Exploring Space Through MATH

Applications in Algebra 2

EDUCATOR EDITION

Recycling Urine In Space

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will

- create and solve equations to make decisions,
- simulate probabilistic situations, and
- make decisions based on probabilistic outcomes.

Prerequisites

Students should have prior knowledge of creating and solving equations that model real-life scenarios, random sampling, and creating and interpreting graphical displays.

Background

This problem applies mathematical principles in NASA's human spaceflight.

The International Space Station (ISS) can accommodate up to six crewmembers for an extended period of time. During crewmember stays, basic resources such as air, food, and water (which are critical to crew survival and mission success) must be resupplied from the ground. Resupply of these resources, especially water, can be very costly. In an effort to allow humans to venture further into space without the need for resupply, NASA has developed innovative ways to use systems onboard the vehicle.

One such ISS system is the Water Recovery System. In this system, the Urine Processor Assembly (UPA) recovers water from the crew's urine so that it can be processed using the Water Processor Assembly (WPA). The WPA mixes this water with condensed atmospheric moisture to process water that is suitable for drinking by the crew.

During this process, urine is collected, preserved, and then boiled and condensed to produce pure water and a concentrated salt solution called brine. The pure water is fed into the WPA while the brine is collected in a tank called the Advanced Recycle Filter Tank Assembly (ARFTA). At the end of each process cycle, the ARFTA is removed from the system and is drained. If the concentration of salts in the brine solutions gets too high, precipitates (or solids) can form. These solids can potentially obstruct the plumbing in the UPA, causing damage to other system components.

Key Concepts
Probability and simulation

Problem Duration
80 minutes

Teacher Prep Time
15 minutes

Technology
Computer with Internet access and projector, TI-Nspire™ handhelds

Materials
- TI-Nspire Student Edition*: Alg2-ST_Nspire_RecUrine.pdf*
- TI-Nspire Student document: *Alg2-ST_RecUrine.tns*
- Video: *Our World: Recycling on the International Space Station*

Skills
Proportional reasoning, solve equations, random sampling on the TI-Nspire, manipulate graphical displays on TI-Nspire

NCTM Principles
- Data Analysis and Probability
- Algebra
- Problem Solving
- Communication
- Connections

Common Core Standards
- Statistics and Probability
- Algebra

An example of this occurred on the ISS in 2009, when calcium sulfate precipitation formed in the distillation assembly, requiring a shutdown of the UPA. While the UPA was offline, the crew was required to use the reserve water supply—replenished only by the launch of additional water. Prevention of precipitate formation in the UPA is critical because replenishing the water and repairing the system can prove to be extremely costly.

