# Developing Students' Conceptual Understanding and Reasoning about Fraction Division

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## Today's Session

- Welcome and introductions
- What students should know before operating with fractions; CCSS standards
- Meanings for division
- Models for division of fractions
- Contexts for division of fractions
- Discussion




## What Students Need to Know Well Before Operating With Fractions

- Meaning of the denominator (number of equal-sized pieces into which the whole has been cut);
- Meaning of the numerator (how many pieces are being considered);
- The more pieces a whole is divided into, the smaller the size of the pieces;
- A fraction can have many different names (equivalence);
- Meanings for whole number operations




## CCSS Standards Related to Division of Fractions: Grade 5

- In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors . . . and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.




## CCSS Standards Related to Division of Fractions: Grade 6

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.
   For example, create a story context for 2/3 ÷ 3/4 and use a visual fraction model to show the quotient; . . .



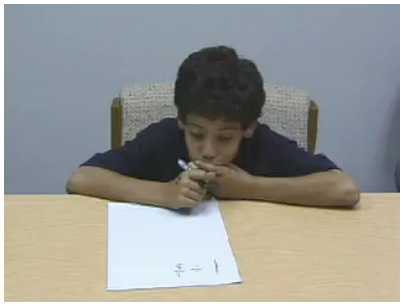

## Connecting with Students' Thinking

- Eliot solves these two problems:

$$1 \div \frac{1}{3} \quad \text{and} \quad 1\frac{1}{2} \div \frac{1}{3}$$

- What does Eliot understand?
- What concepts is he struggling with?
- How could we help him understand how to model and reason about the problem?

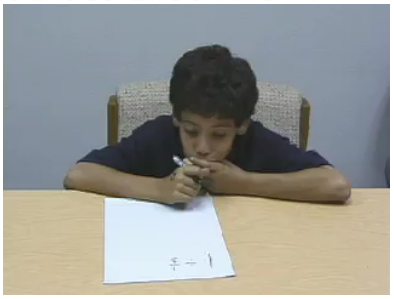
## What Does Eliot Understand About Division of Fractions?



$1 \div \frac{1}{3}$ and $1\frac{1}{2} \div \frac{1}{3}$




## What Does Eliot Know?



- What does Eliot understand?
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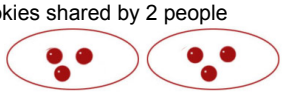
## What Does Division Mean?

- Whole number meanings for division

$$6 \div 2 = 3$$

  - Sharing ("partitive division")
    - Six cookies shared by 2 people



  - Measurement / repeated subtraction
    - 6 cookies, give two to each person. How many people?




## The Need for Coherence

- "Marta gave two friends an equal number of beads. If she started with 6 beads, how many beads did each friend get?"
- 6 beads ÷ 2 friends = 3 beads for each friend. (6 ÷ 2 = 3)
- Might there be a situation where Marta would share her beads with a fraction of a friend?




## Consider $6 \div \frac{1}{2}$

- How many halves are there in six? (measurement)

Or possibly,

- Six is half of what number? (sharing)
- Each interpretation has consequences for context and modeling.



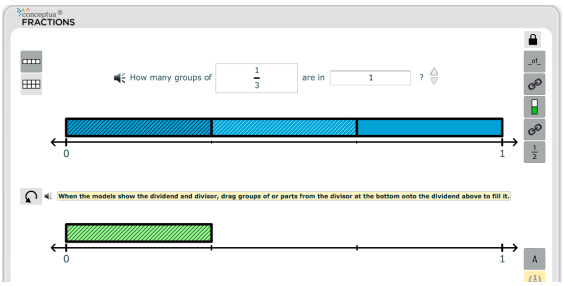

## Reasoning About Division With Eliot's Fractions in Context

- If we use the sharing meaning for division:

$$1 \div \frac{1}{3}$$

  - One is shared by one-third of a group.
  - How many in the whole group?
  - Does this make sense?

