

Strategies for Moving Students toward Fact Fluency

Grades K-5

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Mastery for ALL Students

Fortunately we know quite a bit about helping children develop fact mastery, and it has little to do with the quantity of drill or the drill techniques.



What does “mastery” mean?

“A child can give a quick response without resorting to nonefficient means, such as counting.”



Van de Walle & Lovin, 2006, p. 94

Who can “master” the facts?

“All children are able to master the basic facts – including children with learning disabilities.”



Van de Walle & Lovin, 2006, p. 94

What do we know about “mastering” the facts?

- *Little to do with QUANTITY of drill*
- *Develop understanding of number relationships and properties of operations*
- *Develop efficient STRATEGIES for fact retrieval through practice*
- *Drill use and SELECTION of strategies*

The Role of Number and Operation Concepts

Composing & Decomposing Single-Digit Numbers

- *“Flash” Activities*
- *Problem Solving Activities*

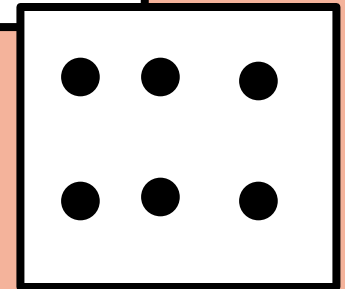
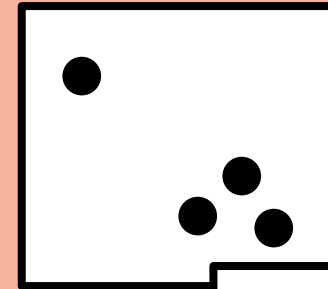
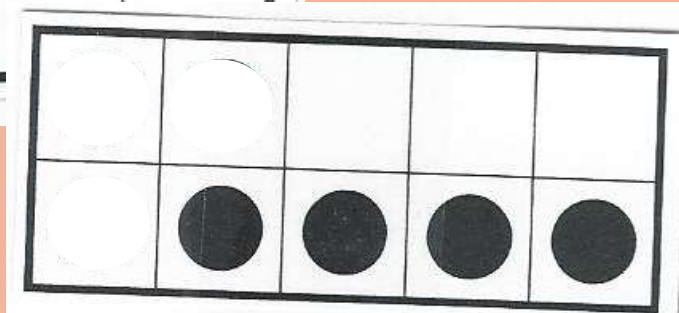
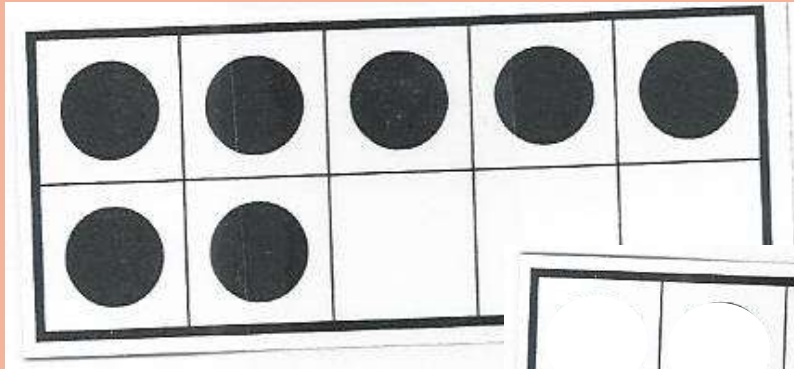
Meanings & Properties of Operations

- *Story Problems for operation sense*
- *Read Expressions with Comprehension*
- *Identifying Patterns & Relationship*

Number Activities #1

“Flash” with Ten Frames & Dot Cards

How many do you see?
How do you see them?

