Number Stories: Then and Now

Johnny W. Lott
Past President, NCTM

Number Stories of Long Ago—D. E. Smith

"The story of our numbers, of the world's attempts to count, of the many experiments in writing numerals, and of the difficulties encountered through the ages in performing our everyday computations—all this is so interwoven with the history of humanity so as to have an interest for every thinking person."

Number Stories of Long Ago

"As the world has grown, so the work with numbers has grown; when the world has faced the mysteries of the universe, numbers have assisted in solving its problems; when commerce and science have shown new needs in computation, arithmetic has always been ready to lend a hand. The history of mathematics is no small part of the history of civilization."

Number Stories of Long Ago

"…it seems proper to relate at least some portion of the story of numbers to the pupils in our schools."

p. v

Number Stories of Long Ago

- Earliest stories have people counting 1, 2, many.
- Later stories have people solving number problems in the early 1900s and before.

Number Stories of Long Ago

As world changed, so did the uses of numbers and the need for them.

Consider a number problem of today!

Consider number work in the example!

- Large and small numbers
- Decimal
- Per cent
- Ratio and proportion
- Probability
- Odds

Why did I do this long and complicated problem?

- Study of numbers allows me to do it
- Problem displays some of power of numbers
- **As homework, scale this up to 2014.**

In 2014--about 1645 billionaires

What Does *Common Core* Expect of Students?

- Kindergarten—set the stage!

- Family Circus Cartoon here with PJ counting by 1s.

What Does *Common Core* Expect of Students?

- Kindergarten
  - Know number names and the count sequence
  - Understand addition as putting together and subtraction as taking apart.

- Now more than the early stories of *Number Stories of Long Ago!*

What Does *Common Core* Expect of Students?

- Kindergarten (comparable to early Roman numerals)

- What is the number that becomes one more when one is taken away from it?

Titus asked Caius in *Number Stories of Long Ago*

Solution of *Number Story*

- Think IX and XI.

What Does *Common Core* Expect of Students?

- Grade 1
- Use place value understanding and properties of operations to add and subtract.

What Does *Common Core* Expect of Students?

- Think about the Babylonians using base 60.

What Does *Common Core* Expect of Students?

- Babylonians used base 60. Principles are the same as base 10.

Example of Babylonian number with place value here.

What Does *Common Core* Expect of Students?

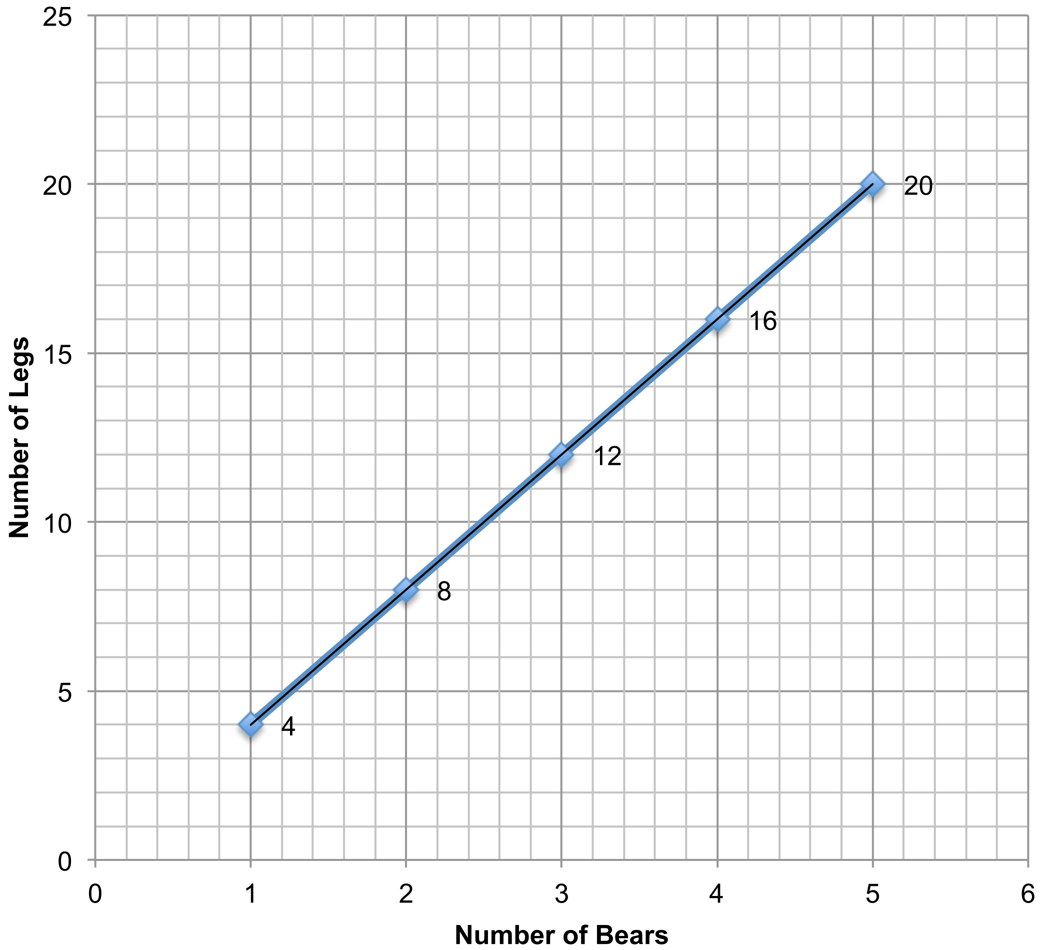
- Grade 2
- Work with equal groups of objects to gain foundations of multiplication.

