Matching, Sorting & Exploring

DISCOVERING FUNCTION FAMILIES

Sami Briceno, Manager of School Partnerships, TX Lead
Carnegie Learning, Inc.
sbriceno@carnegielearning.com
1-888-851-7094 x492

Kasey Bratcher, Director of Educational Services
Carnegie Learning, Inc.
kbratcher@carnegielearning.com
1-888-851-7094 x416
Follow me on Twitter: @kbratcher31

Resources LINK: http://www.carnegielearning.com/landingpages/2014conferences/

Carnegie Learning
Professional Development

A Picture is Worth a 1000 Words

Matching Graphs & Scenarios

Something's Fishy

Candice is a building manager for the Crowley Enterprise office building. One of her responsibilities is cleaning the office building's 200-gallon aquarium. For cleaning, she must remove the fish from the aquarium and drain the water. The water drains at a constant rate of 10 gallons per minute.

Smart Phone, but Is It a Smart Deal?

You have had your eye on an upgraded smart phone. However, you currently do not have the money to purchase it. Your cousin will provide the funding, as long as you pay him interest. He tells you that you only need to pay $1 in interest initially, and then the interest will double each week after that. You consider his offer and wonder: is this *really* a good deal?

Can't Wait to Hit the Slopes!

Andrew loves skiing—he just hates the ski lift ride back up to the top of the hill. For some reason the ski lift has been acting up today. His last trip started fine. The ski lift traveled up the mountain at a steady rate of about 83 feet per minute. Then all of a sudden it stopped and Andrew sat there waiting for 10 minutes! Finally, the ski lift began to ascend up the mountain to the top.

It's Magic

The Amazing Aloysius is practicing one of his tricks. As part of this trick, he cuts a rope into many pieces and then magically puts the pieces of rope back together. He begins the trick with a 20-foot rope and then cuts it in half. He then takes one of the halves and cuts that piece in half. He repeats this process until he is left with a piece so small he can no longer cut it. He wants to know how many total cuts he can make and the length of each remaining piece of rope after the total number of cuts.

Baton Twirling

Jill is a drum major for the Altadena High School marching band. She has been practicing for the band's halftime performance. For the finale, Jill tosses her baton in the air so that it reaches a maximum height of 22 feet. This gives her 2 seconds to twirl around twice and catch the baton when it comes back down.

Music Club

Jermaine loves music. He can lip sync almost any song at a moment's notice. He joined *Songs When I Want Them*, an online music store. By becoming a member, Jermaine can purchase just about any song he wants. Jermaine pays $1 per song.

A Trip to School

On Monday morning, Myra began her 1.3-mile walk to school. After a few minutes of walking, she walked right into a spider's web—and Myra hates spiders! She began running until she ran into her friend Tanisha. She stopped and told Tanisha of her adventurous morning and the icky spider's web! Then they walked the rest of the way to school.

Jelly Bean Challenge

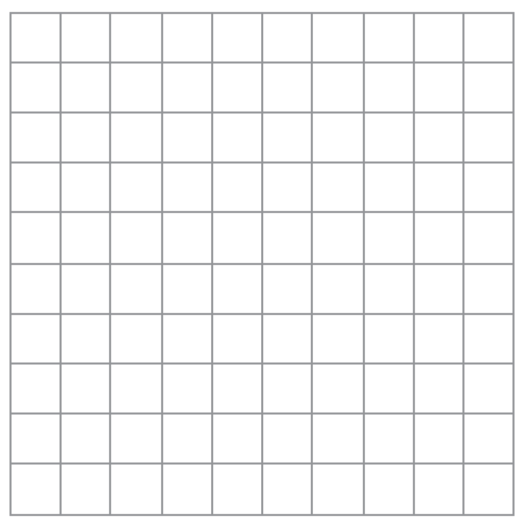
Mr. Wright judges the annual Jelly Bean Challenge at the summer fair. Every year, he encourages the citizens in his town to guess the number of jelly beans in a jar. He keeps a record of everyone's guesses and the number of jelly beans that each person's guess was off by.

Consider the scenario *A Trip to School*.

a. Write a scenario and sketch a graph to describe a possible trip on a different day.