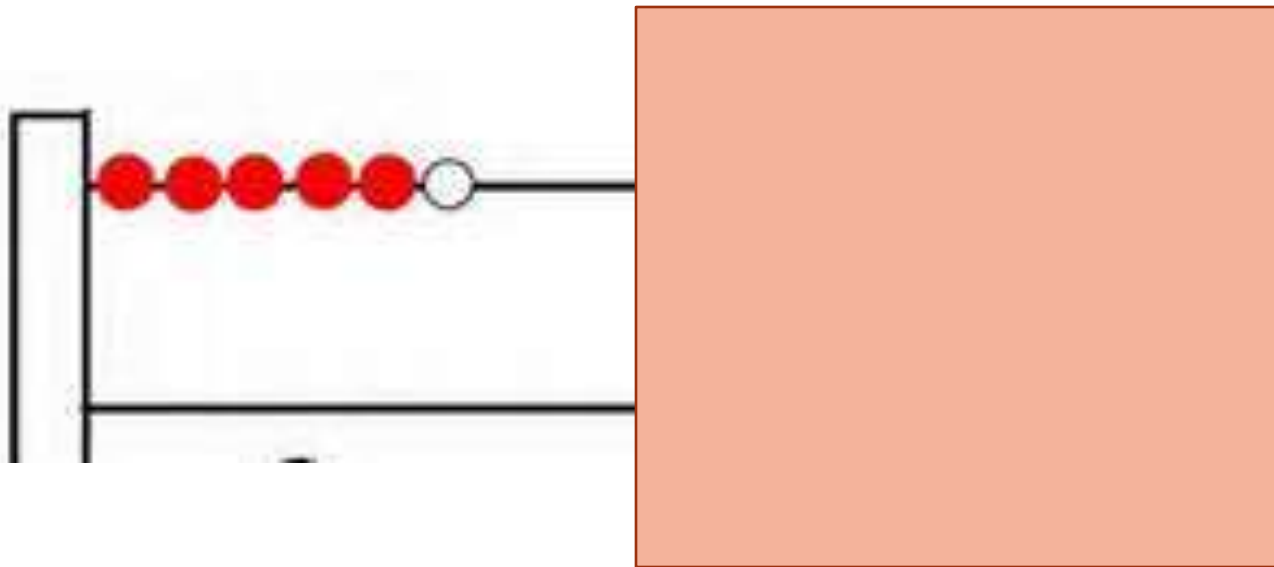




Number Activities #2

“Flash” with Rekenreks

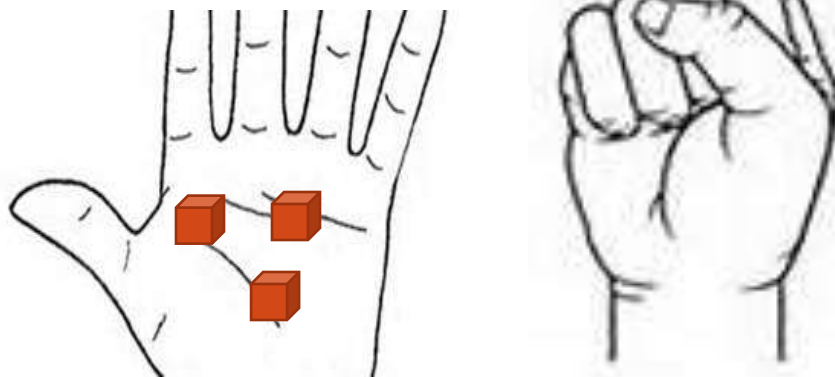
How many do you see?
How do you see them?



Number Activities #3

“Flash” with Small Objects

If we started with 5, how many are hiding?



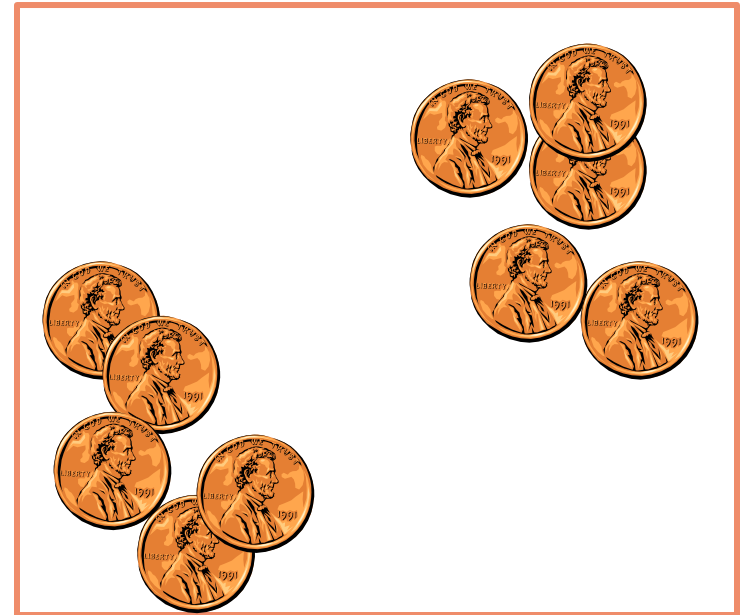
(We also use this to “measure” the fact fluency of kindergartners.) 😊

