USING STORY PROBLEMS TO TEACH TO SPECIFIC MATH GOALS

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The Teacher's Role

1. Identify the next needed learning for the students – set learning goals
2. Analyze and adapt problems in order to further math goals
3. Facilitate the introduction, problem solving time, & closure to focus on the learning goals

"There are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time.

Choosing problems wisely, and using and adapting problems from instructional materials, is a difficult part of teaching mathematics."

(PSSM, 2000, p. 53)

Story Problems in the Common Core

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See also: 3.MD.7b and 3.MD.7d

3.OA.8 Solve 2-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of the answers using mental computation and estimation strategies including rounding.

3.MD.1 Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line.

3.MD.2 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings to represent the problem.

3.MD.8 solve real word and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same ar3ea and different perimeters.

Story Problems in the Common Core

- 4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

- 4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity.

- 4.NF.3d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

- 4.NF.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equation to represent the problem.

- 4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit.

- 4.MD.3 Apply the area and perimeter formulas for rectangles in real word and mathematical problems.

- 4.MD.7 Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Story Problems in the Common Core

- 5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole. Including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem.
- 5.NF.3 solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.ge. By using visual fraction models or equations to represent the problem.
- 5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- 5.NF.7c Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.
- 5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
- 5.G.2 Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Three Learning Goals

- **Introducing New Problem Types**
- **Symbolic Representations for New Problem Types**
- Computation Strategies

Learning Goals #1 & #2: New Problem Types

Understanding new contexts & developing operation sense – understanding what the problem is asking as well as learning ways to represent the problem with physical models, pictures, and equations.

1. **Dramatize – INCLUDED IN LESSONS FOR LEARNING GOAL #1**
   Act out, realistic objects/pictures, visualize, imagine in your "mind's eye"

2. **Model – INCLUDED IN LESSONS FOR LEARNING GOALS #1 & #2**
   Use concrete / physical objects to represent the dramatization, not all the aspects may be present

3. **Pictures / Diagrams – INCLUDED IN LESSONS FOR LEARNING GOALS #1 & #2**
   Use drawings and symbols to represent the dramatization or physical models

4. **Symbolic (Operation Sense Level) – SAVED FOR LESSONS CENTERED ON LEARNING GOAL #2**
   Equations / number sentences

(First Steps in Mathematics: Number Book 2, 2007)

http://det.wa.edu.au/stepsresources/detcms/navigation/first-steps-mathematics/?oid=MultiPartArticle-id-13603817

Learning Goal #1: Introducing New Problem Types

Use "I can" statements to set learning goals

- I can describe the reading strategies I used to take apart and understand the story problem.
- I can represent the story problem with an efficient model and picture.
- I can explain the connections between my picture representation and the story problem.

Use "essential questions" to set learning goals

- What reading skills help us to make sense of story problems?
- How can we represent story problems with efficient models / drawings?
- How can I use my words to explain the connections between my models / representations and the story problem?

Learning Goal #1: Introducing New Problem Types

Choose / create / adapt problems:

- CCSS Table 1 p. 88 & Progressions document for OA
  -http://ime.math.arizona.edu/progressions/
- Initial computation should be relatively simple
- Simple sentences / contexts
- Vary the order the information is given

Learning Goal #1: Introducing New Problem Types

- **Mini-lesson:** Model Math Practice #1 and make sense of this new problem type and, in this case, connect to modeling / representations math practice standard #4.

Tim and Andy went fishing.

Tim caught 5 times as many fish as Andy.

Andy caught 2 fish.

How many fish did Tim catch?

Learning Goal #1: Introducing New Problem Types

- Problem Solving Phase (20-30 minutes) (Partner / Individual)

- Student's Role
  - Find the answer to the original problem and possibly a similar second problem
  - Find a way to draw / record what they did with their objects or what they visualized
  - Answer the questions in the problems

- Teacher's Role
  - Ask students to explain their thinking and actions
  - Prompt students to find a way to record their work in an organized way
  - Determine which 2-3 students' models and / or pictures to share

Learning Goal #1: Introducing New Problem Types

Problem Solving Phase: Students work independently or with partners to make sense of new problem type through modeling and drawing. Teacher supports / scaffolds instruction with questions focused on the learning goals.