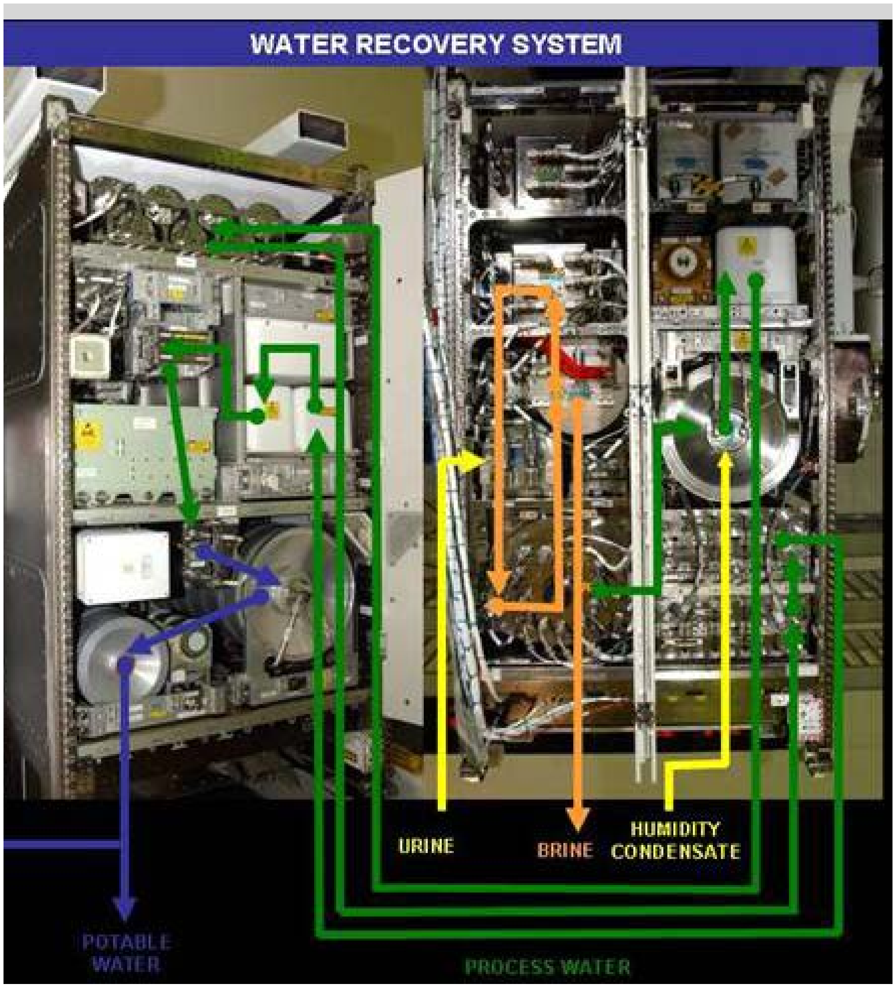


Figure 1: Water Recovery System

NCTM Principles and Standards

Data Analysis and Probability

- Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments
- Understand histograms, parallel boxplots, and scatterplots and use them to display data
- Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions
- Use simulations to construct empirical probability distributions

Algebra

- Use symbolic algebra to represent and explain mathematical relationships

Problem Solving

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other applications

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers and others
- Use the language of mathematics to express mathematical ideas precisely
- Analyze and evaluate the mathematical thinking and strategies of others

Connections

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Common Core Standards

Statistics and Probability

- Make inferences and justifying conclusions
- Use probability to make decisions
- Interpret categorical and quantitative data

Algebra

- Create equations

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

1 – Engage (15 minutes)

- Play the video, *Our World: Recycling on the International Space Station* (7:21 minutes), accessible at: http://youtu.be/2XIK5wXFQDY.
- With students in groups of 3–4, ask them to review and discuss the main points of the background section and the video for several minutes to be sure that they understand the material. Circulate to help facilitate discussion in small groups. Ask if any group needs clarification.

2 – Explore (15 minutes)

- Distribute the TI-Nspire™ handhelds with the loaded student document, *Alg2-ST_RecUrine.tns*.
- Distribute copies of *Recycling Urine in Space* Student Edition.
- Have students work as a class to answer questions on pages 1.3–1.9.

3 – Explain (20 minutes)

- Have students work in groups to answer questions on pages 2.1–2.5.
- Call on students to give their answers and discuss.

4 – Extend (15 minutes)

- Have students work in groups to answer questions on pages 2.7–2.12.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (15 minutes)

- Have students work independently to complete questions 3.3–3.5
- This may be done in class or assigned as homework.

Recycling Urine in Space

Solution Key

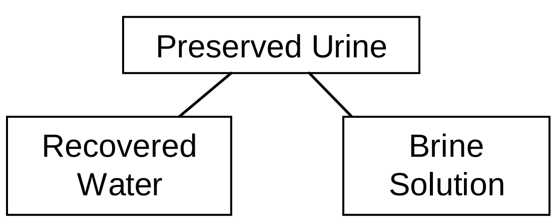
Note to teacher: Instruct students to open TI-Nspire document, Alg2-ST_RecUrine.tns on their handhelds. You may choose to have students record their work directly in their handhelds or on their worksheets in the provided spaces. A solution key is provided below, as well as in the educator's version of the TI-Nspire document, Alg2-ED_RecUrine.tns.

Directions: On the TI-Nspire handheld, open the document, Alg2-ST_RecUrine. Read through the problem set-up (page 1.2) and complete the embedded questions.