Number Activities #4

Sense-Making Activities

What different ways can you hold 10 pennies?

- How many in your left hand?
- How many in your right hand?



Operations Activities #1: Operation Sense

All problem types: Tables 1 & 2 CCSS pages 88-89

TABLE 2. Common multiplication and division situations.⁷

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays, ⁴ Area ⁵	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

Operations Activities #2:

Read with Comprehension

Use meaningful language as you teach expressions / equations with new operation symbols:

- Two bunnies sat on the grass. Three more bunnies hopped there.
How many bunnies are on the grass now?



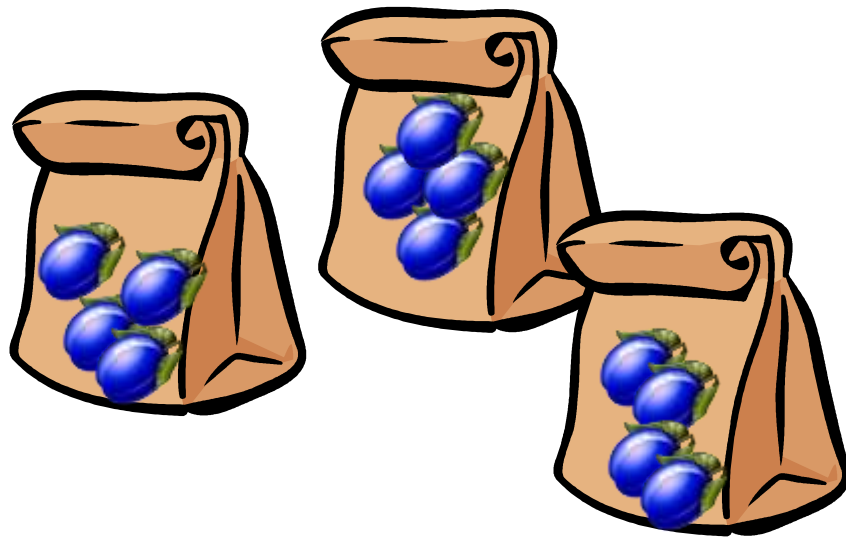
$$2 + 3 = ?$$

“2 bunnies joined with 3 bunnies
is the same amount as how many
bunnies?”

Operations Activities #2: Read with Comprehension

Use meaningful language as you teach expressions / equations with new operation symbols:

There are three bags with four plums in each bag. How many plums are there?



$$3 \times 4 = ?$$

“3 bags of 4 plums is the same amount as how many plums?”

Operations Activities #3: Properties of Operations

Find patterns and make generalizations using contexts:

- $+ 0$ (Additive Identity Property)
- $a + b = b + a$ (Commutative Property of Addition)
- $\times 0$
- $\times 1$ (Multiplicative Identity Property)
- $a \times b = b \times a$ (Commutative Property of Multiplication)

How do we help students develop efficient strategies?

What is a Strategy?

Computation strategy - Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. (CCSS, p. 85)

“A strategy is most useful to students when it is theirs, built on and connected to concepts and relationships they already own.”

(Van de Walle & Lovin, 2006, p. 96)

How do we help students develop efficient strategies?

