Table of Contents

Factor Game Investigation

Introduction:
Does it matter which number you pick to start a game? Does it matter who goes first? Test these ideas with the Factor Game and then analyze data to uncover interesting or unexpected number patterns.

Investigation:

<u>Part 1</u>
1. What patterns in the table might help you extend the data?
2. Record a couple of ideas before you move to other parts of this required investigation.
3. Continue to test, use, and refine your original observations.

<u>Part 2</u>
1. What numbers selected as a first move earn a <u>player</u> more points than the opponent?
2. What patterns do you notice? Record your conjectures on paper.
3. Test your conjectures by continuing the list of information in the table.
4. Finalize a conjecture that describes this type of number.

<u>Part 3</u>
1. What numbers selected as a first move give the <u>opponent</u> more points than the player?
2. What patterns do you notice? Record your conjecture on paper.
3. Test your conjectures referring to your extended table.
4. Finalize your conjecture that describes this type of number.

<u>Part 4</u>
1. Proper factors are all the factors of a number, except itself. Look for relationships between the sum of the proper factors of a number and the number.
2. What patterns do you notice? What is true about the numbers in each group you identify? Record your conjectures on paper. Add numbers as evidence of each conjecture.
3. Test your conjectures.
4. Finalize conjectures that describe the different relationships you identify.

<u>Part 5</u>
1. Return to Part 1.
2. Finalize original conjectures and/or add new ones.

Thanks for being ready to share your discoveries in a whole group discussion.

Reflection
What skills do students need to do this?
How do the skills learned in this investigation related to future learning?
Where does this investigation fit in the learning progression?

Factor Game Rules

1. Player A chooses a number on the game board and circles it.
2. Using a different color, Player B circles all the proper factors of Player A's number. The **proper factors** of a number are all the factors of that number, except the number itself. For example, the proper factors of 6 are 1, 2 and 3. Although six (6) is a factor of itself, it is not a proper factor.
3. Player B circles a new number, and Player A circles all the factors of the number that are not already circled.
4. The players take turns choosing numbers and circling factors.
5. If a player circles a number that has no factors left that have not been circled, that player loses a turn and does not get the points for the number circled.
6. The game ends when there are no numbers remaining with un-circled factors.
7. Each player adds the numbers that are circled with his or her color. The player with the greater total is the winner.

Record this sample game on a game board following the actions of the players. Find an opponent and play the game a couple of times. And then use the data on the Table of First Moves for the investigation.

	Player A	**Player B**
Action	**Score**	**Score**
Player A circles 24. Player B circles factors 1,2,3,4,6,8,12---the proper factors of 24.	24	36
Player B circles 28. Player A circles factors 7, 14---the factors of 28 not already circled.	21	28
Player A circles 27. Player B circles 9---the only factor of 27 not already circled.	27	9
Player B circles 30. Player A circles 5, 10, 15---only factors of 30 not already circled.	30	30
Player A circles 25. All factors of 25 are circled. Player A loses a turn and receives no points for this turn.	0	---
Player B circles 26. Player A circles 13---the only factor of 26 remaining on board.	13	26
Player A circles 22. Player B captures the factor 11	22	11
No numbers remain with uncircled factors. Player B wins. **TOTAL**	126	151

Labsheet 1.1

Factor Game Boards

The Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

The Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

The Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

The Factor Game

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Labsheet 1.2

Table for Recording First Moves

Possible first move	Proper factors	My score	Opponent's score
1	none	lose a turn	0
2	1	2	1
3	1	3	1
4	1, 2	4	3
5	1	5	1
6	1, 2, 3	6	6
7	1	7	1
8	1, 2, 4	8	7
9	1, 3	9	4
10	1, 2, 5	10	8
11	1	11	1
12	1, 2, 3, 4, 6	12	16
13	1	13	1
14	1, 2, 7	14	10
15	1, 3, 5	15	9
16	1, 2, 4, 8	16	15
17			
18			
19			
20			
21			
22			
23			
24			
25			
26			
27			
28			
29			
30			

Toying with Tiles

Introduction

Rectangles, squares, or prime and composite numbers! How can you learn basic facts? What numbers make what shapes? Find out by playing with the tiles and looking for patterns!

Part 1

Accumulate Data: How many rectangles can you make with 7 tiles and 9 tiles? How about 13 tiles? What are the differences among the collections of rectangles? What are the similarities?

To expedite your time, we have created the first set of data for some numbers. In a classroom, students can discover relationships as they are making the rectangles.

Part 2

Finding patterns and relationships

a. What type of numbers has only one rectangular arrangement?

b. What type of numbers has only two rectangular arrangements?

c. What type of numbers has a rectangular arrangement which is a square?

d. How is the number 1 different from all the other numbers that have only one rectangular arrangement? Is 1 prime?

e. Can you find the first number greater than 16 that has exactly two rectangular arrangements? How about 3? How about 4? How about 1?

f. Which numbers have rectangles with two tiles on the side? Write these numbers from smallest to largest. What kind of numbers are these?

g. Which numbers have rectangles three tiles on the side? Write these numbers from smallest to largest. How would you describe this set of numbers?

h. Which numbers have rectangles four tiles on the side? Write these numbers from smallest to largest. How would you describe this set of numbers?

i. Which numbers have rectangles five tiles on the side? Write these numbers from smallest to largest. How would you describe this set of numbers?

j. Which numbers have rectangles that are also squares? Write these numbers from smallest to largest. How would you describe this set of numbers?

k. How many tiles would there be in the next larger square?

l. What is the smallest number with two different rectangles?

m. With three different rectangles? _____

n. What number would be the first with four different rectangles? ___

o. Which numbers have only one rectangle? List them from smallest to largest.

Part 3.
Conjecture: What other ideas are you gathering from this data?

Make a few more rectangles to test your ideas.