Note to teacher: Prior to beginning the lesson, students should seed their calculators with a unique value so that all students' answers will be different. To seed the calculator, on a calculator page, have students use the menu and select ***menu > probability > random > seed****. Then have students enter a unique random number. The syntax will appear as follows: RandSeed 12345. A potential teaching note would be to combine all student data and compute the probabilities based on the large numbers of trials.*

Problem

To recover water from urine, the Urine Processor Assembly uses a purification process, where water is evaporated from the urine. The residual remaining from this process is called brine—a solution that contains a small amount of water and high concentrations of salts, like calcium and sulfate. The UPA can be set to recover different amounts of water from the urine. For example, if it is set at 80 percent water recovery, then 80% of the water will be removed from the urine processed. When the percentage of water recovered increases, the concentrations of the salts in the brine increase. If the concentration of calcium in the brine reaches 950 mg/L, it will form calcium sulfate and precipitate (or fall out of solution).



Embedded Questions

Since successful operation of the UPA is critical to the safety and health of the crewmembers on the ISS, NASA scientists are researching ways to predict when calcium sulfate might cause a problem with the UPA. Urine samples from one hundred ISS astronauts have been collected and analyzed to measure urinary calcium concentrations.

The data provided in the table on page 2.2 models the calcium concentrations in the urine of one hundred astronauts. Because concentrations can vary greatly throughout the day, each value is expressed as the average concentration in a 24-hour period. Concentrations of calcium are provided as mg/L.

It is not practical to compute the exact probability of exceeding the calcium concentration. Instead simulate the random selection of three astronauts for a mission, and then determine the probability of having a calcium concentration too high for successful UPA operation.

1.6 Why is it important to randomize when simulating samples from a population?

Randomization is important in order to achieve a representative sample of all possible combinations of astronauts that could be on the ISS. Each group of three should have an equal opportunity of being chosen.

1.7 Should sampling be performed with or without replacement from this population of astronauts? Explain your response.

An astronaut cannot be selected twice since this does not make sense when making the three-member crews. Therefore, sampling without replacement should be used.

1.8 What other assumptions must be made when taking samples from the inflight calcium concentration data?

Samples from previous calcium concentration data is representative of the concentration of calcium in the urine for the astronauts on any subsequent flight.

Directions: Answer questions 2.1–2.6 in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth, and label with the appropriate units.

2.1 Since there are at least three astronauts on the ISS for any given mission, on page 2.2, generate the random samples of calcium concentrations of three astronauts and calculate the arithmetic mean of your three astronauts. Generate the sample by typing the command **=randsamp(cal_in_urine,3,1)** in the formula cell in column B.

Note to teacher: Discuss with students what the syntax means in the sample command. Three ("3") refers to the number of astronauts; one ("1") indicates that the sample is without replacement.

Students' answers will vary. One example is 104.034 mg/L, 182.548 mg/L, and 193.381 mg/L.

2.3 Calculate the arithmetic mean concentration of calcium in mg/L for the three astronauts.

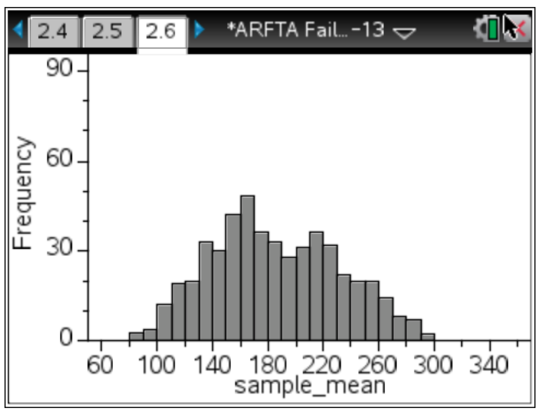
$$104.034\frac{mg}{L}+182.548\frac{mg}{L}+193.381\frac{mg}{L}=479.963\frac{mg}{L}$$

$$479.963\frac{mg}{L}\,/\,3=159.988\frac{mg}{L}$$

2.4 Calculate the theoretical probability of exceeding a certain concentration by summing the number of sample mean concentrations and dividing by the total number of samples in the simulation. To do this, randomly select five hundred groups of three astronauts and compute the average calcium concentration for each group of three.

In the table, on page 2.2, enter **=mean(randsamp(cal_in_urine,3,1))** in cell C1. With your mouse still in C1, select **menu > data > fill**, and press the down arrow until you have selected down to row 500. Then press enter.

