FRAMEWORK for ANALYZING STUDENT WORK

Categories	Characteristics of Categories

Level 0   **No justification given**. When asked to justify, students either said they "just knew" or restated their answers

Level 1:   **Appeal to external authority or rote procedures**. While students at all levels like a teacher's validation, students at this level relied exclusively on external authority and memorized (mis-memorized) procedures. They did not understand, nor were they interested in, why the idea was true

Level 2:   **Naïve reasoning, usually with incorrect conclusions**. Although the students use some deduction, the arguments started with an analogy or with something the students remembered hearing, often incorrectly. As a result, students came to a mostly incorrect conclusion (not just a computational error). If students did reach a correct conclusion, it was for the wrong reasons.

Level 3-A: **Inductive reasoning A (examples, experiments, or empirical demonstrations).** Students concluded that an assertion was valid on the basis of a pattern or on a small number of cases. While students at all levels used examples to understand, students at this level thought that the examples were a proof.

Level 3-B: **Inductive reasoning B (investigating if and why a generalization held).** While still focusing on examples, the students began to generalize by looking for counterexamples, cases of examples, or extreme cases. They showed some doubt that a pattern would necessarily hold indefinitely.

Level 4:   **Transition to formal reasoning (elements of formal reasoning but without the precision)**. Students used an informal version of a Level 5 argument.

Level 5:   **Formal reasoning (acceptable to a mathematician).** Students' arguments were precise and acceptable to a mathematician. Examples include theoretical probability and acceptable proof techniques-- such as proof by cases or proof by mathematical induction.