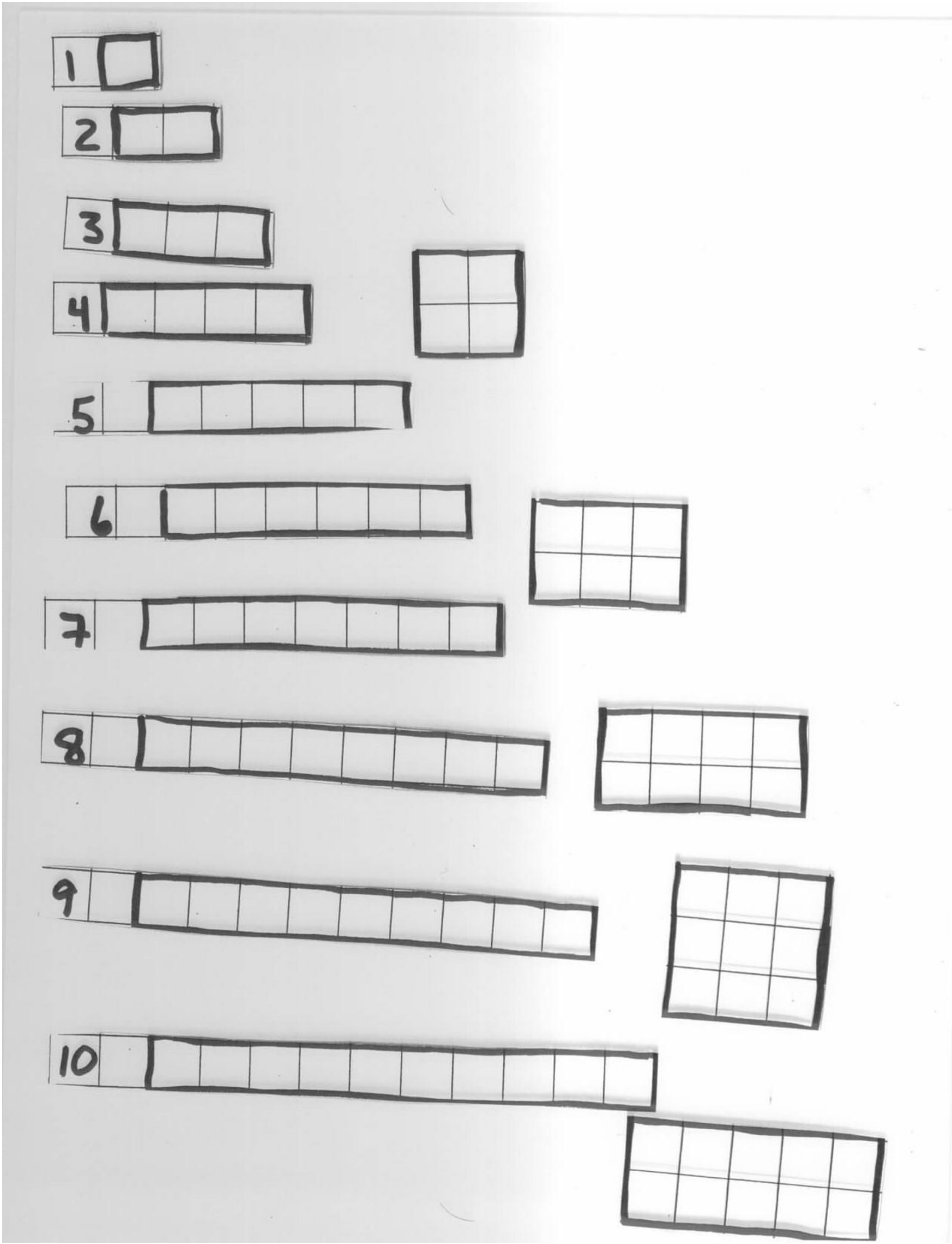
What other ideas are you gathering from this data?