1. Callie has 3 times as many toy trucks as Bob. Bob has 11 toy trucks. How many toy trucks does Callie have?

2. Jerry and Sandy each have a bag of Skittles. Jerry counted 9 Skittles in his bag. Sandy had 4 times as many Skittles as Jerry in her bag. How many Skittles were in Sandy's bag?

Learning Goal #1: Introducing New Problem Types

Closure: Focus sharing on learning goals

- How can we represent story problems with efficient models / drawings?
- How can I use my words to explain the connections between my models / representations and the story problem?
- What reading skills help us to make sense of story problems?

Learning Goal #2:
Symbolic Representations for New Problem Types

Use "I can" statements to set learning goals

- I can represent the story problem with an efficient model and / or picture.
- I can represent what happens in the story and my model with an equation.
- I can explain the connections between my model, picture, and equation and the story problem.

Use "essential questions" to set learning goals

- How can we represent story problems with efficient models and pictures?
- How can we use our models and pictures to help us write equations with variables to represent the unknown?
- How can we use our words to explain the connections between the story problem, our models, and our equations?

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Choose / create / adapt problems:

- **Problem types that students are able to represent with models and pictures**

- **Quantities large enough that students don't just "know" what's missing**

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Warm Up: Mental math strategies for multiplication

3 x 6	4 x 8	3 x 25
30 x 6	4 x 80	3 x 26
3 x 60	4 x 82	3 x 126
3 x 67		

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Mini-lesson: Efficient representations

The homecoming football game was played on a very hot day. The trainers filled 5 glasses with water for each of the 39 football players and 3 glasses of water for each of the 12 cheerleaders. How many glasses of water were poured?

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Problem Solving Phase: Students work independently or with a partner to model and attempt to write equations to match using what they know about operations and grouping symbols such as parentheses.

1. The homecoming football game was played on a very hot day. The trainers filled 5 glasses with water for each of the 39 football players and 3 glasses of water for each of the 12 cheerleaders. How many glasses of water were poured?

2. The homecoming football game was played on a very hot day. The trainers filled 5 glasses with water for each of the 39 football players and for each of the 12 cheerleaders. How many glasses of water were poured?

Learning Goal #2:
Symbolic Representations for "New" Problem Types

- Problem Solving Phase (20-30 minutes) (Partner / Individual)

- Student's Role
  - Solve the two problems and possibly a similar third problem
  - Find a way to picture/model what they visualized.
  - Determine corresponding equations for each problem.
  - Answer the questions in the problems

- Teacher's Role
  - Ask students to explain their thinking and actions
  - Prompt students to find a way to record their work in an organized way
  - Determine which 2-3 students' models and / or pictures to share

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Closure: Focus the discussion on the learning goals

- How can we represent story problems with efficient models and pictures?
- How can we use our models and pictures to help us write number sentences?
- How can we use our words to explain the connections between the story problem, our models, and our number sentence?

Three Learning Goals

- Introducing "New" Problem Types
- Symbolic Representations for "New" Problem Types
- **Computation Strategies**

Computation Strategies in the Common Core

3.OA.5 **Apply properties of operations as strategies** to multiply and divide.

3.OA.7 Fluently multiply and divide within 100 **using strategies** such as the relationship between multiplication and division or properties of operations.

3.NBT.2 Fluently add and subtract within 1000 **using strategies and algorithms** based on place value, properties of operations and / or the relationship between addition and subtraction.

3.NBT.3 Multiply 1-digit whole numbers by multiples of 10 in the range of 10-90 **using strategies** based on place value and properties of operations.