- *Plan simple story problem lessons*
- *Plan lessons to find structures, relationships, and patterns*
- *Avoid temptation to simply “tell” students about a strategy and then have them practice it.*



Drill vs. Practice

- *Drill* — repetitive non-problem-based activity



- *Practice* — problem-based activities in which students are encouraged to develop flexible and useful strategies that are meaningful



Avoid Premature Drill

- *No new information*
- *No new connections*

Practice Strategy Selection

- *“Name the strategy”*
- *Sort by strategy*

Overview of Approach

- *Make strategies explicit*
- *Drill established strategies*
- *Individualize*
- *Practice strategy selection*

- *PROVIDE “HOPE”*



Dos and Don'ts for Teaching Basic Facts

Do:

- Ask children to self-monitor
- Focus on self-improvement
- Drill in short time segments
- Work on facts over time
- Involve families
- Use technology

Don't:

- Use lengthy timed-tests
- Use public comparisons of mastery
- Proceed through the facts in order from 0 – 9
- Work on all the facts at once
- Move to quick-recall activities too soon
- Use facts as a barrier to good mathematics
- Use fact mastery as a prerequisite for calculator use

Resources:

- Van de Walle & Lovin (2006). *Teaching Student-Centered Mathematics: Grades K-3* (ISBN: 0-205-40843-5) = *first Edition*
- Van de Walle, Lovin, Karp, & Bay-Williams (2014). *Teaching Student-Centered Mathematics: Developmentally Appropriate Instruction for Grades PreK-2*. (ISBN: 0-13-282482-5) = *second edition*
- Tiny ten frames: www.ablongman.com/vandewalleseries/
- Contexts for Learning by Catherine Fosnot: contextsforlearning.com
- CCSS Progressions documents for math: <http://ime.math.arizona.edu/progressions/>
- Box of Facts / Book of Facts: <http://www.origoeducation.com/the-box-of-facts/>
- Rekenrek activities:
 - <http://www.k-5mathteachingresources.com/Rekenrek.html>
 - <http://www.enasco.com/c/math/Math+Manipulatives/Counting+%26+Sorting/Rekenreks/>
 - http://bridges1.mathlearningcenter.org/media/Rekenrek_0308.pdf

Strategies for Addition and Subtraction in Grades K-2

1. Zero Property
2. Commutative Property
3. + 1, +2 (Counting On)
4. +10
5. Doubles
6. Subitizing on Ten Frames & Combos to 10
7. Doubles plus
8. Memorize 6+3
9. + 9
10. Go through 10

121 Known Facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

Subtraction Strategies for grades K-8

-	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
0	0	1	2	3	4	5	6	7	8	9	10											
1		0	1	2	3	4	5	6	7	8	9	10										
2			0	1	2	3	4	5	6	7	8	9	10									
3				0	1	2	3	4	5	6	7	8	9	10								
4	K				0	1	2	3	4	5	6	7	8	9	10							
5						0	1	2	3	4	5	6	7	8	9	10						Gr 2
6							0	1	2	3	4	5	6	7	8	9	10					
7								0	1	2	3	4	5	6	7	8	9	10				
8									0	1	2	3	4	5	6	7	8	9	10			
9										0	1	2	3	4	5	6	7	8	9	10		
10											0	1	2	3	4	5	6	7	8	9	10	

- -0
- -1 & -2 (Count back)
- Back through 5 (e.g., $6 - 3 = 6 - 1 - 2$)
- Back through 10 (e.g., $13 - 4 = 13 - 3 - 1$)
- Up through 10 (e.g. $13 - 9$ “count on 1 to get 10 and 3 more to get to 13, so 9 is 4 away from 13”)
- Think addition / Think addition doubles ($13 - 9 = ?$ Is the same as $9 + ? = 13$)

Strategies for Addition and Subtraction for Grades 3-8

1. Zero Property
 2. + 1 (Counting On)
 3. + 10 (teen numbers)
 4. Doubles
 5. Commutative Property
 6. + 2 (Counting On)
 7. Subitizing on 10 Frame / Combos to Make 10
 8. Go through 10
 9. + 5 / Go through 5
(Visualize the 10 frame)
 10. Memorize 6 + 3
- 121 Known Facts

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

Multiplication Strategies Grades K-8

x	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

1. Zero Property
2. Identity Property
3. Commutative property
4. Doubles
5. Tens facts

1. Ten and half
2. Doubles + a group
3. Double – Double
4. Nines
5. Double – Double – Double

1. **6x6**
2. **6x7, 7x6**
3. **7x7**
4. **Distributive Property w/known facts**

121 Known Facts