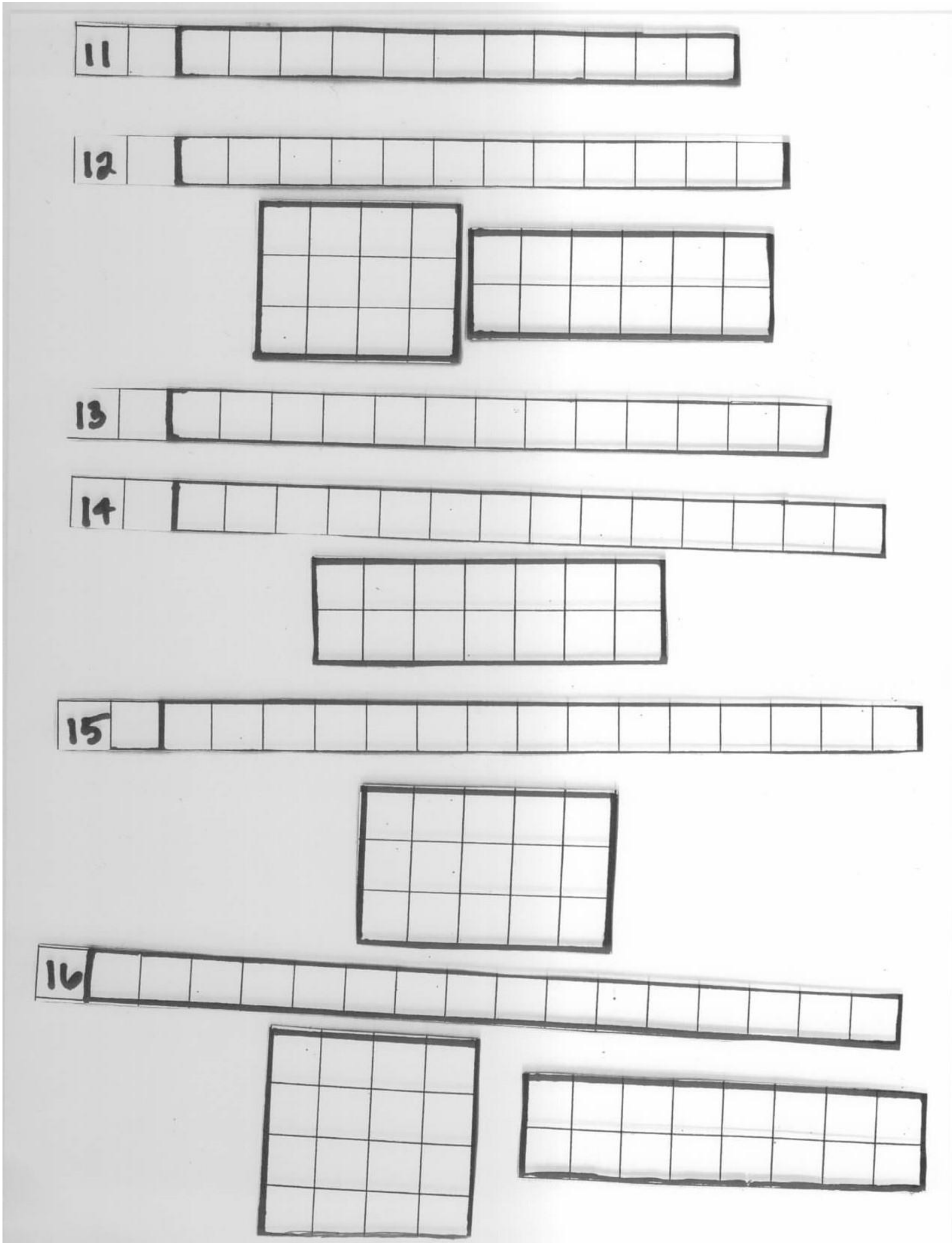
Reflection

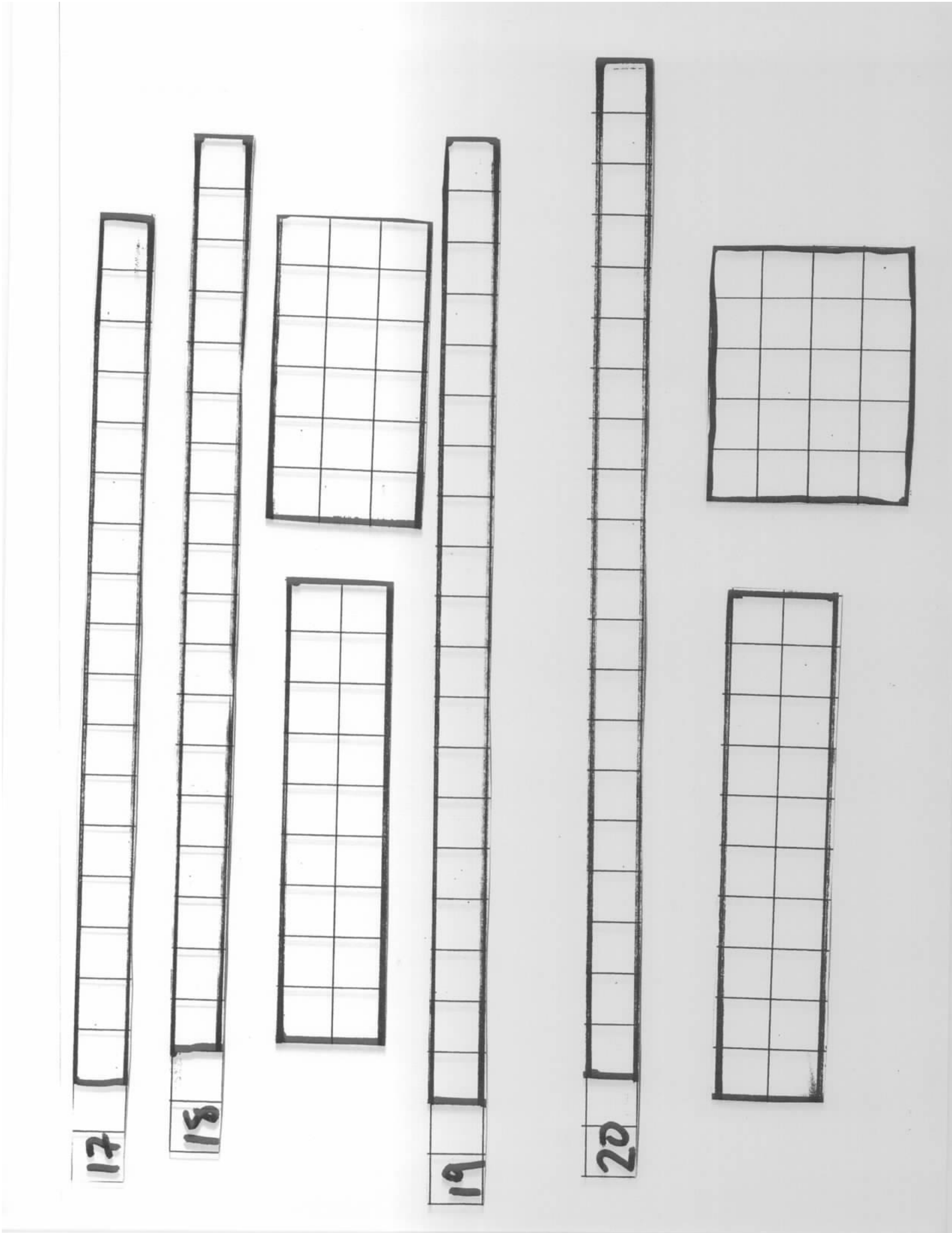
1. Where does activity fits in the learning progressions?

