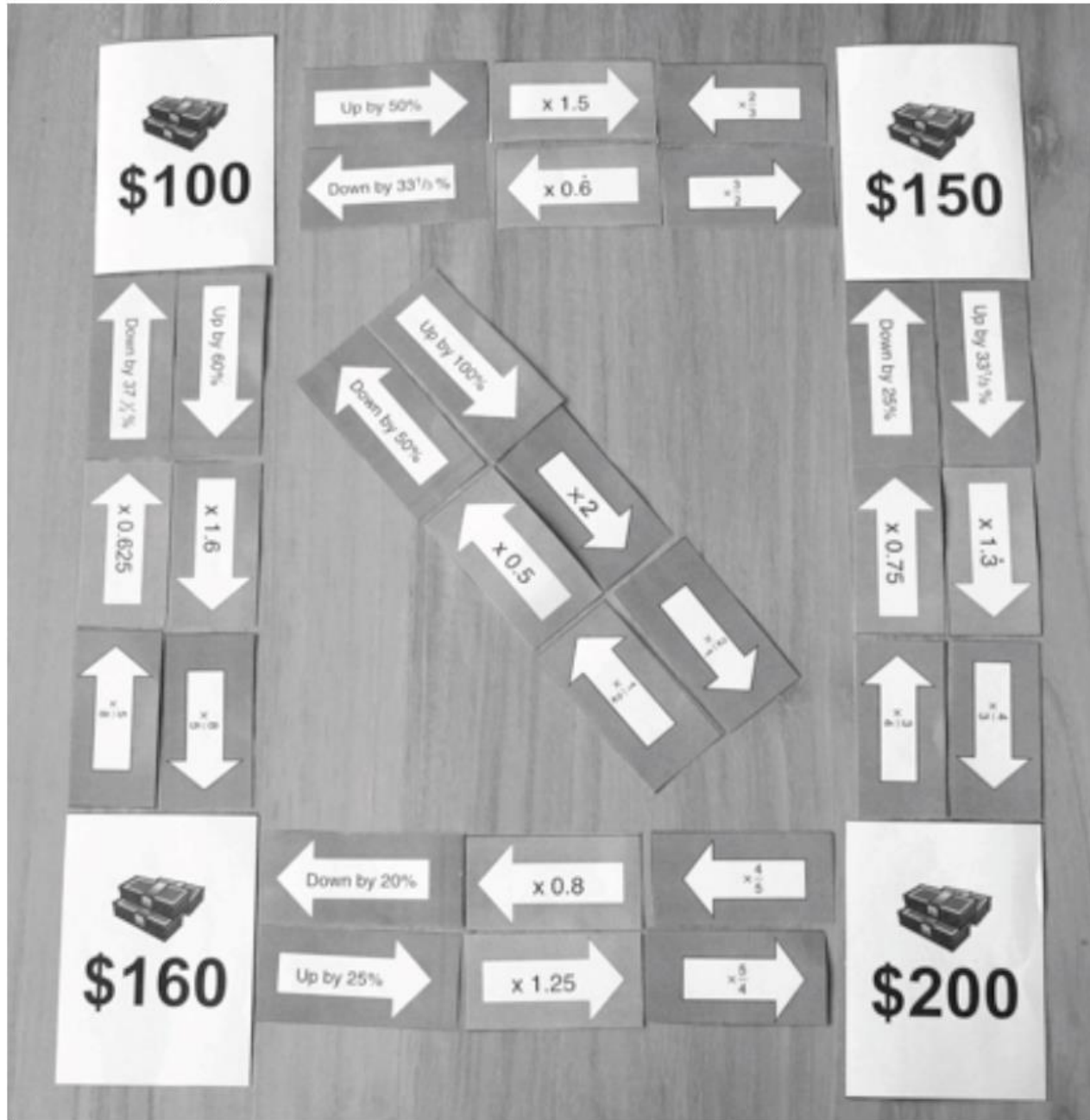
\$150

Moving around to groups, taking notes, asking questions and answering questions with questions!