Computation Strategies in the Common Core

4.NBT.4 Fluently add and subtract multi-digit whole numbers **using the standard algorithm.**

4.NBT.5 Multiply a whole number of up to 4 digits by a 1-digit whole number, and multiply two 2-digit numbers, **using strategies** based on place value and the properties of operations. **Illustrate and explain the calculation** by using equations, rectangular arrays, and/or area models.

4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, **using strategies** based on place value, the properties of operations, and/or the relationship between multiplication and division**. Illustrate and explain** the calculation by using equations, rectangular arrays, and/or area models.

4.NF. Add & subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an **equivalent fractions**, and/or by using **properties of operations** and the **relationship between addition and subtraction**.

4.NF.4 **Apply & extend previous understandings of multiplication** to multiply a fraction by a whole number.

4.NF.5 Express a fraction with denominator 10 as an **equivalent fraction** with denominator 100, and use this technique to add two fractions with respective denominators 10 & 100.

Computation Strategies in the Common Core

5.NBT.5 Fluently multiply multi-digit numbers using the **standard algorithm**

5.NBT.6 find whole-number quotients of whole numbers with up to 4-digit dividends and 2-digit divisors, **using strategies** based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

5.NBT.7 Add, subtract, multiply, and divide decimals to hundredths, **using concrete models or drawings and strategies** based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

5.NF.1 Add and subtract fractions with unlike denominators by replacing given fractions with **equivalent fractions** in such a way as to produce and equivalent sum or difference of fractions with like denominators.

5.NF.4 **Apply and extend previous understandings** of multiplication to multiply a fraction or whole number by a fraction.

5.NF.7 **Apply and extend previous understandings** of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

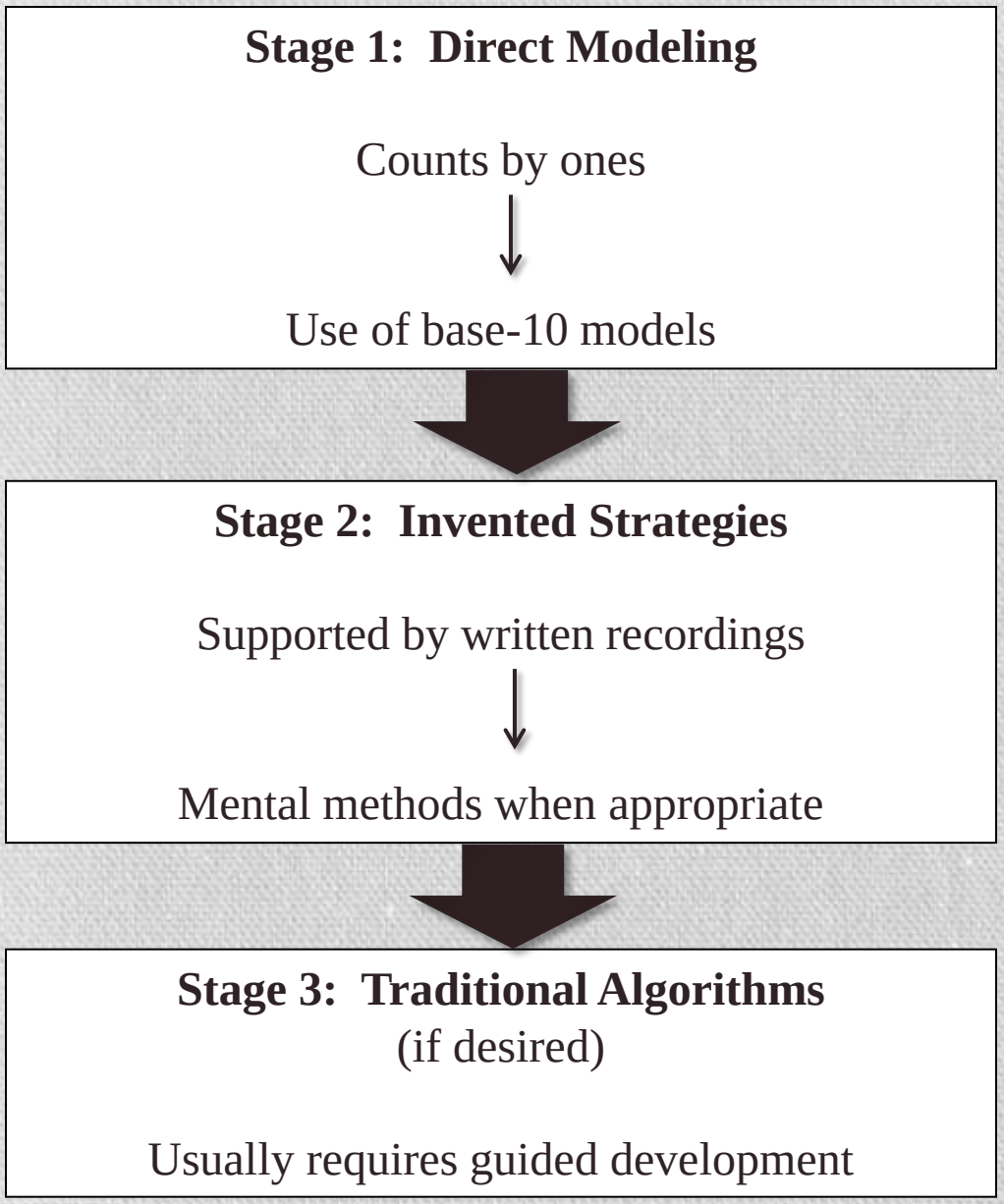
Computation Strategies in the Common Core

