


**Persevering through Algebra  
by Promoting Student  
Reasoning and Understanding**

**Jean S. Lee, University of Indianapolis  
Sarah A. Roberts, Iowa State University**  
NCTM Regional 2014, Indianapolis



What do we know about  
implementing good tasks?

## Principles to Action: Mathematics Teaching Practices



- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student learning.

## Skyscraper Windows Task

*A building is 12 stories high and is covered entirely by windows on all four sides. Each floor has 38 windows on it. Once a year, all the windows are washed.*

*The cost for washing the windows is \$2.00 for each first-floor window, \$2.50 for each second-floor window, \$3.00 for each third floor window, and so on.*

*How much will it cost to wash the windows of this building?*

*What if the building is 30 stories tall?  $n$  stories tall?*

*(Driscoll 1999, p. 70)*



Share and discuss your strategies!



Student Solution #1:

Students produce a table

12 stories		cost per all 12 stories	
12	\$7.50 = \$285	= \$2,166	
11	\$7.00 \$260	cost per all 30 stories	
10	\$6.50 \$247	= \$10,545	
9	\$6.00 \$228		
8	\$5.50 \$209		
7	\$5.00 \$190		
6	\$4.50 \$171		
5	\$4.00 \$152		
4	\$3.50 \$133		
3	\$3.00 \$114		
2	\$2.60 \$95		
1	\$2.00 per window \$70		

11	2.00 = 70	16	9.5 = 301
2	2.5 95	17	10.5 320
3	3 114	18	10.5 329
4	3.5 133	19	11 348
5	4 152	20	11.5 367
6	4.5 171	21	12 386
7	5 190	22	12.5 405
8	5.5 209	23	13 424
9	6 228	24	13.5 443
10	6.5 247	25	14 462
11	7 266	26	14.5 481
12	7.5 285	27	15 500
13	8 304	28	15.5 519
14	8.5 323	29	16 538
15	9 342	30	16.5 557

30 stories

## Student Solution #2: Finding patterns in a table

floor(n)	#/window $\times$	windows $=$	Total F(n) price/floor	Total $\Delta$ price/floor
1	\$2	38	$\$76 = 76 + 19(0)$	19
2	\$2.50	38	$95 = 76 + 19(1)$	19
3	\$3	38	$114 = 76 + 19(2)$	19
4	\$3.50	38	$133 = 76 + 19(3)$	19
5	\$4	38	$152 = 76 + 19(4)$	19
6	\$4.50	38	$171 = 76 + 19(5)$	19
7	\$5	38	$190 = 76 + 19(6)$	19
8	\$5.50	38	$209 = 76 + 19(7)$	19
9	\$6	38	$228 = 76 + 19(8)$	19
10	\$6.50	38	$247 = 76 + 19(9)$	19
11	\$7	38	$266 = 76 + 19(10)$	19
12	\$7.50	38	$285 = 76 + 19(11)$	19
			<u>2166</u>	

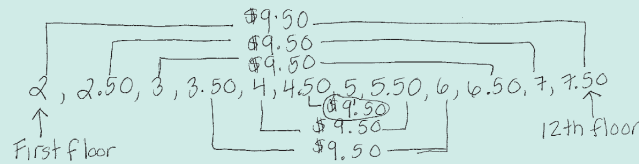
## Student Solution #3: Creating an equation for a single floor

What I know:

- add \$0.50 per floor per window
- let  $X =$  Number of floors
- # of windows remains constant (38 per floor)
- for each additional floor, cost is additional \$19.

linear relationship  $1.50 + 0.50n =$  cost for that floor so:  $38(1.50 + 0.50n)$

### Student Solution #4: Using a pairing strategy to identify a sum



\$9.50 for 6 pairs of 12 floors  
 $6(\$9.50) = \$57$  per vertical column  
 $\$57 \times 38$  windows = \$2166.00

Therefore, Total cost =  $\frac{n}{2}(a_1 + a_n) \cdot 38$   
 # of pairs common per floor sum

$n$  = # of floors  
 $a_1$  = first floor  
 $a_n$  = last floor

### Student Solution #5: Using the price of washing first and last floors to establish a mean

$$\text{last story} = 1.50 + (\text{story} \cdot .50)$$

$$\frac{(\text{first story} + \text{last story})}{2} \cdot \text{story} \cdot 38$$

$$\text{30 story: cost} = 1.50 + (30 \cdot .50) = \$16.50$$

$$\frac{(2.00 + 16.50)}{2} \cdot 30 \cdot 38$$

$$= \$10,545$$

n stories:

$$= \frac{(2.00 + [1.50 + .50n])}{2} \cdot n \cdot 38$$

## Student Solution #6: Using the arithmetic sequence and series

$$\begin{aligned}
 \text{Total Cost} &= \sum_{n=1}^k 38(1.5 + .5n), \quad k = \# \text{ of floors} \\
 &= 38 \left[ \sum_{n=1}^k 1.5 + \sum_{n=1}^k .5n \right] \\
 &= 38 \left[ 1.5k + \frac{.5k(k+1)}{2} \right] \quad \text{b/c } \sum_{n=1}^k n = \frac{k(k+1)}{2} \\
 &= 38(1.5k) + 38 \left( \frac{.5k(k+1)}{2} \right) \\
 &= 57k + \frac{19k(k+1)}{2} \\
 &= \frac{114k + 19k(k+1)}{2} \\
 &= \frac{114k + 19k^2 + 19k}{2} \\
 &= \frac{19k^2 + 133k}{2} \\
 &= \frac{19k(k+7)}{2}
 \end{aligned}$$

## Audience Reflection

1. How do you use this task and uphold the *Principles to Action Mathematics Teaching Practices*?
2. What do teachers need to attend to when implementing this specific task?
3. How can we promote student reasoning and understanding as a way to persevere in solving this task?

## Questions?

Jean S. Lee	jslee@uindy.edu
Sarah A. Roberts	sroberts@education.ucsb.edu

Driscoll, M. (1999). *Fostering Algebraic Thinking: A Guide for Teachers, Grades 6-10*. Portsmouth, NH: Heinemann.

Roberts, S.A. & Lee, J.S. (2013). A skyscraping feat. *Mathematics Teacher*, 104(4), 258-264.



## Tips for a great conference!

Rate this presentation on the conference app **[www.nctm.org/confapp](http://www.nctm.org/confapp)**

Download available presentation handouts from the Online Planner!  
**[www.nctm.org/planner](http://www.nctm.org/planner)**

Join the conversation! Tweet us using the hashtag **#NCTMIndy**