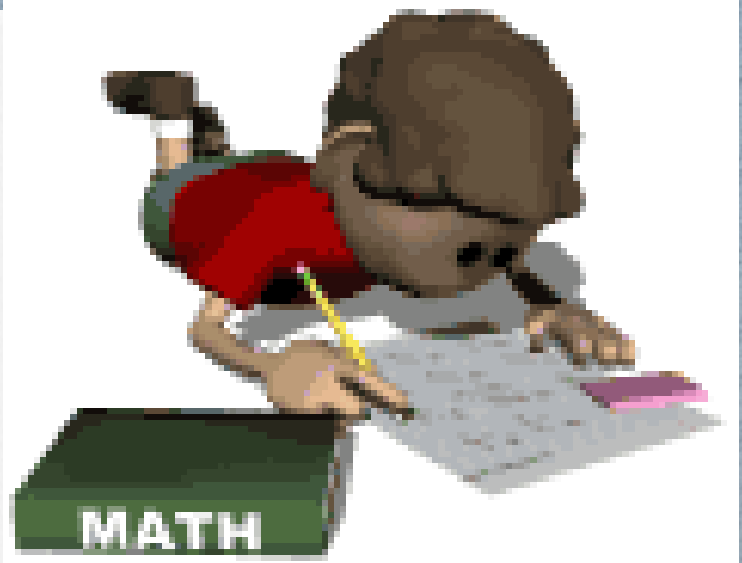


\$160

Collaborative activity



Independent Work



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Post-Assessment



Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
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Common Core Standards

- **7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.**
- **7.NS.2 Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers.**
- **7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.**

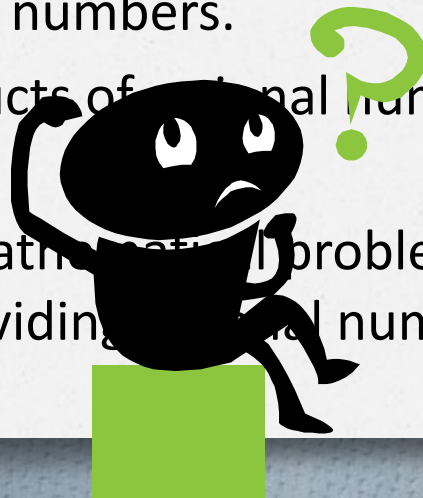


Learning Targets of Unit

- Compute unit rates associated with ratios of fractions in like or different units.

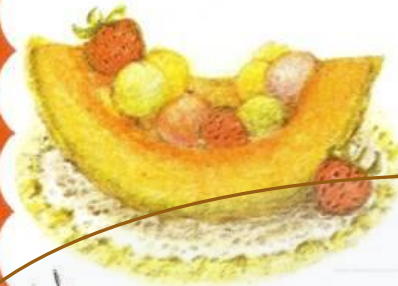
If I gave an exit slip, quiz question, or other short cycle formative assessment for each of these targets, would it tell me if they really got and were able to do the standard?

- Apply the properties of operations, particularly distributive property, to multiply rational numbers.
- Interpret the products of rational numbers by describing real-world contexts.
- Solve real-world mathematical problems by adding, subtracting, multiplying, and dividing rational numbers, including complex fractions.



From the kitchen of...

Susan Wunderlich



Red Velvet Cake with Cream Cheese Frosting

$2\frac{1}{2}$ c. flour
 $1\frac{1}{2}$ c. sugar 1 tsp. soda
2 c. salad oil 1 tsp. salt
2 eggs beaten 2 Tbsp. cocoa
1 tsp. vinegar 1 c. buttermilk
2 oz. red food coloring

Cream sugar & oil in bowl. Add eggs & beat well. Add vinegar & food coloring. Beat well. Sift flour, soda, salt & cocoa together. Add to creamed mixture alternately with buttermilk. Add vanilla & beat well. Pour into 2(3) greased & floured cake pans. Bake at 350° 30-35 minutes.

Frosting - cream 1 stick margarine & 8 oz. cream cheese. Add 1 Tbsp. vanilla, then 1 box confec. sugar. Then 1 c. chopped nuts if desired.

How does the FAL structure provide for differentiation & productive struggle?



