Exploring Fraction Division: Why We Flip and Multiply!

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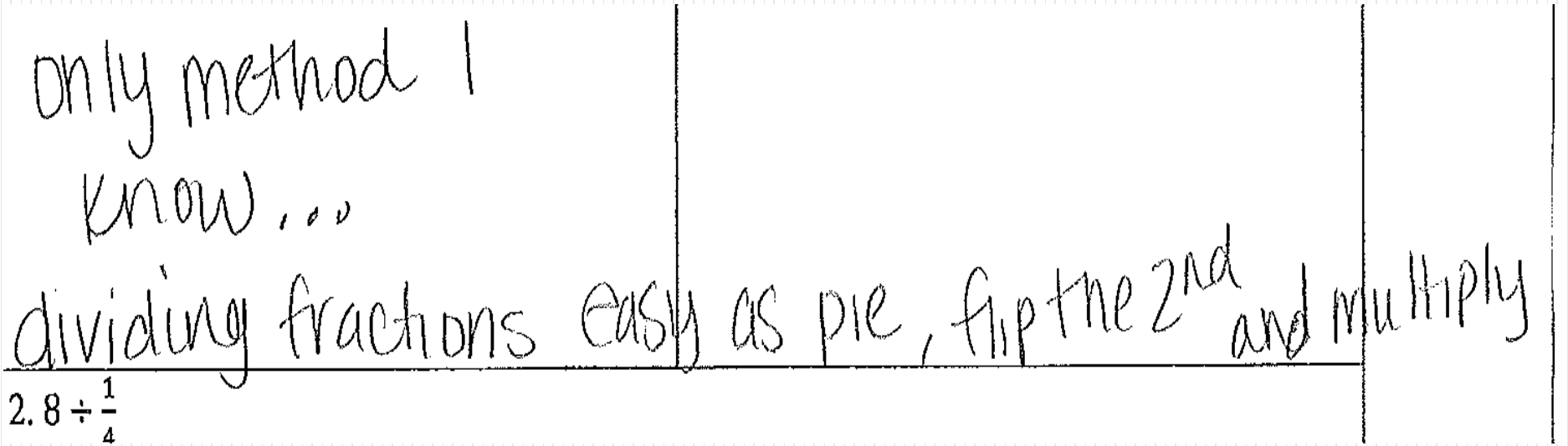
Why are we interested?



Why are we interested?

- Retention
- Problem Solving
- Reasonableness of answers

Other Reasons to be Interested?

This is the age of accountability!

STANDARDS

- NCTM Principles and Standards
- Common Core State Standards

NCTM Representation Standard for Grades 6-8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

CCSS

Grade 6 **The Number System**

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Two Views of Division

Partitive Division

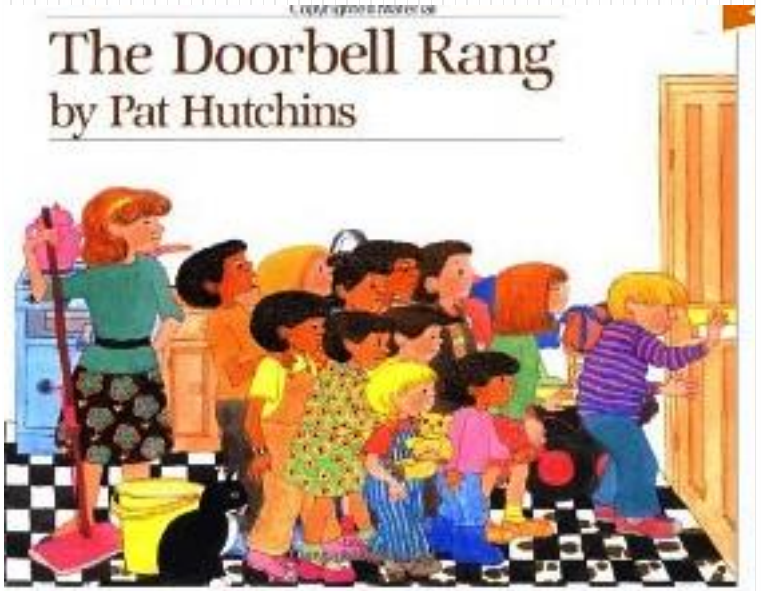
How many in each group?

Measurement Division

How many groups?

Recognizing Partitive Division Situations

The Doorbell Rang by Pat Hutchins (1986)



Where is the "measurement" version of *The Doorbell Rang*?

