The Unit Circle

Math Objectives

- Students will describe the relationship between the unit circle and the sine and cosine functions.
- Students will describe the shape of the sine and cosine curves after unwrapping the unit circle.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will attend to precision (CCSS Mathematical Practice).

Vocabulary

- continuous function
- periodic function
- unit circle

About the Lesson

- This lesson involves using a slider to change the measure of the central angle of a unit circle and making connections with right triangle trigonometry.
- As a result, students will:
  - Discover the relationships between the right triangle and the line segments creating the sine and cosine functions.
  - Observe that as the measure of the central angle changes line segments representing the length of a highlighted leg of the right triangle in the unit circle appear on the screen.
  - Need prerequisite knowledge of right triangle trigonometry and continuous functions.

Related Lessons

- Prior to this lesson: Radian Measure
- After this lesson: Trigonometric Transformations

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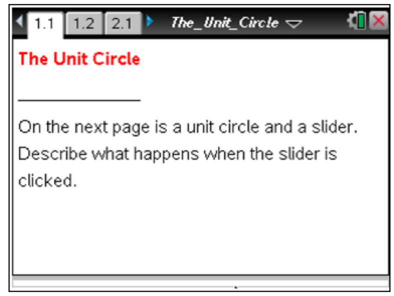
- Use Quick Poll to assess students' understanding.
- Use Class Capture to share students' work.
- Collect student documents and analyze the results.
- Utilize Class Analysis to display students' answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- The_Unit_Circle_Student.pdf
- The_Unit_Circle_Student.doc

TI-Nspire document

- The_Unit_Circle.tns

Discussion Points and Possible Answers

> **Tech Tip:** Press esc to hide the entry line if students accidentally press tab or ctrl G to reveal the entry line.

> **Tech Tip:** If students tap the screen twice in the white space of a graph, the entry line will appear so that a function can be graphed. If students inadvertently do this, they can simply tap on the screen and the entry line will be removed.

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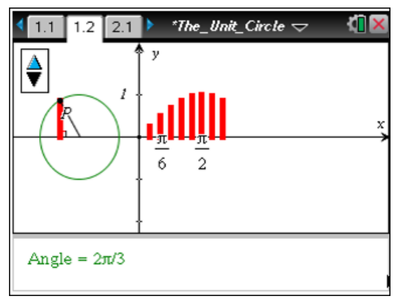

1. The circle pictured is called a unit circle. Why is that term used?

   <u>Answer:</u> It is called a unit circle because its radius is one unit.

2. Use the slider, to make three segments appear. What is the relationship between the right triangle in the unit circle and the vertical line segments?

   <u>Answer:</u> The length of the highlighted vertical leg in the triangle is equal to the length of the corresponding line segment on the graph to the right.

> **CCSS Mathematical Practice:** Students will reason abstractly and quantitatively. Mathematically proficient students are able to explain how the triangle in the unit circle and the vertical segments on the sine curve are connected and what they mean, not just how to draw the sine curve.

3. Will the lengths of the line segments continue to increase? Why or why not?

   <u>Answer:</u> The lengths of the line segments will increase until the angle becomes $\frac{\pi}{2}$. Then the lengths of the line segments will decrease until the angle becomes π. This occurs because the length of the highlighted leg decreases after it reaches the "top" of the circle.

4. Continue to use the slider until you obtain values of θ such that $\frac{\pi}{2} < \theta < \pi$. Are any of the line segments the same size? Why or why not?

Answer: Yes. The line segments that are the same height have the same reference angle. Students might also say that the lengths of the legs in the circle have matching lengths when the triangle is reflected over the *y*-axis.

5. Use right triangle trigonometry to explain the relationship between the angle θ and the highlighted leg of the right triangle in the unit circle. What trigonometric function can be represented by the length of the leg of the right triangle?

Answer: $\sin\,\theta = \dfrac{\textbf{length of vertical leg}}{\textbf{hypotenuse}}$

$$\sin\,\theta = \frac{y}{1}$$

$$y = \sin\,\theta$$

Teacher Tip: Remind students that the hypotenuse is equal to 1, because it is a unit circle and has a radius of 1.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

6. Use the slider until you obtain values of θ such that $\pi < \theta < \frac{3\pi}{2}$. Explain the placement of the line segments.

Answer: The line segments lie below the x-axis but are the same lengths as before gradually increasing in length until $\frac{3\pi}{2}$. The line segments that are the same height have the same reference angle (even though they are below the x-axis).