What Does *Common Core* Expect of Students?

- Tie material to previous work.
  - Think of a teddy bear.
    - How many legs?
  - Think of two teddy bears.
    - How many legs?
  - Build a table.

Bears	1	2	3	4	5	6	7	8
Legs	4	8	12	16	20	24	28	32

Tie Sequence to Graph to Multiplication Table



What Does *Common Core* Expect of Students?

- Grade 3
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

What Does *Common Core* Expect of Students?

Think about *Number Stories of Long Ago*

Two examples of ancient multiplication here.

What Does *Common Core* Expect of Students?

- Grade 4
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

What Does *Common Core* Expect of Students?

Hagar the Horrible fraction cartoon here.

What Does *Common Core* Expect of Students?

- Think of *Number Stories of Long Ago.*
- Ahmes along the Nile used only unit fractions to name other fractions.
  - For example,

$$1/2 = 1/3 + 1/6$$

What Does *Common Core* Expect of Students?

- Think of *Number Stories of Long Ago.*
- Ahmes along the Nile used only unit fractions to name other fractions.

- Homework: Write 1/7 as a sum of unit fractions.

What Does *Common Core* Expect of Students?

- Grade 5
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

What Does *Common Core* Expect of Students?

From Number *Stories* of *Long Ago*

Scratch Division—two versions

Scratch Division Example here.

What Does *common Core* Expect of Students?

- Grade 6
- Apply and extend previous understandings of numbers to the system of rational numbers

What Does *common Core* Expect of Students?

- Think about *Number Stories of Long Ago. Simon Stevin wrote decimals first.*

- *Example of Simon Stevin Decimals here.*

What Does *Common Core* Expect of Students?

Pearls Before Swine
Decimal Cartoon here.

What Does *Common Core* Expect of Students?

- Grade 7
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

What Does *Common Core* Expect of Students?

- *Number Stories of Long Ago* does a chapter on puzzles.

- *Take any number, multiply it by 6, add 12, divide by 3, subtract 2, divide by 2, subtract the original number and add 9.*

- *I can tell you the result.*

What Does *Common Core* Expect of Students?

- Sidney Harris Speed Limit Cartoon here.

What Does *Common Core* Expect of Students?

- Grade 8
- Know that there are numbers that are not rational, and approximate them by rational numbers.

What Does *Common Core* Expect of Students?

- Think *Number Stories of Long Ago.*
- *If you walk halfway to the door and then walk half the remaining distance, and then half of what is left, and so on continually, how long will it take you to get out of the room?*

What Does *Common Core* Expect of Students?

- *Foxtrot Cartoon here.*

What does *Common Core* Expect from Students?

- High School
- Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation.

What does *Common Core* Expect from Students?

Solve the following equations:	Students may say:	Purpose of each equation:
a) $x+1=0$	a) I cannot find a solution in the natural numbers. I should find the solution in the integers.	a) The solution is a negative integer
b) 3x-1=0	b) I cannot find a solution in the integers. I should find the solution in the rational numbers.	b) The solution is a rational number
c) $x^2-5=0$	c) I cannot find a solution in the rational numbers. I should find the solution in the real numbers.	c) The solutions are irrational and real numbers
d) $x^2+4=0$	d) I cannot find a solution in the real numbers.	d) The solutions involve the square root of a negative number

What does *Common Core* Expect from Students?