Let's focus on Grandma from the story:

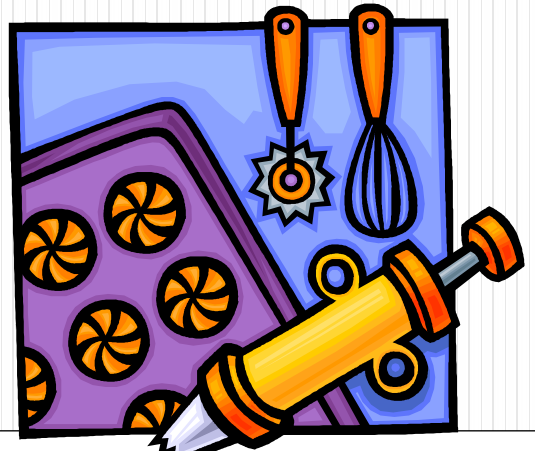
It was Grandma with an enormous tray of cookies.

So Grandma must have been baking all day – with more cookies to share as a result!

What could we consider to construct our measurement cookie story?

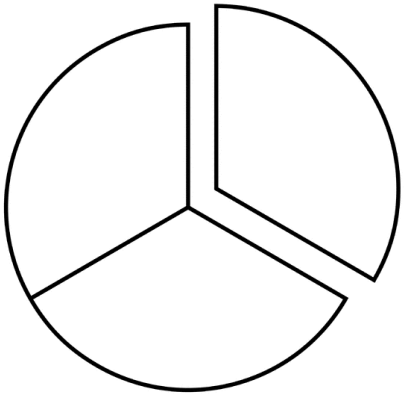
Sugar?

Butter?



The Reading Club is hosting a pizza party for all students at Central Middle School who have read at least ten books this semester. Help them decide how many pizzas they should order if they are expecting 150 students to attend the party.

They predict that each student will want to eat one-third of a pizza for lunch. Use your pizza pieces to complete the table.

# of Pizzas	Serving Size = $\frac{1}{3}$	Picture/Process	# of Servings
1	How many $\frac{1}{3}$ pizza servings are in 1 pizza?		3

2	in 2 pizzas?		
3	in 3 pizzas?		
4	in 4 pizzas?		

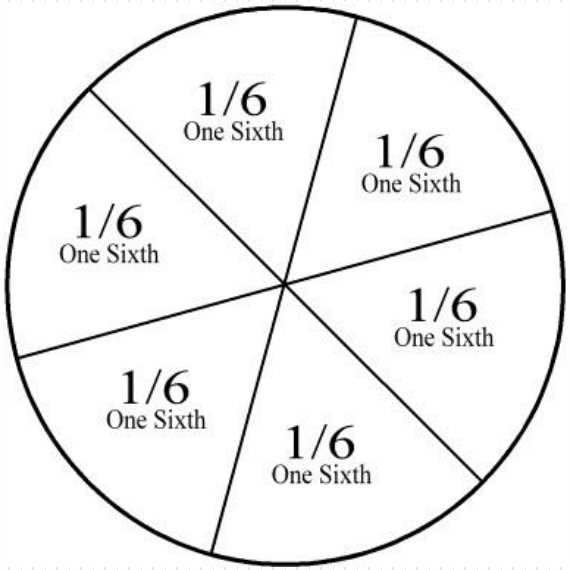
Use the results from your table to answer the following questions:

1) How many servings (each one-third of a pizza) could they get from 10 pizzas?

2) How many pizzas do they need to order for 150 students if each student gets one-third of a pizza?

The club president doesn't want to order that many pizzas. She suggests they limit each person to one-sixth of a pizza. How many servings can they get from 1 pizza?

Include a sketch in your explanation.



4) How many pizzas do they need to order for 150 students if each student gets one-sixth of a pizza?

1 : 6 **? : 150**

÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

Summary:

5) The process of dividing a pizza into servings of *1/3* pizza can be written symbolically as *1 ÷ 1/3*. Since there are 3 servings of *1/3* pizza in one whole pizza, we know that *1 ÷ 1/3=3*.

a) How many thirds are in 2 pizzas?

_____

So, *2 ÷ 1/3* = _____.

.

b) How many thirds are in 8 pizzas?

_____

So, $8 \div 1/3 =$ _____.

c) How many thirds are in one-third of a pizza? _____

So, $1/3 \div 1/3 =$ _____.

d) How many thirds are in one-half of a pizza, including part of a serving of size one-third?

_____

So, $1/2 \div 1/3 =$ _____.

