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# *Be The Square, Become a Parallelogram: Enacting Linear Transformations*

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# TODAY's AGENDA

- Introduction (10 minutes)
  - Brief Introduction to materials
  - Motivation for development
- Engage with Activity (45 minutes)
  - Groups – take turns
  - Engage in Activity (Don't just read it!)
- Reflection and Discussion (15 minutes)
  - Prompts – small and whole group discussion
  - Discussion of other activities

# Making the Connections Project

- Funded by the National Science Foundation (DUE- 9981029)
- Originally developed for courses for prospective secondary mathematics teachers
- Developed into a broader project with the Education Development Center – *Seeing the Connections* – expanded the content to include higher algebra and advanced mathematical thinking

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# Seeing the Connections: Promoting Profound Understanding of Secondary Mathematics

- Collaborative Curriculum Project from, Education Development Center, UNH, and Stony Brook University
- Funded by NSF DUE
- http://www2.edc.org/connect
- The *Seeing the Connections* materials are the "off-spring" of three NSF-funded proof-of-concept projects:
  - Making the Connections: Higher Algebra to School Mathematics (Cuoco, Greenes, Findell, Benson)
  - Making Mathematical Connections in Programs for Prospective Teachers (Portnoy, Graham, Grundmeier)
  - Gateways to Advanced Mathematical Thinking (Cuoco)

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# Goals for *Making the Connection*

- Develop a series of content-based activities that provide prospective teachers with the opportunity to make connections between two mathematical areas (transformational geometry and linear algebra) and school and university mathematics.

- Develop a series of pedagogical activities that prospective teachers explore within the context of the developing mathematical understandings above.

# Content Modules

**1- Isometries of the Plane**

- Discover the four basic isometries (rotation, reflection, translation, and glide).
- Reinforce the place of definition in mathematics. Sharing definitions and the ensuing discourse is likely to bring out the importance of careful wording.
- Identify similarity transformations.
- Make connections between functions and geometric transformations.

**2- Rotations, Reflections, Translations, and Glides**

- Discover basic properties of various isometries.
- Understand definitions and invariants of each isometry.

**3- Compositions**

- Discover that the class of isometries is preserved by composition.
- View isometries as functions.

# Content Modules (continued)

**4- Proof with Isometries**

- Be familiar with the use of isometries in proof.
- Consider basic Euclidean postulates.

**5- The Human Vertices**

- Enable students to make connections (physically) between transformational geometry and linear algebra.
- Linear transformations are functions.
- Non-invertible transformations collapse **R**2 to **R**1 or to {0}.
- Sign of the determinant indicates orientation.

**6- Isometries and Linear Algebra**

- This activity is meant to bring closure to the mathematical ideas connecting transformational geometry and linear algebra by introducing the idea of a group structure.

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# Implementation

***The materials have been used in the following settings:***

- Undergraduate geometry courses for prospective teachers and other mathematics majors
- Masters level courses for teachers

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# Making Mathematical Connections in Programs for Prospective Teachers

**Activity 5:The Human Vertices:** Investigate transformations of a square of side one in the coordinate plane. Each of you will have a starting coordinate. You will connect the vertices with taught rope for straight lines.

e.g. $t_1$ (x,y) = (x, -y)

$t_2$ (x,y) = (x-2, y+1)

| Person | Second Starting Coordinate |
|---|---|
| A | (1,0) |
| B | (0,0) |
| C | (1,1) |
| D | (0,1) |

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# *HUMAN VERTICES*
# ENGAGING IN A SAMPLE ACTIVITY

- Engage in the activity as a learner
- As a group keep track of observations and questions
- Change roles – i.e. vertices and observers
- Think about how students (prospective teachers) might engage in this activity

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# Reflection/Discussion

- What are the main mathematical themes?
- What are some of the connections your group made?
- How might a future secondary teacher engage with the activity?
- What mathematical practices might be enhanced through this type of activity?
- What would you do differently?

# Pedagogical Modules

A – Examination of Mathematical Tasks

B – Classroom Observations

C – Guidelines for Creating a Class Activity

*These activities involve the prospective teachers in the analysis of pre-college mathematical curricula and tasks, the analysis of classroom observations conducted in middle school and/or high school classrooms, and the development, implementation, and evaluation of a class activity focused on transformational geometry.*

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# An Example: Activity A

1) First, complete the mathematical task from the high school curriculum. Work on the task individually for five to ten minutes. Then discuss with you group any variation in individual interpretations and solution strategies. Agree on a solution.

2) Here is a list of issues a teacher might consider when previewing a mathematical task. Consider the task you just finished and identify how each of these issues might come into play when a teacher is preparing to use the task with a class.

- Student preparation issues (prior knowledge and ability, scaffolding (providing appropriate incremental support), variation in student abilities)
- Type of instruction (direct, whole class discussion, small groups or pairs, individual)
- Resources (calculators, manipulatives, tools (straight edge, compass, etc.), materials (paper, colored pencils, etc.)

# Activity A (continued)

3) Consider the mathematical task you just completed. Here are some questions a teacher might ask about any mathematical task. Discuss these questions and make a compile a group response to each.

a) How does this task fit into the unit?

b) Should this task be moved to elsewhere in the unit, or should some other task be placed before it?

c) How does this task foreshadow upcoming ideas?

d) How can we use this task to take advantage of opportunities to explore important mathematical ideas?

e) Where are opportunities to develop problem solving skills?

# Student Reaction 1

*It was a lot of fun. It was a good experience to get into the classroom. I'm doing my student teaching next semester, and I just thought it was really beneficial. I feel badly…or not badly…I don't know …the people who aren't going to be teachers in this class aren't going to feel the same benefit as me, so I feel…but at the same time, I know that some people might really enjoy the experience because they aren't going to be teachers, and there's an opportunity to get into the classroom. So, I don't know what you think about that, but for myself, it was really interesting.*

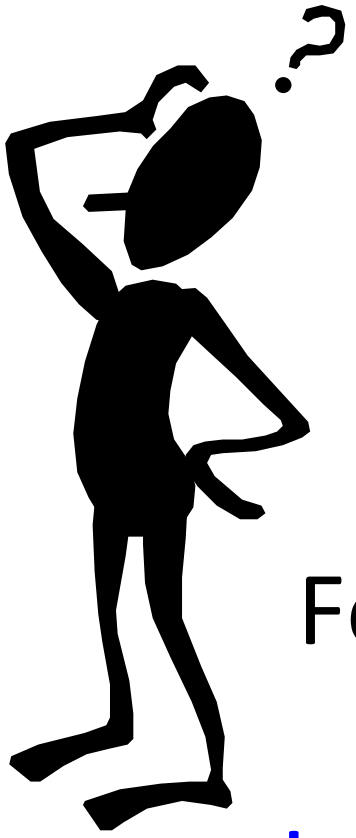
# Student Reaction 2

From the Final Course Reflection:

*The way the material was presented gave be direction, motivation, and all the tools necessary to learn more than I ever thought I would about geometry. I am not planning to be a teacher, but learning the material and looking at lessons from a different point of view helped me form new ideas on how to convey my ideas to others and obviously that is an invaluable tool that will be useful no matter what I do in life.*

# Student Reaction 4

*One thing that was beneficial was when we were evaluating, using the activities that you gave us, we talked about scaffolding and the importance of giving certain information that leads us to high learning and we were able to do that in our lesson planning….*



For copies of the activities contact:

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Thank You

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