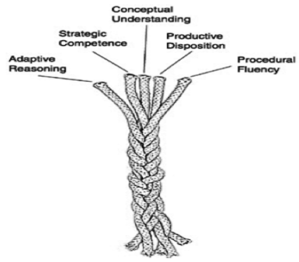

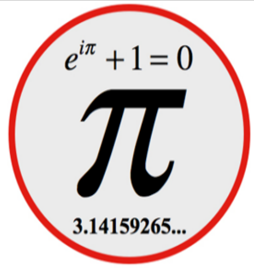

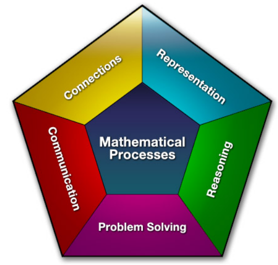

The Mathematical Practices of Finding Structure and Making Connections

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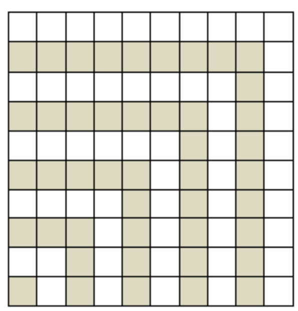
Number and Geometry



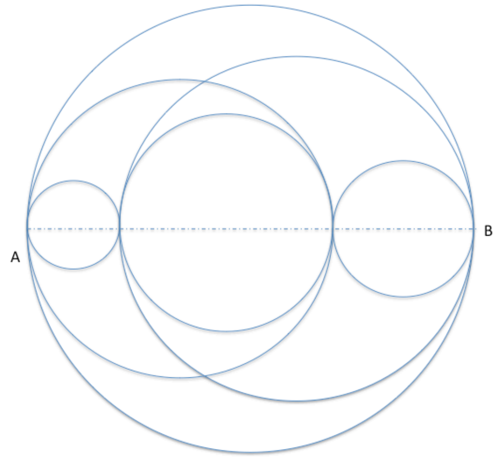
- Use this picture to show that:

  1+3+5+7+9+11+13+15+17+19 = 100

- Can you use your idea to show that, for any whole number n, <u>the sum of the first n odd numbers is n^2</u> ?

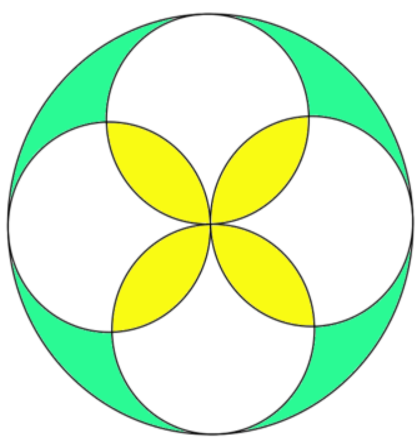
Geometric measure and linear functions

Circle Park has a network of circular trails for cyclists to use (see Figure below). The trails have bridges so that they meet only along the diameter AB. Which is the shortest way to travel from A to B using this network of circular paths?



Geometric measure and combinatorics

Which area is more: The green or the yellow? Explain your answer.



Examine these four problems.
What do they have in common mathematically?
In what ways are they different?

CUISENAIRE RODS

How many ways can you build a train that is the same length as the purple rod?
How many of each type of train are there? (i.e., 1-car trains, 2-car trains, 3-car trains, 4-car trains)

MAKING CHANGE
You have pennies, nickels, and dimes. How many ways can you make 10¢ ? What are the different exchanges you can make with these coins?

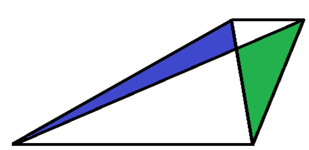
BINOMIAL EXPANSION
What are the coefficients in the binomial expansion of $(x + y)^3$?

RIBBON CUTTING
You have a 4-inch ribbon. In how many ways can you cut it into pieces each of length a whole number of inches? How many such cuttings are there of each type? (i.e. 1 piece, 2 pieces, 3 pieces, 4 pieces)

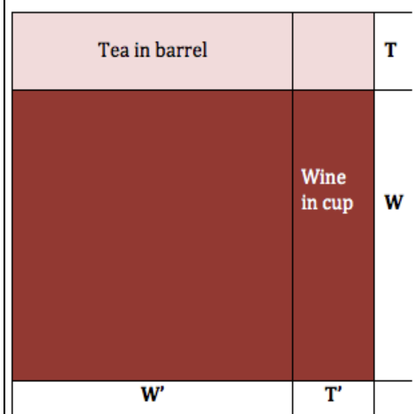
Can you find a common structure in these problems?