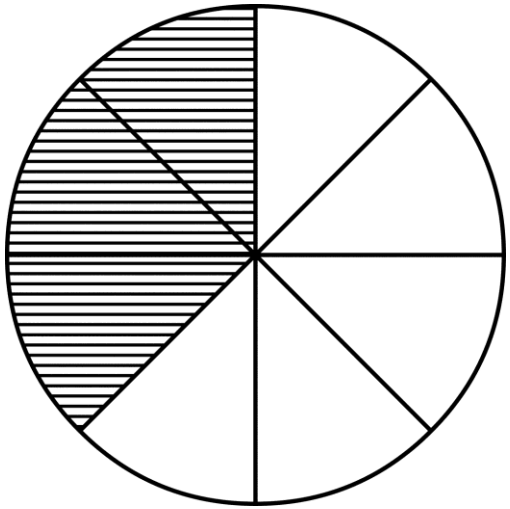
e) How many thirds are in one-fourth of a pizza? (Be sure to include partial servings).

So, $1/4 \div 1/3 =$ _____.

6) Explain how to divide a number by a unit fraction like *1/3*.

Talk with a tablemate – how would you communicate your understanding? What might we expect from students as they discuss these ideas? Is there vocabulary we want to encourage?

The treasurer complains that 1/6 of a pizza is not enough, and besides, the pizzeria cuts each pizza into 8 slices and not 6. He suggests they allow each student to have 3/8 of a pizza. Use this information to complete the table. Include partial servings in your answer.

# of Pizzas	Serving Size = $\frac{3}{8}$	Picture/Process	# of Servings
1	How many $\frac{3}{8}$ pizza servings are in 1 pizza?		

# of Pizzas	Serving Size = $\frac{3}{8}$	Picture/Process	# of Servings
2			
3			
5			

7) About how many pizzas should they order for all 150 students if each student is served 3/8 of a pizza?

What kind of answers would be acceptable here? Are we looking for an exact answer?

÷ ÷

Summary:

8) The process of dividing a pizza into servings of *3/8* pizza can be written symbolically as *1 ÷ 3/8*. Since there are *2 2/3* servings in one whole pizza, we know that *1 ÷ 3/8=2 2/3*. Rewriting the mixed number as an improper fraction, we see that *1 ÷ 3/8= 8/3*. Use the idea that 1 pizza has *8/3* of a group (serving) of *3/8* to answer the following questions. Write any mixed numbers as improper fractions.

a) How many *3/8* are in 2 pizzas? _____

So, *2 ÷ 3/8* = _____.

b) How many *3/8* are in 3 pizzas? _____

So, *3* ÷ *3/8* = _____.

c) How many *3/8* are in five pizzas? _____

So, *5* ÷ *3/8* = _____.

d) How many *3/8* would there be in one-half of a pizza, including part of a serving of size one-third?

_____

So, *1/2* ÷ *3/8* = _____.

e) How many *3/8* are in one-fourth of a pizza?

_____ (Be sure to include partial servings.)

So, *1/4* ÷ *3/8* = _____.

9) Describe how to divide a number by a non-unit fraction like 3/8.

Talk with a tablemate – how would you communicate your understanding? What might we expect from students as they discuss these ideas? Is there vocabulary we want to encourage?

The secretary is making bookmarks for each of the children out of lengths of ribbon donated by ClothMart. Use fraction strips to help you determine the number of bookmarks she can cut each piece of ribbon. She starts with a piece of ribbon that is *1/2* yard long.

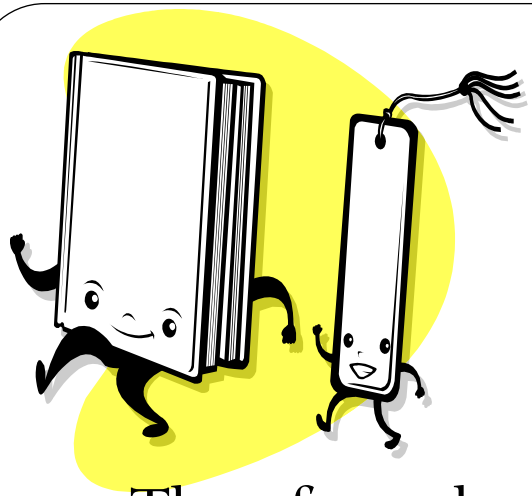
If each bookmark needs to be *1/5* yard long, then she can cut _____ whole bookmarks, with _____ yard of ribbon leftover.

She could cut another _____ part of a bookmark from the leftover ribbon.

So *1/2* yard would make _____ bookmarks (including partial bookmarks).