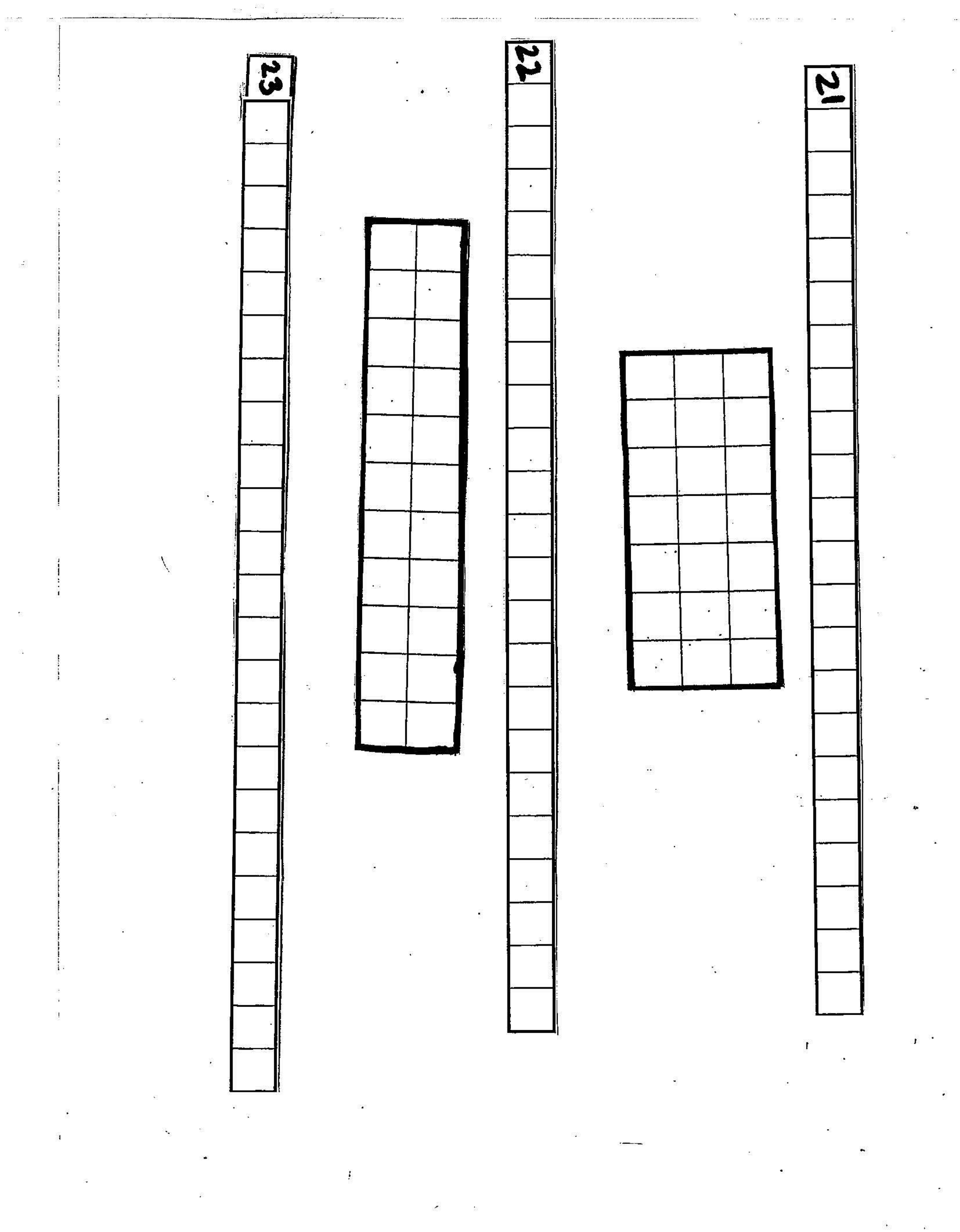
2. What skills do students need to do this?

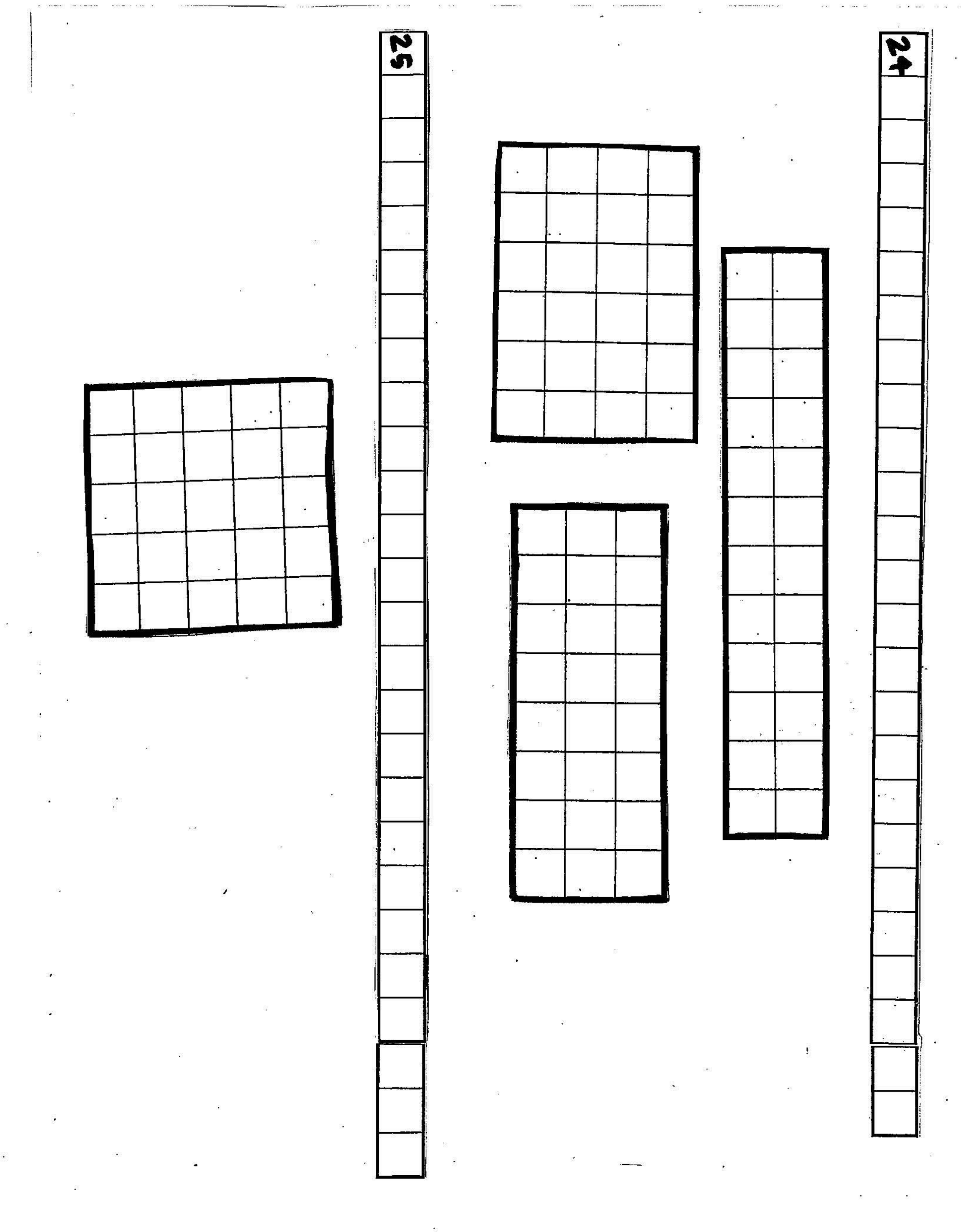
3. How do the concepts/skills learned in this activity relate to future skills?