2.5 Create a histogram on page 2.6 using your list of samples. Set the bin width at 10, and align the graph in multiples of 10. Do this by selecting **ctrl > menu > Bin Settings**.



Directions: Answer questions 2.7–2.12 in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth, and label with the appropriate units.

2.7 To compute the probability of exceeding the calcium concentration required to cause a failure, use the histogram to count the number of times the average concentration exceeds the threshold value on page 2.8. The threshold value in column B refers to the maximum concentration of calcium in the urine that would cause precipitation to occur. Complete the percent risk column for each level of percent recovery listed in the table. Use the number of times an average greater than the threshold occurred divided by the total number of samples in the simulation.

Note to teacher: Students may need to be informed to hover over each set of data on the histogram to collect the total number of times an average is greater than the threshold.

2.6 | 2.7 | 2.8 | *ARFTA Fail...-13

	A recovery	B threshold	C risk	D
◆				
1	85.	140.	77.6	
2	80.	190.	55.4	
3	75.	240.	22.2	
4	70.	290.	0.4	
5	65.	330.	0.	
6				

A recovery

Students' answers will vary.

2.9 What is the probability of UPA failure if the water recovery rate is 85%?

Students' answers will vary.

2.10 What is the probability of UPA failure if the water recovery rate is 70%?

Students' answers will vary.

Failure of the UPA is attributed to precipitation in the Advanced Recycle Filter Tank Assembly (ARFTA). Since the ARFTA is changed every 15 days, the risks that were calculated on page 2.8 pertain to a 15-day period.

2.11 Using your answers from page 2.8, compute the probability of *not* having an ARFTA failure during a 6-month mission if the recovery rate is 70%.

Students' answers will vary slightly. However, using the risk above of 0.4%: $p(\text{no failure in 6 months}) = (1-0.004)^{12} = 0.996^{12} = 0.953$ *or 95.3% chance of no failure*

2.12 The UPA operated for six months on the ISS without a failure while recovering 85% of the urine volume as water. Given the probability of failure you calculated using 85% recovery, is it surprising the ISS did not experience a failure? Explain your answer.

Assuming that the months are independent, the probability of going six months without a failure is computed as $(1-0.776)^{12} = 0.224^{12}$ *or* 1.596×10^{-8} *(basically zero). So, it is surprising that the UPA functioned properly for that many months without a failure.*

Directions: Complete questions 3.3–3.5 independently. Round all answers to the nearest whole number, and label with the appropriate units.

Use the following information to answer the questions. Decide the percent recovery at which the UPA should operate to save the most time for the astronauts, but still have the least risk of failure.

- It costs $50,000 per kilogram of mass moved from Earth to the ISS.
- Water weighs 1 kilogram per liter.
- Each astronaut needs 2.5 liters of water per day.
- Each ARFTA costs approximately $1 million dollars to produce.
- Only three ARFTAs exist and all three are located on the ISS.
- Each ARFTA has a mass of about 50 kilograms.
- An additional 50 kilograms of mass is added to each ARFTA when taking one from Earth to the ISS.

3.3 How much would it cost to replenish the water consumed from the reserves if an ARFTA failed and was down for two days before the three-person crew could replace it?

(2.5)(3)(2)(50,000) = $750,000

3.4 How much would it cost to replace the failed ARFTA with a new one from Earth?

$1,000,000 + 50(50,000) + 50(50,000) = $6,000,000

3.5 Using the probabilities of failure for each recovery percentage, and assuming six months between replenishing missions from Earth, decide the percent recovery at which the UPA should operate to maximize the efficiency, while keeping in mind the consequences of a failed ARFTA. Justify your response using the probabilities you calculated in the table on page 2.8.

Considering a failed ARFTA costs about $6 million dollars to replace (and if the system is down for even two days), add another $750,000 to replace the water. To err on the side of caution, choose either 65% or 70% recovery due to the low risk of failure.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

NASA Experts

Daniel B. Gazda, Ph.D – Senior Environmental Scientist, Water and Food Analysis Laboratory, NASA Johnson Space Center

Torin McCoy – Environmental Operations Lead, Toxicologist, NASA Johnson Space Center

Mathematics Educator

Curtis Brown – Statistics Coordinator, National Math and Science Initiative