Scenario

Graph



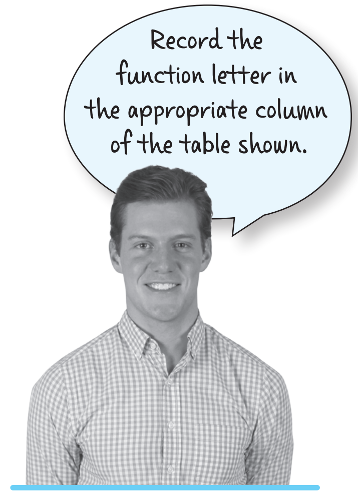
PROBLEM 2 Up, Down, or Neither?

In the previous lesson, you determined which of the given graphs represented functions. Gather all of the graphs from the previous lesson that you identified as functions.

A function is described as increasing when the dependent variable increases as the independent variable increases. If a function increases across the entire domain, then the function is called an **increasing function**.

A function is described as decreasing when the dependent variable decreases as the independent variable increases. If a function decreases across the entire domain, then the function is called a **decreasing function**.

If the dependent variable of a function does not change or remains constant over the entire domain, then the function is called a **constant function**.



1. Analyze each graph from left to right. Sort all the graphs into one of the four groups:
   - increasing function,
   - decreasing function,
   - constant function,
   - a combination of increasing, decreasing, or constant.

Increasing Function	Decreasing Function	Constant Function	Combination of Increasing, Decreasing, or Constant

2. Each function shown represents one of the graphs in the increasing function, decreasing function, or constant function categories. Enter each function into a graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x$

- $f(x) = \left(\frac{1}{2}\right)^x - 5$

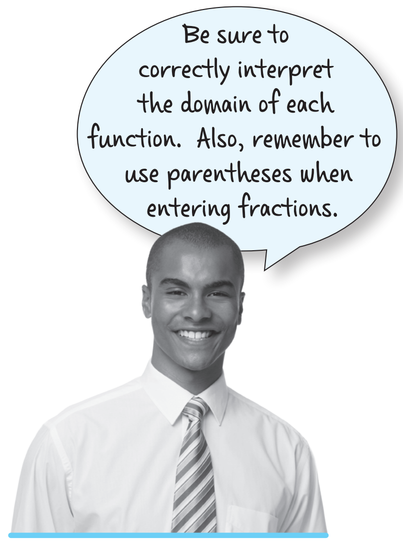
- $f(x) = 2^x$, where x is an integer

- $f(x) = -\frac{2}{3}x + 5$

- $f(x) = -x + 3$, where x is an integer

- $f(x) = \left(\frac{1}{2}\right)^x$

- $f(x) = 2$, where x is an integer



3. Consider the seven graphs and functions that are increasing functions, decreasing functions, or constant functions.

a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

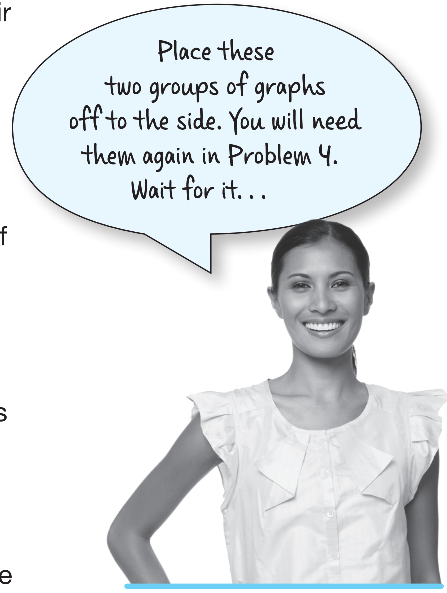
Group 1	Group 2

b. What is the same about all the functions in each group?

Congratulations! You have just sorted the graphs into their own *function families.* A **function family** is a group of functions that share certain characteristics.

The family of **linear functions** includes functions of the form $f(x) = mx + b$, where m and b are real numbers.

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but is not equal to 1.



4. Go back to your table in Question 3 and identify which group represents linear and constant functions and which group represents exponential functions.

5. If $f(x) = mx + b$, represents a linear function, describe the m and b values that produce a constant function.

PROBLEM 3 Least, Greatest, or Neither?