1
2
3
4
5
6
7
8
9
10


11
12
13
14
15
16


17
18
19
20


23
22
21


25
24

Product Game Investigation

Introduction:
Math becomes valuable when you want to win a game. Try your hand at the Product Game. Analyze the relationships among factors, products and multiples to win and also to construct a new product game board, which you believe others will enjoy playing.
Investigation:

Part 1 –Play the Game
Read the rules. ON PAGE 2 follow the description of play for Players A and B. Record their moves on the adjacent game board; complete the game. Play additional games with a human opponent; start each game with a different product. Play the game against an electronic opponent at
http://calculationnation.nctm.org/

Rules
1. Player A puts a paper clip on a number in the factor list. Player A does not mark a square on the product grid because only one factor has been marked---it takes two factors to make a product.
2. Player B puts the other paper clip on any number in the factor list (including the same number marked by Player A) and then shades or covers the product of the two factors on the product grid.
3. Player A moves *either one* of the paper clips to another number in the factor list and then shades or covers the new product.
4. Each player, in turn, moves one paper clip and marks a product. If a product is already marked, the player does not get a mark for that turn. The winner is the first player to mark four squares in a row---up and down, across, or diagonally.

Part 2 Analyze the Game
Complete the Product Game Question Sheet.

Part 3 Make a Game
Make your own product game.

1. Choose factors to include in your factor list.
2. Determine the products you need to include on the game board.
3. Sketch a game board that will accommodate all the products.
4. Decide how many squares a player must get in a row to win.

Reflection
What skills do students need to do this?
How do the skills learned in this investigation related to future learning?
Where does this investigation fit in the learning progression?

Product Game Investigation
page 2

Record and complete the sample game. You will need two paper clips or other items to use as makers for the Factors listed below the game board.

Player A	Player B
Places paperclip on 6	•Places paperclip on 8; •Says, "A factor of 6 multiplied by the factor 8 equals product 48." •Records 6 x 8 = 48
•Moves paper clip from 8 to 7. • Says, "A factor of 6 multiplied by the factor 7 equals product 42." •Records 6 x 7 = 42	•Moves paper clip from 7 to 5 •Says, "A factor of 6 multiplied by the factor 5 equals product 30." •Records 6 x 5= 30
•Moves paper clip from 6 to 8 •Says, "A factor of 5 multiplied by the factor 8 equals product 40." •Records 8 x 5 = 40	

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:
1 2 3 4 5 6 7 8 9

Player A O	Player B X
	6 • 8 = 48
6 x 7 = 42	6 • 5 = 30
6 * 8 = 48	

Product Game Question Sheet

1. You have played the Product Game. Suppose one of the paper clips is on 4. What products can you make by moving the other paper clip? What do you know about this group of numbers?

2. You marked 12 on the game board. On which factors could the paper clips be? List all possibilities. What are other products that could be made in more than one way?

3. Record this partial game on a game board and then answers the following questions.

Player A	Player B
Chooses paper clip 8	Chooses paper clip 2 8 x 7 = 56
7 x 4 = 28	7 x 3 = 21
3 x 6 = 18	6 x 5 = 30

a. List the possible moves you could make.
b. Which moves would give you three markers in a row?
c. Which move(s) would allow you to block your opponent?
d. Which move would you make Explain your strategy?

4. Which factors were used to create this Product Game Board?

4	6	14
9	21	49

Factors _______ _________ _________

5. What factors were used to create this Product Game board? Which number is missing from the grid?

9	15	18	----------
21	?	30	35
-------	36	42	49

Factors _______ _________ _________ ___________

The Product Game Board

Name
Date

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:
1 2 3 4 5 6 7 8 9

The Product Game

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

Factors:
1 2 3 4 5 6 7 8 9

Divisibility

Introduction:

Studying the rules of divisibility highlights the relationship between multiplication and division. In order for one whole number to be divisible by another whole number, the divisor must be a factor of the dividend. Therefore, the multiples of a number are also divisible by that number. In this activity, you will investigate lists of multiples. You might just uncover something interesting or unexpected.

Investigation:

Part 1

1. Here are the first 20 multiples of 2.

0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38

2. What patterns do you notice? Write you conjectures down.
3. Test your conjectures by continuing your list of multiples.
4. Do your conjectures hold true? Construct a viable argument for divisibility of 2.
5. Repeat steps 1 – 4 with 5 and 10. (Optional)

Part 2

1. The divisibility rule for 3 is:
- A positive integer is evenly divided by 3 if the sum of its digits is divisible by 3.
2. Test several numbers to verify this is true.
3. List the first 20 multiples of 6.
4. What patterns do you notice? Write down your conjectures.
5. Test your conjectures by continuing your list of multiples.
6. Construct a viable argument for divisibility by 6.