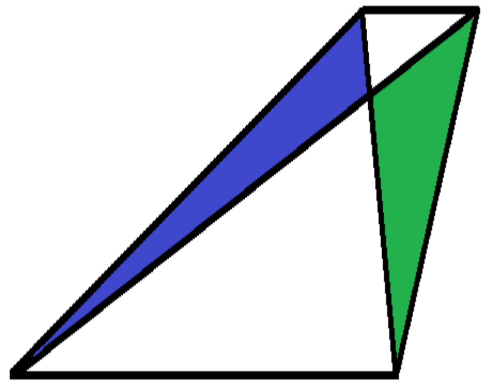
- (**Tea & wine**) I have a barrel of wine, and you have a cup of green tea. I put a teaspoon of my wine into your cup of tea. Then you take a teaspoon of the mixture in your teacup, and put it back into my wine barrel. Question: Is there now more wine in the teacup than there is tea in the wine barrel, or is it the other way around?

- (**Trapezoid diagonals**) The diagonals a trapezoid divide the trapezoid into four triangles. What is the relation of the areas of the two triangles containing the legs (non parallel sides) of the trapezoid?



If two things have the same measure, then what they don't have in common have the same measure





The 3-Permutation CSPS

1. What are all three-digit numbers that you can make using each of the digits 1, 2, and 3, and using each digit only once?

2. Angel, Barbara, and Clara run a race. Assuming there is no tie, what are all possible outcomes of the race (first, second, third)?

3. You are watching Angel, Barbara, and Clara playing on a merry-go-round. As the merry-go-round spins, what are all the different ways that you see all three of them in order, from left to right?

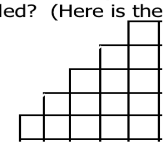
4. In a 3 x 3 grid square, color three of the (unit) squares blue, in such a way that there is at most one blue square in each row and in each column. What are all ways of doing this?

5. Find all of the symmetries(*) of an equilateral triangle

Hand-shakes CSPS (n-choose-2)

(a) (Consecutive sums) Find the sum of the integers from 1 to n-1: 1+2+3+. . . +(n-1

(b) (Side area of a staircase) In an (n-1)-step staircase made from unit cubes, how many cubes are needed? (Here is the 5-step staircase.)



(c) How many points (x, y) in the plane are there such that x and y are integers and $0 < y < x < n$?

(d) (Hand shakes) In a group of n people, each person shakes hands with each of the others. How many hand-shakes occur?

(e) (Two person delegation) From a group of n voters, how many ways are there to select a two-person delegation to attend the party caucus?

(f) (Diagonals of a polygon) In a convex polygon with n vertices how many diagonals (lines joining non-adjacent vertices) are there?

(g) (Pizza pieces) Using a pizza knife, what is the largest number of pieces of pizza that can be made with n-1 cuts? (Equivalently, what is the largest number of regions of the plane that can be carved out by n-1 lines?)

(h) (Recursion) Let f(n) be a function of an integer variable $n \geq 0$ such that f(1) = 2, and f(n) = f(n-1) + n for all $n \geq 1$. What is f(n-1)?

Expanded Usiskin CSPS

1. Find all ways to express ½ as the sum of two unit fractions
2. Find all rectangles with integer side lengths whose area and perimeter are numerically equal.
3. Nan can paint a house in n days, and Mom can paint it in m days (n and m positive integers). Working together they can paint the house in 2 days. What are the possible values of n and m?
4. Given a point P in the plane, find all n such that a small circular disk centered at P can by covered by non-overlapping congruent tiles shaped like regular n-gons that have P as a common vertex.
5. For which positive numbers s does $p(x) = x^2 - sx + 2s$ have integer roots?
6. The base b and corresponding height h of a triangle are integers. A 2 x 2 square is inscribed in the triangle with one side on the given base, and vertices on the other two sides. What are the possible values of the pair (b, h)?