7. Continue to use the slider until $\theta = 2\pi$ to graph a continuous function. What do the coordinates of the points on the continuous function represent?

Answer: The *x*-coordinates represent the measure, in radians, of θ, the central angle. The *y*-coordinates represent the measure of sin θ.

8. Write an equation of the continuous function graphed.

   Answer: $y = \sin x$

9. If we continued to graph the function for values of θ such that $2\pi < \theta < 4\pi$, describe what you would expect to see. Explain your reasoning.

   Answer: We would expect to see the curve repeat another cycle as the point continues around the circle again. Note that the sine function is periodic.

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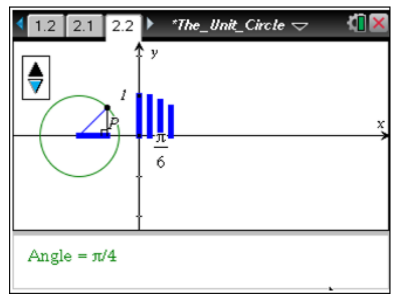

10. Use the slider until three segments appear. What is the relationship between the right triangle in the unit circle and the vertical line segments?

    Answer: The length of the highlighted leg in the triangle is equal to the length of the corresponding line segment on the graph to the right.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

11. Use right triangle trigonometry to explain the relationship between the angle θ and the highlighted leg of the right triangle in the unit circle. What trigonometric function can be represented by the length of the leg of the right triangle?

    Answers:

    $$\cos\theta = \frac{\text{length of horizontal leg}}{\text{hypotenuse}}$$

    $$\cos\theta = \frac{x}{1}$$

    $$x = \cos\theta$$

12. Use the slider until you obtain values of θ such that $\frac{\pi}{2} < \theta < \pi$. Explain the placement of the line segments.

Answer: The line segments lie below the x-axis. The *y*-coordinates of the endpoints of the line segments are equal to the *x*-coordinates of the point as it moves around the unit circle. When the point reaches the second quadrant, its *x*-coordinates are negative.

13. Continue to use the slider until $\theta = 2\pi$ to graph a continuous function. What do the coordinates of the points on the continuous function represent?

    Answer: The *x*-coordinates represent the measure, in radians, of θ, the central angle. The *y*-coordinates represent the measure of cos θ.

14. Write an equation of the continuous function graphed.

    Answer: $y = \cos x$

15. How would you explain to a friend how to graph this function accurately on graph paper without using technology?

    Sample Answers: Answer will vary. However, students should be led to conclude that the function $y = \cos \theta$ has a maximum value of 1 when $x = 0$ and when $x = 2\pi$. When $x = \pi$, the cosine function reaches its minimum value of –1. The cosine function has zeroes at $x = \pi/2$ and $x = 3\pi/2$. Once the student plots these points, it is fairly simple to "connect the dots" and produce a cosine curve.

    CCSS Mathematical Practice: Students will attend to precision. Mathematically proficient students will be able to communicate precisely to others how to graph the cosine function.

16. In terms of θ, represent the *x*- and *y*-coordinates of the point moving around the unit circle. Explain your reasoning.

    Answer: The *x*-coordinate of the point on the unit circle represents cos θ and the *y*-coordinate represents sin θ. Students might say that the sine curve was formed by connecting the endpoints of segments whose lengths were equal to the lengths of the highlighted vertical leg of the right triangle and that $y = \sin \theta$. The cosine curve was formed by connecting the endpoints of segments whose lengths were equal to the lengths of the highlighted horizontal leg of the right triangle and $x = \cos \theta$.

    Teacher Tip: Students might want to refer to their answers for questions 5 and 11 to help answer this question.

Wrap Up:

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The relationship between the unit circle and the sine and cosine functions.
- The shape of the sine and cosine curves created by unwrapping the unit circle.

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Note 1

Questions 3, 4, 5, *Quick Poll*

You can send a Quick Poll to assess students' understanding of the unit circle as it is unfolding. Challenge students to answer the following:

- When will the line segments start to decrease, and when will they increase again?
- List two values of θ that have line segments of the same length.
- Which trigonometric function represents the length of the leg of the right triangle?

Note 2

Question 11, *Quick Poll*

You can send a Quick Poll in advance of question 11 to see if students can relate the length of the leg to the trigonometric function, cos θ.