Part 3

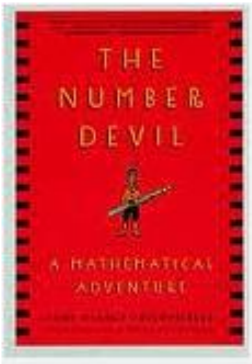
1. Divisibility by 4 has several rules. A positive integer is evenly divisible by 4:
- if the number formed the by the last two digits is divisible by 4.
- if the tens digit is even and the ones digit is 0, 4, 8 OR if the tens digit is odd and the ones digit is 2 or 6.
- if twice the tens digit plus the ones digit is divisible by 4.
2. Test several numbers to verify these rules.
3. List the first 20 multiples of 12.
4. What patterns do you notice? Write down your conjectures.
5. Test your conjectures by continuing your list of multiples.
6. Construct a viable argument for divisibility by 12.

Part 4

Choose a number – 8, 14, 15, 16, 18, 20 – to investigate.

Reflection

1. What skills do students need to do this?
2. How do the skills learned in this investigation related to future learning?
3. Where does this investigation fit in the learning progression?



The Number Devil

Introduction:
Prime numbers are defined as having exactly two unique factors. Being able to determine whether a number is a prime number or not is an important skill. In The Number Devil, Robert learns about prima donna numbers and how to test for them. In this activity, you will do a similar method and look for patterns in the numbers.

Investigation:
Part 1 – *The Number Devil* is a book about a boy, Robert, who hates Math. He encounters a magical being in his dreams that introduces him to the wonder of numbers. Read chapter 3, The Third Night, starting on page 49.

Part 2 – Construct a 6-column color-coded sieve.
1. You will need a piece of grid paper that is 6 columns across and 17 rows down. Place the numbers 1 through 100 in the grids in order.
2. Outline the box with the number 2 in yellow; outline the box with the number 3 in orange; outline the box with the number 5 in blue; outline the box with the number 7 in pink.
3. Highlight all the numbers that are multiples of 2 in yellow. Highlight all the numbers that are multiples of 3 in orange. Highlight all the numbers that are multiples of 5 in blue. Highlight all the multiples of 7 in pink.
4. If a number has more than one multiple, use a diagonal line to color half of it in one color and the other half in the other color. The first three rows should look like this:

5. If a number has more than two multiples, split the box on the other diagonal and color all three or four colors.

Part 3
1. What patterns do you notice? Write down your conjectures.
2. Extend your chart to test your conjectures.

Reflection
1. What skills do students need to do this?
2. How do the skills learned in this investigation related to future learning?
3. Where does this investigation fit in the learning progression?

Interesting Divisibility Rules

2 Last digit is even
Example: 123,456 is divisible by 2 because the last digit, 6, is even.

3 Sum of the digits is divisible by 3
Example: 123,456 is divisible by 3 because 1 + 2 + 3 + 4 + 5 + 6 = 21, which is divisible by 3

3 The difference of the quantity of the digits 2,5,8 in the number from the quantity of the digits 1,4,7 in the number is a multiple of 3
Example: 123,456 is divisible by 3 because it has two 2,5,8's and two 1,4,7's so 2-2=0 and 0 is a multiple of 3.

4 The number formed by the last two digits is divisible by 4
Example: 123,456 is divisible by 4 because 56 is divisible by 4.

4 The tens digit is even and the ones digit is 0,4,8 or the tens digit is odd and the ones digit is 2,6
Example: 123,456 is divisible by 4 because the tens digit is odd and the ones digit is 6.

4 Twice the tens digit plus the ones digit is divisible by 4
Example: 123,456 is divisible by 4 because 5 x 2 = 10; 10 + 6 = 16; 16 is divisible by 4.

5 Last digit is 5 or 0
Example: 12,345 is divisible by 5 because the last digit is 5.

6 Divisible by both 2 and 3
Example: 123,456 is divisible by 6 because it is divisible by both 2 and 3.

7 Subtract 2 times the last digit from the rest
Example: 12,334 is divisible by 7 because 4 x 2 = 8 and 1,233 – 8 = 1225. Then repeat the process with 5 x 2 = 10 and 122-10 = 112. Repeat again 2 x 2 = 4 and 11 – 4 = 7 which is definitely divisible by 7.

7 Add 5 times the last digit to the rest
Example: 12,334 is divisible by 7 because 4 x 5 = 20 and 1233 + 20 = 1253; Then repeat the process with 3 x 5 = 15 and 125 + 15 = 140 which is divisible by 7.

7 Multiply each digit from left to right with the corresponding number in the following pattern -2, -3, -1, 2, 3, 1 and sum the results
Example: 12,334 is divisible by 7 because (1 x -2) + (2 x -3) + (3 x -1) + (3 x 2) + (4 x 3) = -2 + -6 + -3 + 6 + 12 = 7 which is divisible by 7.

8 Last 3 digits is divisible by 8
Example: 123,456 is divisible by 8 because 456 ÷ 8 – 57

8 If the hundreds digit is even, look at last two digits

Example: 123,456 is divisible by 8 because the hundreds digit is 4 (even) and the last two digits are 56 and 56 is divisible by 8.

8 If the hundreds digit is odd, look at the last two digits plus 4

Example: 123,552 is divisible by 8 because the hundreds digit is 5 (odd) and the last two digits are 52. 52 + 4 = 56, which is divisible by 8.

8 Add the last digit to twice the rest

Example: 456 is divisible by 8 because 45 x 2 = 90 and 90 + 6 = 96. Repeat the process 9 x 2 = 18 and 18 + 6 = 24, which is divisible by 8.

8 Add four times the hundreds digit to twice the tens digit to the ones digit

Example: 123,456 is divisible by 8 because (4 x 4) + (2 x 5) + 6 = 16 + 10 + 6 = 32 which is divisible by 8.

9 If the sum of the digits is divisible by 9

Example: 123,453 is divisible by 9 because 1 + 2 + 3 + 4 + 5 + 3 = 18 which is divisible by 9.

