

What Does Number Sense for Rational Numbers Look Like?

Terry Wyberg
University of Minnesota

Sue Ahrendt
University of Wisconsin – River Falls

NCTM Annual Meeting
New Orleans, LA

April 11, 2014

- www.cehd.umn.edu/ci/rationalnumberproject/
- <http://www.cehd.umn.edu/ci/rationalnumberproject/rnp1-09.html>
- <http://www.cehd.umn.edu/ci/rationalnumberproject/rnp2-09.html>

A. Watching Children

Student	Problem	Student knows ...	Student does not know ...
Madison (5 th grade) #356 25 seconds	$\frac{1}{2} + \frac{1}{3}$		
Sean (5 th grade) #370 1 min 35 seconds	$\frac{1}{2} + \frac{1}{3}$		
Johanna (3 rd grade) #352 1 min 3 seconds	$\frac{1}{2} + \frac{1}{4}$		
Felicia (2 nd grade) #329 1 min 39 seconds	$\frac{3}{4} + \frac{1}{2}$		
Leah (5 th grade) 1 min 40 seconds	$\frac{3}{4} + \frac{1}{6}$		

B. Developing Mental Images

Using Manipulatives

Name _____

Lesson 2
Student Page A

Directions:

The class will work together in groups or in pairs on these problems. Answers are to be given orally or by drawing a picture. On some of the problems children may want to use the fraction circles to help solve the problem.

1. The yellow piece is the unit.
How many blues cover the yellow piece? _____
1 blue is _____ of the yellow.
(say the word)
2. The blue piece is the unit.
How many reds cover the blue piece? _____
1 red is _____ of the blue.
(say the word)
3. The brown piece is the unit.
How many reds cover the brown piece? _____
1 red is _____ of the brown.
(say the word)
4. What color is 1-half of the blue? _____
5. What color is 1-third of the yellow? _____
6. Draw a picture of a pizza. Show on your drawing the pizza cut into 2 fair shares.

Each fair share is _____ of the whole pizza.
(say the word)

Name _____

Lesson 6
Student Page B

Directions:

Use fraction circles to compare the two fractions. Circle the larger fraction.

$\frac{1}{3}$

$\frac{2}{3}$

$\frac{1}{4}$

$\frac{3}{4}$

$\frac{4}{5}$

$\frac{3}{5}$

$\frac{2}{7}$

$\frac{2}{6}$

$\frac{6}{7}$

$\frac{2}{7}$

$\frac{4}{12}$

$\frac{4}{15}$

$\frac{8}{12}$

$\frac{11}{12}$

$\frac{6}{7}$

$\frac{3}{7}$

$\frac{2}{7}$

$\frac{2}{9}$

$\frac{9}{10}$

$\frac{3}{10}$

$\frac{4}{8}$

$\frac{4}{6}$

Try these without
manipulatives.

$\frac{2}{3}$

$\frac{2}{8}$

$\frac{13}{100}$

$\frac{27}{100}$

$\frac{7}{10}$

$\frac{7}{9}$

$\frac{9}{10}$

$\frac{9}{100}$

Informal Ordering Strategies

Determine the larger number by picturing them in your mind. Circle the larger number in each pair.

I. Same Numerator

$$\frac{3}{17} \quad \frac{3}{23}$$

II. Transitive (1 minute 14 seconds)

$$\frac{7}{8} \quad \frac{1}{4}$$

III. Residual (2 minutes)

$$\frac{2}{3} \quad \frac{11}{12}$$

IV. Same Denominator

$$\frac{7}{23} \quad \frac{5}{23}$$

Formal Ordering Strategies

V. Using Equivalent Fractions

$$\frac{3}{5} \quad \frac{2}{3}$$

VI. Using Cross-Multiplication

$$\frac{7}{13} \quad \frac{5}{11}$$

C. Progression for Finding Exact Sums and Differences

CCSS.4.NF.B.3.A Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

CCSS.4.NF.B.3.B Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.

CCSS.4.NF.B.3.C Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

CCSS.4.NF.B.3.D Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

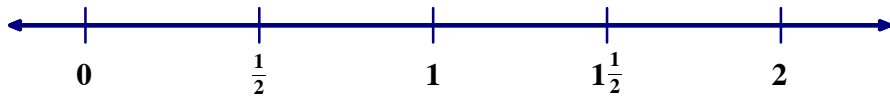
CCSS.5.NF.A.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example,* $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. *(In general,* $a/b + c/d = (ad + bc)/bd$.*)*

CCSS.5.NF.A.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result* $2/5 + 1/2 = 3/7$, *by observing that* $3/7 < 1/2$.

Problem A

Jayna has $\frac{3}{8}$ of a cup of flour. Sarah gives her $\frac{1}{4}$ of a cup of flour. How much flour does Jayna have now?

I. Estimate



II. Exact Using Fraction Circles

III. Using Language

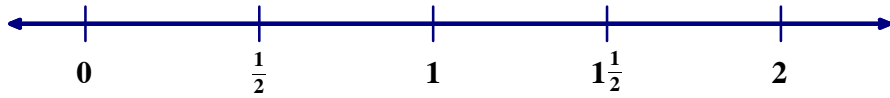
IV. Exact Using Algorithm

V. Exact Using a Number Line

Problem B

Malita has $1\frac{3}{4}$ gallons of apple juice. She gives $\frac{1}{3}$ of a gallon of juice to LaRue. How much juice does Malita have left?

I. Estimate



II. Exact Using Fraction Circles

III. Using Language

IV. Exact Using Algorithm

V. Exact Using a Number Line