# TEACHING RATIONALLY
## DEVELOPING UNDERSTANDING OF FRACTION OPERATIONS

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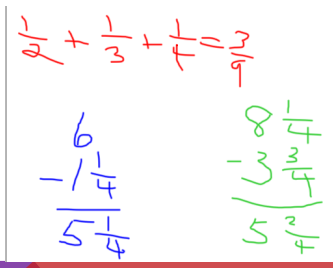
## NATIONAL ASSESSMENT OF EDUCATIONAL PROGRESS

Estimate the answer: $\frac{12}{13} + \frac{7}{8}$

| Responses | Percent responding (Age 13) |
|---|---|
| 1 | 7 |
| 2 | 24 |
| 19 | 28 |
| 21 | 27 |
| I don't know | 14 |

NAEP 1981

## STUDENT WORK




## ADDITION OF FRACTIONS WITH CONCEPTUAL UNDERSTANDING

IMAP: Integrating Mathematics and Pedagogy
San Diego State University
http://www.sci.sdsu.edu/CRMSE/IMAP/video.html

## CCSS FOR MATHEMATICS

GRADES 4 - 7

Apply and extend
previous understandings
of operations with whole numbers
to add, subtract, multiply, and divide fractions.

## CCSS MATHEMATICAL PRACTICES

MP 4: Model with Mathematics

- Apply mathematics to solve problems arising in everyday life
- Interpret mathematical results in the context of the situation
- Reflect on whether the results make sense

## FRACTION OPERATIONS

How do we develop conceptual understanding?

- Meaning of operations
- Contextual problems
- Models
- Algorithms only after students understand meanings, and can use models to solve problems in context.

## CUISENAIRE RODS – LET'S GET TO KNOW THEM

What is the relationship between orange and yellow? Between orange and red? Between orange and white? Explore other relationships.

Suppose brown is the whole.
If brown is 1, what could you call purple?
red? white?

ETA

## CUISENAIRE RODS – LET'S GET TO KNOW THEM

Choose a rod and challenge your neighbor:
If this rod is 1, how can you name other rods?

I'm thinking of a rod.
Purple is two-thirds of my rod.
What color is my rod?

ETA

## CUISENAIRE RODS – LET'S GET TO KNOW THEM

Let's invent a new rod: red-orange.
We can call it *rorange*!

Suppose rorange is 1.
How can you name some of the other rods?

ETA

## CUISENAIRE RODS – LET'S GET TO KNOW THEM

Suppose rorange is 1.
How can you name some of the other rods?

ETA

## ADDITION

Build unit fractions.
Define the whole.
Find a rod to represent the fraction.
Make a train.

$$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$$

$$\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$$

What is the pattern?
Can you generalize a rule for adding fractions with the same denominators?

## CONTEXTUAL PROBLEM

Lisa lives 1/2 of a mile from the French Quarter.
Jenny lives 1/3 of a mile farther.
How far is Jenny from the Quarter?

How would you use Cuisenaire rods to model this?

- What is the operation?
- How can you model 1/2? 1/3?
- What's the whole? How do you choose?

## ADDITION

Add fractions with unlike denominators.

$$\frac{1}{4}+\frac{1}{2}$$

$$\frac{1}{2}+\frac{1}{3}$$

What do you have to consider?
Can you generalize a rule?

## CONTEXTUAL PROBLEM

Marie lives 1/2 of a mile from the beach. Joe lives 1/4 of a mile from the beach. How much closer to the beach is Joe than Marie?

What is the meaning?

How can you model it?

## SUBTRACTION

What does 10 - 4 mean?

- Take away or separate
- Comparison
- Missing addend

## SUBTRACTION

Write a contextual problem that could be solved by subtracting

$$\frac{1}{2}-\frac{1}{3}$$

## SUBTRACTION

$$\frac{1}{2}-\frac{1}{3}$$

Pat has one half of a pie left in the refrigerator.
For lunch Pat eats one third of the pie.
How much pie does Pat have left?

## MULTIPLICATION

What does 4 x 6 mean?