Complete the table to help her decide how many bookmarks she can cut from each piece of ribbon based on the lengths provided. Leave the last column blank for now.

Length of Ribbon	Length of Bookmark	# of Whole Bookmarks	Length of Leftover Ribbon	Part of a Bookmark	Division Sentence
$\frac{1}{2}$ yard	$\frac{1}{5}$ yard	2 bookmarks	$\frac{1}{10}$ yard	$\frac{1}{2}$ bookmark	$\frac{1}{2} \div \frac{1}{5} =$
$\frac{3}{4}$ yard	$\frac{1}{6}$ yard				
$\frac{2}{3}$ yard	$\frac{1}{4}$ yard				
$1\frac{3}{4}$ yard	$\frac{1}{3}$ yard				
$1\frac{1}{3}$ yard	$\frac{1}{8}$ yard				



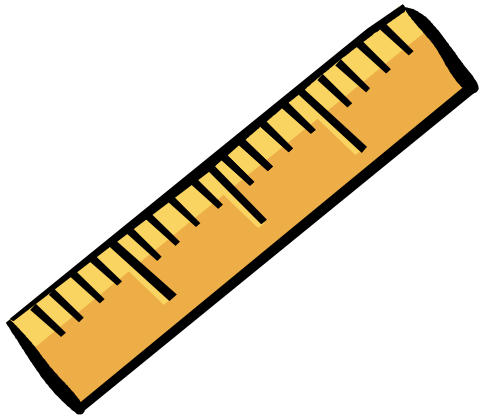
10) Notice the secretary was able to make 2 or more bookmarks from each piece of ribbon but always had ribbon leftover (remainder) that was too short to make another whole bookmark.

Therefore, the answer to each of these division problems could be written as a mixed number.

Complete the last column in the table by writing the answer (# of bookmarks) as a **mixed number**.

Does the fractional part of each mixed number answer refer to the length of leftover ribbon or to part of a bookmark?

Using a Ruler: Many word problems involving fraction division can be solved using a ruler. Try these sample problems. Start by drawing a segment that has the same length as the dividend. Then determine how many groups of the divisor are in the dividend by marking off segments that are the same length as the divisor. Be careful with the remainder!



11) I have 5 ½ yards of ribbon. If each bow requires 1 ½ yards of ribbon, how many bows can I make?

To solve, let 1 inch represent 1 yard of ribbon. Draw a 5 ½ inch-long segment. Mark the segment every 1 ½ inches to determine the number of bows I can make.

# of bows: _____

(include part of a bow in your answer)

Number Sentence: 5 ½ ÷ 1 ½ =

12) Ari is making root beer. His recipe makes 4 ¾ gallons of root beer. How many ½ gallon bottles can he fill?

# of bottles of root beer: _____

(include part of a bottle in your answer)

Number Sentence:

13. Are these division problems for *3/4÷1/2*? For those that are, which interpretation of division is used (measurement or partitive division). For those that are not, determine how to solve the problem if it can be solved.

a) Beth poured ¾ cup of cereal in a bowl. The cereal box says that one serving is ½ cup. How many servings are in Beth's bowl?

b) Beth poured ¾ cup of cereal in a bowl. Then Beth took ½ of that cereal and put it into another bowl. How many cups of cereal are in the second bowl?

c) If ¾ cup of flour makes ½ batch of cookies, then how many cups of flour are required for a full batch of cookies?

d) If ½ cup of flour makes a batch of cookies, then how many batches can you make with ¾ cup of flour?

e) If ¾ cup of flour makes a batch of cookies, then how much flour is in ½ of a batch of cookies?

RESOURCES used in presentation

- Printable Ruler Template

http://www.donteatthepaste.com/2012/07/printable-book-report-and-ruler.html

- Fraction Strip Template

http://www.teachervision.fen.com/arithmetic/printable/6190.html

- Dana Center Activity Packet

http://www.utdanacenter.org/mathtoolkit/instruction/lessons/7_divide.php

Related Articles

- **Oranges, Posters, Ribbons, & Lemonade**

  Author: Christopher M. Kribs-Zaleta

  Mathematics Teaching in the Middle School, April 2008

- **Measurement and Fair-Sharing Models for Dividing Fractions**

  Authors: Jeff Gregg and Diana Underwood Gregg

  Mathematics Teaching in the Middle School, May 2007

- **Take a Bite Out of Fraction Division**

  Authors: Nesrin Cengiz and Margaret Rathouz in

  Mathematics Teaching in the Middle School, October 2011