# Reasoning About Division With Fractions







# Reasoning About Division With Eliot's Fractions in Context

- This time, using a measurement / repeated subtraction meaning: $1 \div \frac{1}{3} =$
  - How many one-thirds are contained in one?
  - How many times can one-third be subtracted from one?
  - How does this work?
  - What happened when Eliot encountered a left over with $1\frac{1}{2} \div \frac{1}{3} =$




# Materials for Modeling Division of Fractions

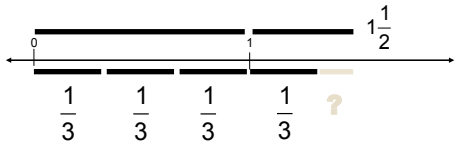
- How might you use these materials to model $1\frac{1}{2} \div \frac{1}{3}$
  - Paper strips
  - Fraction circles
- You could also use:
  - Pattern blocks
  - Fraction bars / fraction strips




# Using a Linear Model With a Measurement Interpretation

$$1\frac{1}{2} \div \frac{1}{3}$$

**How many one-thirds are in one and one-half?**

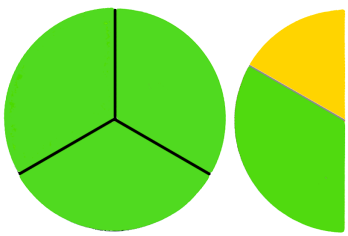






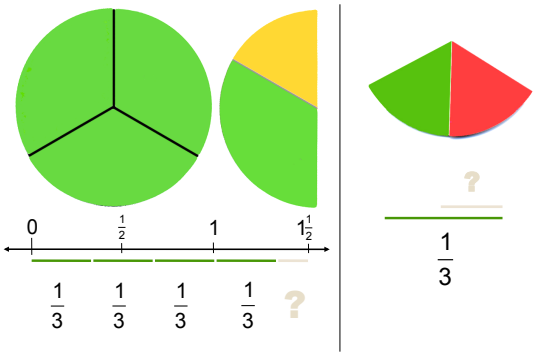
# Using an Area Model With a Measurement Interpretation

- Representation of $1\frac{1}{2} \div \frac{1}{3}$ with fraction circles.






# How Many Thirds?

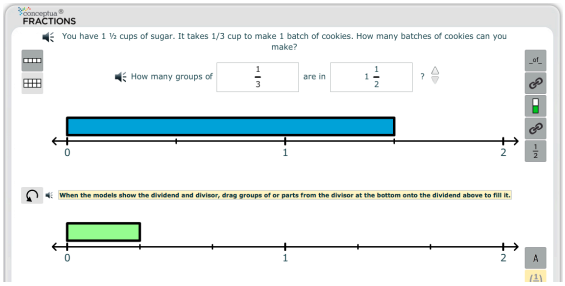


## A Context For Division of Fractions

- You have $1\frac{1}{2}$ cups of sugar. It takes $\frac{1}{3}$ cup to make 1 batch of cookies. How many batches of cookies can you make?
  - How many cups of sugar are left?
  - How many batches of cookies could be made with the sugar that's left?




## Modeling the Context For Fraction Division







## Another Context For Division of Fractions

You have $1\frac{1}{2}$ yards of ribbon. It takes $\frac{2}{3}$ of a yard to make one bow. How many packages can be made?

- How much of a yard of ribbon is left?
- How much of a bow will that amount make?




## Model Using Your Materials

- Use your materials to model

$$1\frac{1}{2} \div \frac{2}{3}$$




## Reasoning About Division of Fractions

- What do you notice about these problems and their solutions?

$$1\frac{1}{2} \div \frac{1}{3} = \qquad 1\frac{1}{2} \div \frac{2}{3} =$$

So what would $1\frac{1}{2} \div \frac{4}{3} = ?$



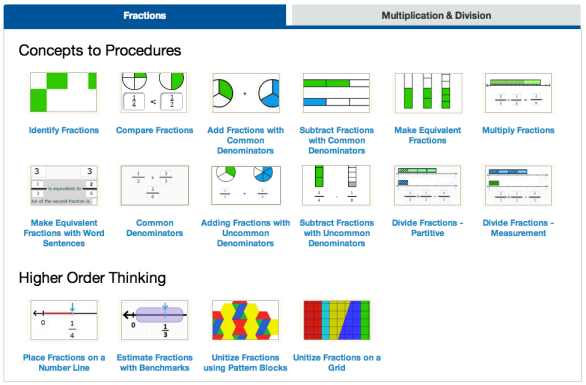

## Thinking More Deeply About Contexts for Division of Fractions

- What contexts work best for division of fractions?
- What aspects of these contexts allow them to work better than others?
- Develop your own new context for the problem we just modeled.

# Thinking More Deeply About Division of Fractions

- Estimating and judging a reasonable answer.
- Recognizing situations involving division of fractions.
- Considering and identifying other contexts where the division of fractions occurs.
- Making thoughtful number choices when considering examples.




http://www.conceptuamath.com/fractions.html




# Questions/Discussion

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