Don't Prevent Students' Mistakes, Prepare for Them

Today's Learning Targets



- o I can experience productive struggle and explain its purpose and benefits.
- o I can explain the purpose of Formative Assessment Lessons (FALs).
- o I can describe the process of implementing a FAL.
- o I can access and use the materials to plan and perform a FAL.

home

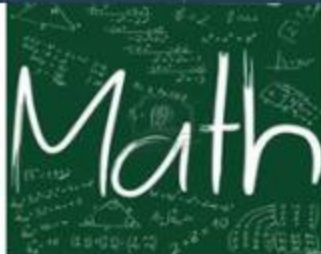
Networks

Math

SocialStudies

Professional Learning

Video/Tech



Everyone is a genius. But if you judge a fish on its ability to climb a tree, it will live its whole life believing that it is stupid.

-A Einstein



Debbie Waggoner - Instructional Specialist, *Kentucky Department Education* CKEC Region - Mathematics and Social Studies Emphasis

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[Kentucky Department of Education](#)
Office of Next Generation Learners

[Central Kentucky Educational Cooperative](#)

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More info About Me, [click here](#)

[Links to regional specialists, click here.](#)

Keep up with KDE's social media channels!

Facebook: www.facebook.com/kydeptofed Twitter: www.twitter.com/kydeptofed

Links to latest meetings/presentations

--NCTM National Conference New Orleans
Productive Struggle - April 10th, 2014

--CKEC Social Studies Network Mar. 25th

-->>SS Balanced Assessment Survey<<--

--CKEC ISLN Mar. 20th

--MS/HS Math Cadre Mar. 19th

--KCM Ratios & Proportions session Mar. 10th

CKEC/CKSEC upcoming trainings...



view upcoming events
and [register here](#)

home

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Math

Social Studies

Professional Learning

Video/Tech



Elementary Grades K-5

Secondary Grades 6-12

Formative Assessment Lessons

Great K-12 Math Resources

CCSS math documents

Math Practice Standards

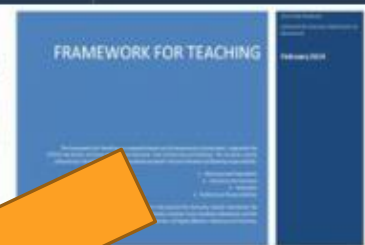
RTI Math Strategies MATs

Common Core Conversations Math Resources

Math Differentiation

Assessments

Math APPs



MS/HS FALs website

Elementary FALs

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Keep up with KDE's social media
Facebook: www.facebook.com/kydeptofeducation

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CKEC/CKSEC upcoming trainings...

view upcoming events and [register here](#)



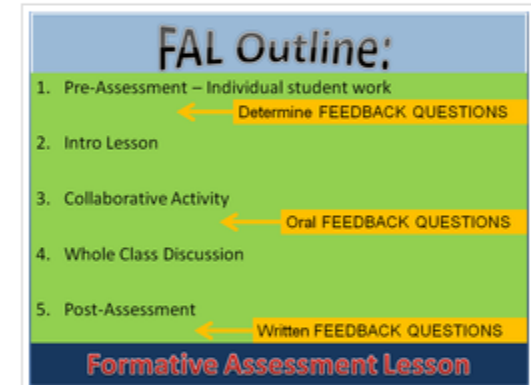
Elementary Formative Assessment Lessons



Created by Kentucky Department of Education Mathematics Specialists and field tested by Kentucky teachers participating in the Mathematics Leadership Network.

These versions have been through at least one revision. If you encounter errors or would like to make suggestions, please use the contact form below.

>> **Formative Assessment Lesson Outline slide** <<



Printable List of all Elementary Mathematics Formative Assessment Lessons Sept. 2012

Concept-Focused Formative Assessment Lessons

Kindergarten: Counting Dots



This lesson unit is intended to help you assess how well students are able to count objects up to 10 no matter how they are arranged and also how well they are able to represent their counts with written numerals. It will help you to identify students who have the following difficulties: Not being able to track where they start/stop counting, Arranging objects to be counted, One-to-one correspondence. This lesson involves *mathematical content* in the standards from across the grade, with emphasis on: Counting and Cardinality K.CC Know number names and the count sequence. Count to tell the number of objects. This

Who are we?



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