10 Last digit is a 0

Example: 123,450 is divisible by 10 because the last digit is 0.

11 Form the alternating sum of the digits from left to right

Example: 123,453 is divisible by 11 because 1 – 2 + 3 – 4 + 5 – 3 = 0 and 0 is divisible by 11.

11 Add the digits in blocks of two from right to left

Example: 123,453 is divisible by 11 because 53 + 34 + 12 = 99 which is divisible by 11.

11 Subtract the last digit from the rest

Example: 123,453 is divisible by 11 because 12,345 – 3 = 12,342 and 1,234 – 2 = 1232 and 123 – 2 = 121 and 12 – 1 = 11.

11 If the number of digits is even, add the first and subtract the last digit from the rest

Example: 123,453 is divisible by 11 because there are 6 digits (even) so 2345 +1 – 3 = 2343. Repeat the process 34 + 2 – 3 = 33 which is divisible by 11.

11 If the number of digits is odd, subtract the first and last digit from the rest

Example: 12,342 is divisible by 11 because it has 5 (odd) digits so 234 – 1 – 2 = 231 and 3 – 2 – 1 = 0, which is divisible by 11.

12 Divisible by both 3 and 4

Example: 123,456 is divisible by 12 because it is divisible by 3 and 4.

12 Subtract the last digit from twice the rest

Example: 123,456 is divisible by 12 because (12,345 x 2) – 6 = 24,684; (2468 x 2) – 4 = 4932; (493 x 2) – 2 = 984; (98 x 2) – 4 = 192; (19 x 2) – 2 = 36 which is divisible by 12.

13 Add four times the last digit to the rest

Example: 123,448 is divisible by 13 because 12,344 + 32 = 12,376; 1237 + 24 = 1261; 126 + 4 = 130 which is divisible by 13.

13 Multiply each digit from right to left by the digit in the following pattern -3, -4, -1, 3, 4, 1 and sum the results

Example: 123,448 is divisible by 13 because (8 x -3) + (4 x -4) + (4 x -1) + (3 x 3) + (2 x 4) + (1 x 1) = -26 which is divisible by 13.

14 Divisible by both 2 and 7

Example: 12,334 is divisible by 14 because it is divisible by both 2 and 7.

14 Add the last two digits to twice the rest'

Example: 12,334 is divisible by 14 because (123 x 2) + 34 = 280 which is divisible by 14.

15 Divisible by both 3 and 5

Example: 12,345 is divisible by 15 because it is divisible by 5 and 3.

16 Add the last two digits to four times the rest

Example: 123,456 is divisible by 16 because (1234 x 4) + 56 = 4992 and (49 x 4) + 92 = 288 and (2 x 4) + 88 = 96 which is divisible by 16.

16 Examine the last 4 digits

Example: 123,456 is divisible by 16 because 3456 ÷ 16 = 216.

17 Subtract 5 times the last digit from the rest

Example: 123,454 is divisible by 17 because 12,345 – 20 = 12,325; 1232 – 25 = 1207; 120 – 35 = 85 which is divisible by 17.

18 Divisible by both 2 and 9

Example: 123,444 is divisible by 18 because it is divisible by both 2 and 9.

19 Add twice the last digit to the rest

Example: 123,443 is divisible by 19 because 12,344 + 6 = 12,350; 1235 + 0 = 1235; 123 + 10 = 133; 13 + 6 = 19 which is divisible by 19.

20 Divisible by 10 and tens digit is even

Example: 123,440 is divisible by 20 because its last digit is 0 and the tens digit is 4 (even).

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78

79	**80**	**81**	**82**	**83**	**84**
85	**86**	**87**	**88**	**89**	**90**
91	**92**	**93**	**94**	**95**	**96**
97	**98**	**99**	**100**		

Watch One, Do One, Teach One: GCFs and LCMs

Introduction: Finding the greatest common factor (GCF) and the least common multiple (LCM) of two whole numbers is a skill that is a fundamental prerequisite to understanding rational number operation. For example, GCF(24,16)=8 plays an important role in constructing equivalent fractions such as:

$$\frac{24}{16}=\frac{3}{2}$$

And, LCM(24,16)=48 plays an important role in reasoning about adding fractions such as:

$$\frac{13}{24}+\frac{7}{16}=\frac{26}{48}+\frac{21}{48}$$

How can we help students "make sense" of GCFs and LCMs? How can we help students "visualize" GCFs and LCMs? What algorithms are available for finding GCFs and LCMs? We will approach these questions in this exploration.

Investigation: In this investigation we will ask you to engage in the protocol called, "Watch one, do one, teach one". In this protocol, the student is asked to play the role of observer by "watching one" as a mentor produces a product, explains a skill, or demonstrates a procedure. The student's role then shifts to participant by "doing one" through some demonstration of their newly acquired understanding, skill or procedure. Finally, the student's role shifts to mentor by "teaching one" where they explain their understanding of what they have just learned to another student, teacher or the whole class.

Watch One: There are four video segments that present opportunities for you to "watch one". Each video is 6-8 minutes long and presents a different procedure for finding LCM(24,16) and GCF(24,16) and the reasoning behind the procedure. Make a group decision who will watch which video. Once video assignments have been decided upon, watch the video you have been assigned.

Do One: Once you have watched a video, shift to "try one". Try the following three problems using the method that you have learned. Solutions to each of the "try one" problems are found the reverse of this sheet.

1. LCM(12,15) and GCF(12,15)
2. LCM(16,28) and GCF(16,28)
3. LCM(25,27) and GCF(25,27)

Teach One: Pair up with another person who watched a different video and take turns "teaching one". Use the following three problems. Solutions to each of the "teach one" problems are found on the reverse of this sheet.

1. LCM(14,16) and GCF(14,16)
2. LCM(12,18) and GCF(12,18)
3. LCM(36,27) and GCF(36,27)

Reflection

1. What skills do students need to do this?

2. How do the skills learned in this investigation related to future learning?
3. Where does this investigation fit in the learning progression?