- Equal groups: 4 groups of 6
- Repeated addition: 6 + 6 + 6 + 6
- Area


## MULTIPLICATION

What is the meaning of each?

Can you build a model? Can you tell a story?

$$3 \times \frac{1}{4}$$

$$2 \times 1\frac{3}{5}$$

## MULTIPLICATION

What is the meaning of each?

Can you build a model? Can you tell a story?

$$\frac{1}{4} \times 8$$

$$\frac{2}{3} \times 9$$

## MULTIPLICATION

What is the meaning of each?

Can you build a model? Can you tell a story?

$$\frac{1}{3} \times \frac{3}{5}$$

$$\frac{2}{3} \times \frac{3}{5}$$

## HOW MUCH IS $\frac{3}{4}$ OF $\frac{2}{3}$ ?

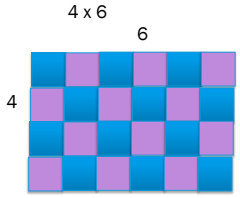
John and his little brother mow the lawn together. John agrees to mow $\frac{2}{3}$ of the lawn. When they stop for lunch, John sees that he has finished $\frac{3}{4}$ of his section. How much of the lawn has John done?

Start with a model of $\frac{2}{3}$. Partition it into fourths.

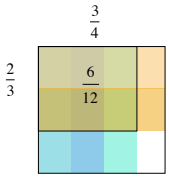
Hint: rorange is a good whole to start with.

## AREA MODEL

4 x 6




$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$



## DIVISION

What does 12 ÷ 3 mean?

- Equal groups: How many groups of 3 are in 12?
- Fair shares: How can 12 be shared fairly among 3?
- Repeated subtraction: How many 3s can I take from 12?

What does 3 ÷ $\frac{1}{2}$ mean?

Can you model it? Can you tell a story?

## CONTEXTUAL PROBLEM

At the end of the day, a bakery had one-half of a loaf of french bread left. The three employees split it up, with each taking home the same amount. How much of a loaf did each employee take home?

•What is the operation?
•How would you use Cuisenaire rods to model this?

## CONTEXTUAL PROBLEM

A highway crew can resurface $\frac{2}{3}$ of a mile of highway a day.

There are $8\frac{1}{4}$ miles in the project.

How long will the job take them?

•What is the operation?
•How would you use Cuisenaire rods to model this?

## DEVELOPING THE ALGORITHM: DIVISION

How can you think about

*A ÷ B ?*

How many groups of *B* are in *A*?

## DEVELOPING THE ALGORITHM: DIVISION

$1 \div \frac{1}{2}$ $\quad$ $1 \div \frac{3}{4}$

$1 \div \frac{1}{3}$ $\quad$ $1 \div \frac{3}{5}$

$1 \div \frac{2}{3}$ $\quad$ $1 \div \frac{4}{5}$

What is the pattern?
Can you develop an algorithm?

## DEVELOPING THE ALGORITHM: DIVISION

$$A \div B = \frac{A}{B}$$

$$\frac{A}{B} = \frac{A \bullet \frac{1}{B}}{B \bullet \frac{1}{B}} = \frac{A \bullet \frac{1}{B}}{1} = A \bullet \frac{1}{B}$$

$$A \div B = A \bullet \frac{1}{B}$$

## DEVELOPING THE ALGORITHM: DIVISION

$$\frac{A}{B} \div \frac{C}{D} = \frac{A \bullet D}{B \bullet D} \div \frac{B \bullet C}{B \bullet D} = \frac{AD \div BC}{BD \div BD}$$

$$AD \div BC = \frac{AD}{BC} = \frac{A \bullet D}{B \bullet C} = \frac{A}{B} \bullet \frac{D}{C}$$

ALGORITHM: $\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \bullet \frac{D}{C}$

## ALTERNATIVE ALGORITHM

What does this mean? $\frac{2}{3} \div \frac{1}{6}$

Divide fractions by finding common denominators:

$$\frac{2}{3} \div \frac{1}{6} = \frac{4}{6} \div \frac{1}{6} = \frac{?}{?}$$