Gamow's Problem: A treasure is hidden on an island. On the island, there are just two trees, one oak and one pine. All you know is that if you start from a point, walk all the way to the oak tree, make a 90° turn right, take the same number of steps, and put a spike (S1) in the ground. Now return to the initial point, walk to the pine, make a 90° turn left, take the same number of steps, and put another spike (S2) in the ground. If the hidden treasure is on the midpoint of the segment between two spikes then where is the initial point? (See Nahin, 1998.)

If Smith had written *Number Stories of Long Ago*

Based on what we know about *Common Core*, what might we see that is different?

If *Number Stories of Long Ago were written now*

- We would still have
  - Counting and symbolism for numerals
  - Algorithms and arguments over multidigit work
  - Fraction complications
  - Decimals as representations of numbers
  - Complex numbers in high school?

Today's *Number Stories of Long Ago*

- The following problem is long and involved but does encompass much of the work that is recommended for students in grade 8 and before.

- However, the reasoning is more explicit now.
- The technology is more essential now.

Today's *Number Stories of Long Ago*

- In very old days, we would have taken the problem to a counting table (or today's counters) in a shop.
- We might not have gotten the type of answer that we will today.

Today's *Number Stories of Long Ago*

- Here is the problem:
- Suppose that it is 2006 and a male is 43 years old (meaning he was born in 1963) and was a high school dropout. What are the odds that this man has a net worth of $50 million?

- Write your estimate now!

Today’s *Number Stories of Long Ago*

- NO!
  - You cannot assume that he is a trust-fund person.
  - You cannot assume that he just inherited the money.
  - You cannot assume that he just won the lottery.
  - This is an old style problem in today’s world.

Odds of a male H.S. dropout with a net worth of $50 million

- Posed for a boy born in 1963 at age 43 in 2006
  - According to US Census in 2000 (when kid would be 37), there were 4.02% males in the 8.07% of the age 35-39 population for a percentage of

    4.02/8.07 = 49.814% of the population.
  - Also there were 4,098,020 live births in 1963 so

    49.814% x 4,098,020 = 2,041,388 males
  - In 1979 (when male was 16), there were 102,000 male dropouts, so the odds of dropping out then were

    102,000/2,041,388 = 5%

Odds of a male H.S. dropout with a net worth of $50 million

- How many males were 43 in 2006?
  - Remember 5% of males at age 16 dropout
- How many people make $50 million?
  - In 2006 according to Forbes: the World's Richest People, there were 18 people at least 43 years old worth at least $1 billion.
  - In 2006, 34% of the population was 43 year old males so there were

$$0.34 \times 296{,}500{,}000 = 100{,}800{,}000$$

or about 100 million males

Odds of a male H.S. dropout with a net worth of \$50 million

- How many males were 43 in 2006?
  - Remember 5% of males at age 16 dropout
  - Remember 100 million males in 2006
- The proportion of billionaires is 18 per 100 million.
- The Pareto distribution allows us to estimate the probability of people whose income is greater than or equal to minimum worth x. And we can set up a ratio to scale to the 18 of 100 million.

$$(1/1 \text{ billion})^k \geq 18/100 \text{ million}$$

$$k = 0.75$$

Odds of a male H.S. dropout with a net worth of $50 million

- How many males were 43 in 2006?
  - Remember 5% of males at age 16 dropout
  - Remember 100 million males in 2006
  - $k = 0.75$
- To get the probability with $50 million net worth we have

$$(1/50 \text{ million})^{0.75} = 0.00000168$$

probability is about 0.000168%

Odds of a male H.S. dropout with a net worth of $50 million

- How many males were 43 in 2006?
  - Remember 5% of males at age 16 dropout
  - Remember 100 million males in 2006
  - Remember P(at least $50 mill) = 0.000168%

$$5\% \times 0.000168\% = 0.000000084$$

- Dropouts earn less– about 73% of graduates' salaries.
- A dropout's share of $1 is 0.73/1.73 = 0.422
- Multiplying odds by this factor yields 0.422 x 0.000000084 = 0.000000035 or about 1/28,000,000
- The odds of dropping out at 16 and having $50 million in 2006 was about 1 in 28 million.

What can we do to get ready for problems like this?

- Join NCTM and study journals
- Come to conferences
- Join Learn and Reflect sessions

- Look at NCTM's Mathematics Education Trust Grants and Awards; we fund work in different areas.

For a copy, email me

jlott@mso.umt.edu

References

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