A function has an **absolute minimum** if there is a point that has a y-coordinate that is less than the y-coordinates of every other point on the graph. A function has an **absolute maximum** if there is a point that has a y-coordinate that is greater than the y-coordinates of every other point on the graph.

1. Sort the graphs from the Combination category in Problem 2 into three groups:

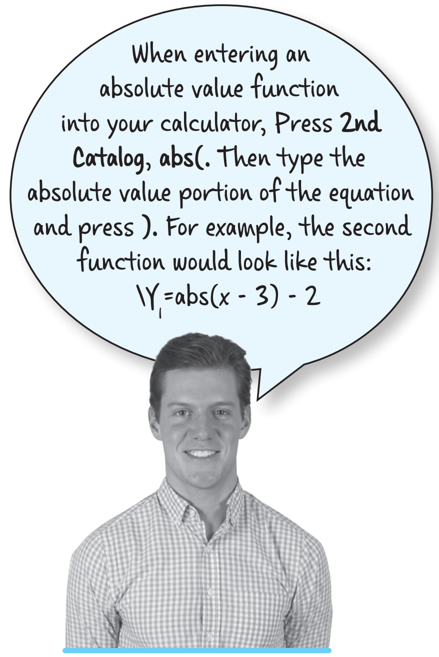
- those that have an absolute minimum value,
- those that have an absolute maximum value, and
- those that have no absolute minimum or maximum value.

Then record the function letter in the appropriate column of the table shown.

Absolute Minimum	Absolute Maximum	No Absolute Minimum or Absolute Maximum

2. Each function shown represents one of the graphs with an absolute maximum or an absolute minimum value. Enter each function into your graphing calculator to determine the shape of its graph. Then match the function to its corresponding graph by writing the function directly on the graph that it represents.

- $f(x) = x^2 + 8x + 12$
- $f(x) = |x - 3| - 2$
- $f(x) = x^2$
- $f(x) = |x|$
- $f(x) = -|x|$
- $f(x) = -3x^2 + 4$, where x is integer
- $f(x) = -\frac{1}{2}x^2 + 2x$
- $f(x) = -2|x + 2| + 4$



3. Consider the graphs of functions that have an absolute minimum or an absolute maximum. (Do not consider Graphs A and C yet.)

   a. Sort the graphs into two groups based on the equations representing the functions and record the function letter in the table.

Group 1	Group 2

   b. What is the same about all the functions in each group?

Congratulations! You have just sorted functions into two more function families.

The family of **quadratic functions** includes functions of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers, and a is not equal to 0.

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.

4. Go back to your table in Question 3 and identify which group represents quadratic functions and which group represents linear absolute value functions.

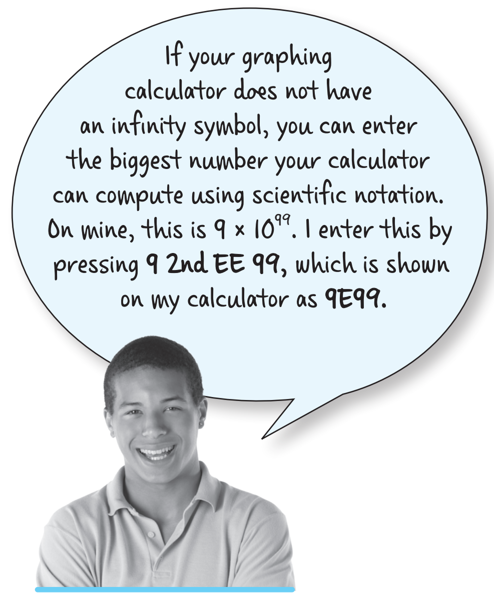
PROBLEM 4 Piecing Things Together

Analyze each of the functions shown. These functions represent the last three graphs of functions from the no absolute minimum and no absolute maximum category.