Computation Strategy

Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Computation Algorithm

A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.



Van de Walle & Lovin, 2006, p. 158

Learning Goal #3: Computation Strategies

Use "I can" statements to set learning goals

- I can represent story problems with equations.
- I can represent my calculations and thinking with number phrases and/or expressions.
- I can justify my adding or subtracting strategy using words and my number phrases.

Use "essential questions" to set learning goals

- How can I represent the story problem with an equation?
- How can I represent my thinking and calculations with numerical expressions?
- How can I use my words and expressions to justify my adding and subtracting strategy?

Learning Goal #3: Computation Strategies

- **Content Standard: 3.NBT.2**

  Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations and / or the relationship between addition and subtraction.

Fluent =

- Efficient
- Flexible
- Accurate

Learning Goal #3: Computation Strategies

Choose / create / adapt problems

- Contexts that contain easily recognized operations
- Quantities that move students forward to more complex / efficient strategies
- Tools that support students thinking as they move forward to more complex / efficient strategies

Learning Goal #3: Computation Strategies

Warm Up: mental math with money

25¢ + 26¢

76¢ + 27¢

$1.25 + 75¢

$5.10 + $3.06

$1.98 + $2.02

Learning Goal #3: Computation Strategies

Problem Solving Phase: Students work independently or with partners to develop computation strategies and ways to record their thinking.

Garth and his sister Christine both play basketball. Garth scored 198 points this season and Christine scored 204 points this season. How many points did they score altogether?

Garth and his sister Christine both play basketball. Garth scored 198 points this season and Christine scored 204 points this season. Who scored more points? How many more points?

At Riverview the chicks started hatching on May 1. There were 235 chicks out of their shells by the end of the day. The next morning there were 301 chicks out of their shells. How many more chicks got out of the shells over night?

Learning Goal #3: Computation Strategies

Closure: Focus sharing on learning goals

- How can I represent the story problem with an equation?
- How can I represent my thinking and calculations with numerical expressions?
- How can I use my words and expressions to justify my adding and subtracting strategy?

References and Resources

- Progressions documents for CCSS:

  ime.math.arizona.edu/progressions

- Van de Walle & Lovin, 2006. *Teaching Student-Centered Mathematics, grades K-3. Pearson: New York.*
- *CCSS, Tables 1 & 2, pp. 88-89*

- *Contact me with questions / comments:*

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