Try One Solutions

1. LCM(12,15)=60 and GCF(12,15)=3
2. LCM(16,28)=112 and GCF(16,28)=4
3. LCM(25,27)=675 and GCF(25,27)=1

Teach One Solutions

1. LCM(14,16)=112 and GCF(14,16)=2
2. LCM(12,18)=36 and GCF(12,18)=6
3. LCM(36,27)=108 and GCF(36,27)=9

Handshake Problem

Introduction: Number theory can become really interesting when you have time to discover patterns and unexpected relationships! We will explore some relationships with this activity. Hopefully you will discover some unexpected ones, maybe some you already know!

Investigation:

Part 1.

If everyone in this room shakes hands with everyone else, how many handshakes will there be? Make a table to keep track.

People	Handshakes

Part 2.

1. Predict the number of handshakes more than _____, less than ________.
2. How did you determine the total number of handshakes?
3. Describe patterns you noticed in the investigation. Any unexpected relationships?
4. How can you find the number of handshakes for any number (n) of people?

Reflection

1. What skills do students need to do this?
2. How do the skills learned in this investigation related to future learning?
3. Where does this investigation fit in the learning progression?

Testing for Primality…. What should I check? When can I stop?

Introduction: Prime numbers are important in mathematics. They form the basic building blocks for the natural numbers through the Fundamental Theorem of Arithmetic. They are important for efficiently finding greatest common factors and least common multiples. They form the basis for cryptology which allows us to safely make purchases on the internet. But how can we "test" a number to determine if it is prime or not, that is, how can we test for "primality"?

Investigation: Consider the number 97. Is it prime? Is it composite? How can we tell? One way to "test" a number to see if it is prime is through a process called trial division. In trial division, a candidate for primality is divided by a whole number less than itself to test if the whole number is a factor. So, for the number 97 we might carry out the following sequence of trial divisions:

n	$97 \div n$	n	$97 \div n$	n	$97 \div n$	n	$97 \div n$	n	$97 \div n$
1	97	21	4.619	41	2.366	61	1.59	81	1.198
2	48.5	22	4.409	42	2.31	62	1.565	82	1.183
3	32.333	23	4.217	43	2.256	63	1.54	83	1.169
4	24.25	24	4.042	44	2.205	64	1.516	84	1.155
5	19.4	25	3.88	45	2.156	65	1.492	85	1.141
6	16.167	26	3.731	46	2.109	66	1.47	86	1.128
7	13.857	27	3.593	47	2.064	67	1.448	87	1.115
8	12.125	28	3.464	48	2.021	68	1.426	88	1.102
9	10.778	29	3.345	49	1.98	69	1.406	89	1.09
10	9.7	30	3.233	50	1.94	70	1.386	90	1.078
11	8.818	31	3.129	51	1.902	71	1.366	91	1.066
12	8.083	32	3.031	52	1.865	72	1.347	92	1.054
13	7.462	33	2.939	53	1.83	73	1.329	93	1.043
14	6.929	34	2.853	54	1.796	74	1.311	94	1.032
15	6.467	35	2.771	55	1.764	75	1.293	95	1.021
16	6.063	36	2.694	56	1.732	76	1.276	96	1.01
17	5.706	37	2.622	57	1.702	77	1.26	97	1
18	5.389	38	2.553	58	1.672	78	1.244		
19	5.105	39	2.487	59	1.644	79	1.228		
20	4.85	40	2.425	60	1.617	80	1.213		

1. Is 97 prime? Is 97 composite? How do you know?

2. Why was it unnecessary to trial divide by 4 after having trial divided by 2? Cross out other numbers that were unnecessarily by a similar argument. What type of numbers are left?

3. Look closely at the cells n=9 and n=10. What happens at this point in the trial division process? Is it necessary to check any numbers greater than 9? Why or why not?

Putting It Together: The previous exploration was meant to give you an opportunity to discover an efficient method for testing for primality. See if you can complete the following two statements to put-into-words your discoveries. Use the pink "Discovery Key" to verify your discoveries

Discovery 1: When testing for primality using ordered trial divisions, it is only necessary to test factors that are……because…….

____

____

____

Discovery 2: When testing a number for primality using ordered trial divisions, it is only necessary to test factors that are less than…..because…..

____

____

____

Summarize your discoveries into a single statement that describes an efficient method for testing for primality using trial division:

____

____

Use your method to test the following numbers for primality:

1. 173 Prime / Composite
2. 171 Prime / Composite
3. 331 Prime / Composite
4. 323 Prime / Composite
5. 463 Prime / Composite
6. 441 Prime / Composite
7. 587 Prime / Composite
8. 529 Prime / Composite
9. 887 Prime / Composite
10. 899 Prime / Composite

Reflection

1. What skills do students need to do this?
2. How do the skills learned in this investigation related to future learning?
3. Where does this investigation fit in the learning progression?

Discovery 1: When testing for primality using trial division, it is only necessary to test numbers that are prime. This is because every composite number can be written as the product of primes that precede it. In our example, 2 was not a factor of 97. This result automatically rules out 6 as a factor because every multiple of 6 is also a multiple of 2. This is because $6 = 2 \cdot 3$. So, we do not have to check 6 (or any other even numbers) after we have checked 2. Every composite number is similarly ruled out!

Discovery 2: When testing a number for primality using trial division, it is only necessary to check for factors that are less than the square root of the number. Let's look at why this is true using our test case of 97. First we compute $\sqrt{97} \approx 9.85$. The table demonstrates that dividing 97 by a number less than 9 results in a quotient greater than 9. Conversely, dividing 97 by a number greater than 9 results in a quotient less than 9. So, 9 is the "tipping point" in terms of how the divisor and quotient "share" their portions of 97. Why can we stop checking at 9? Well, because any number that evenly divides 97 will have a quotient that also evenly divides 97, after 9 all the available quotients are eliminated because no number less than 9 evenly divides 97. So, we can stop checking after 9.