- $f(x) = \begin{cases} -2x + 10, & -\infty \le x < 3 \\ 4, & 3 \le x < 7 \\ -2x + 18, & 7 \le x \le +\infty \end{cases}$

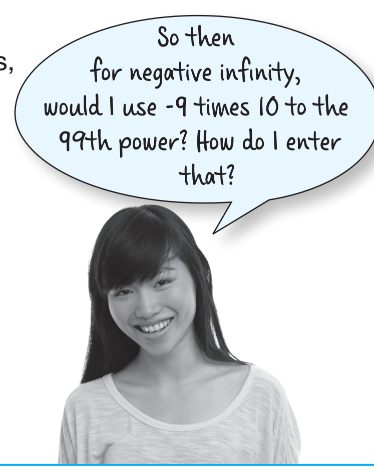
- $f(x) = \begin{cases} -2, & -\infty < x < 0 \\ \frac{1}{2}x - 2, & 0 \le x < \infty \end{cases}$

- $f(x) = \begin{cases} \frac{1}{2}x + 4, & -\infty \le x < 2 \\ -3x + 11, & 2 \le x < 3 \\ \frac{1}{2}x + \frac{1}{2} & 3 \le x \le \infty \end{cases}$



These functions are part of the family of *linear piecewise functions*. **Linear piecewise functions** include functions that have equation changes for different parts, or pieces, of the domain.

Because these graphs each contain compound inequalities, there are additional steps required to use a graphing calculator to graph each function.



Let's graph the piecewise function:

$$\begin{cases} -2x + 10, & -\infty \le x < 3 \\ f(x) = 4, & 3 \le x < 7 \\ f(x) = -2x + 18, & 7 \le x \le +\infty \end{cases}$$

You can use a graphing calculator to graph piecewise functions.

The first section is $(-2x + 10)$.

Step 1: Press **Y=**. Enter the first section of the function within parentheses. Then press the division button.

The first part of the inequality is $(-\infty \le x)$.

Step 2: Press the (key twice and enter the first part of the compound inequality within parentheses.

Use the **2ND TEST** to locate the inequality symbols. The **TEST** button is on the same key as the **MATH** button.

Step 3: Enter the second part of the compound inequality within parentheses and then type two closing parentheses.

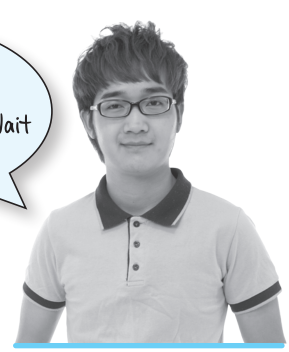
Press **GRAPH** here to see the first section of the piecewise function.

The second part of the compound inequality is $(x < 3)$. The entire line should look like: $\backslash Y_1 = (-2x + 10)/((-\infty \le x)(x < 3))$

Step 4: Enter the remaining sections of the piecewise functions as Y_2 and Y_3.

By completing the first piecewise function, you can now choose the graph that matches your graphing calculator screen.

You will need these graphs again in Problem 5. Wait for it. . .



1. Enter the remaining functions into your graphing calculator to determine the shapes of their graphs.
2. Match each function to its corresponding graph by writing the function directly on the graph that it represents.

Definition

The family of **linear functions** includes functions of the form $f(x) = mx + b$, where m and b are real numbers.

Graphical Behavior

Increasing / Decreasing:

Maximum / Minimum:

Curve / Line:

Linear Functions

Examples

1

Definition

The family of **exponential functions** includes functions of the form $f(x) = a \cdot b^x$, where a and b are real numbers, and b is greater than 0 but not equal to 1.

Graphical Behavior

Increasing / Decreasing:

Maximum / Minimum:

Curve / Line:

Exponential Functions

Examples

Definition

The family of **quadratic functions** includes functions of the form, $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers, and a is not equal to 0.

Graphical Behavior

Increasing / Decreasing:

Maximum / Minimum:

Curve / Line:

Quadratic Functions

Examples

1

Definition

The family of **linear absolute value functions** includes functions of the form $f(x) = a|x + b| + c$, where a, b, and c are real numbers, and a is not equal to 0.

Graphical Behavior

Increasing / Decreasing:

Maximum / Minimum:

Curve / Line:

Linear Absolute Value Functions

Examples

Definition

The family of **linear piecewise functions** includes functions that have equation changes for different parts, or pieces, of the domain.

Graphical Behavior

Increasing / Decreasing:

Maximum / Minimum:

Curve / Line:

Linear Piecewise Functions

Examples