USING WORD PROBLEMS IN EARLY GRADES

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Lori Williams, Ph.D.

dr.loriwilliams@sbcglobal.net

The Teacher's Role

1. Identify the next needed learning for the students – set learning goals
2. Analyze and adapt problems in order to further math goals
3. Facilitate the introduction, problem solving time, & closure to focus on the learning goals

"There are many, many problems that are interesting and fun but that may not lead to the development of the mathematical ideas that are important for a class at a particular time.

Choosing problems wisely, and using and adapting problems from instructional materials, is a difficult part of teaching mathematics."

(PSSM, 2000, p. 53)

Story Problems in the Common Core

- K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations.

- K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

Story Problems in the Common Core

- 1.OA.1 Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

- 1.OA.2 Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Story Problems in the Common Core

- 2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

- 2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

- 2.MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately.

Three Learning Goals

- **Introducing New Problem Types**
- **Symbolic Representations for New Problem Types**
- Computation Strategies

Learning Goals #1 & #2: New Problem Types

Understanding new contexts & developing operation sense – understanding what the problem is asking as well as learning ways to represent the problem with physical models, pictures, and equations.

1. **Dramatize – INCLUDED IN LESSONS FOR LEARNING GOAL #1**
   Act out, realistic objects/pictures, visualize, imagine in your "mind's eye"

2. **Model – INCLUDED IN LESSONS FOR LEARNING GOALS #1 & #2**
   Use concrete / physical objects to represent the dramatization, not all the aspects may be present

3. **Pictures / Diagrams – INCLUDED IN LESSONS FOR LEARNING GOALS #1 & #2**
   Use drawings and symbols to represent the dramatization or physical models

4. **Symbolic (Operation Sense Level) – SAVED FOR LESSONS CENTERED ON LEARNING GOAL #2**
   Equations / number sentences

(First Steps in Mathematics: Number Book 2, 2007)

http://det.wa.edu.au/stepsresources/detcms/navigation/first-steps-mathematics/?oid=MultiPartArticle-id-13603817

Learning Goal #1: Introducing New Problem Types

Use "I can" statements to set learning goals

- I can describe the reading strategies I used to take apart and understand the story problem.
- I can represent the story problem with an efficient model and picture.
- I can explain the connections between my picture representation and the story problem.

Use "essential questions" to set learning goals

- What reading skills help us to make sense of story problems?
- How can we represent story problems with efficient models / drawings?
- How can I use my words to explain the connections between my models / representations and the story problem?

Learning Goal #1: Introducing New Problem Types

Choose / create / adapt problems:

- CCSS Table 1 p. 88 & Progressions document for OA
  -http://ime.math.arizona.edu/progressions/
  - "compare with smaller unknown"

- Single digits

- Simple sentences / contexts

- Vary the order the information is given

Learning Goal #1: Introducing New Problem Types

Warm Up: Modeling "more than" and "fewer than" situations

- Partner 1 has 5 blocks. Partner 2 should have the SAME number of blocks.
- Partner 2 has 5 blocks. Partner 1 has 2 more than Partner 2.
- Partner 1 has 7 blocks. Partner 2 has the SAME number of blocks as Partner 1.
- Partner 1 has 7 blocks. Partner 2 has 3 fewer blocks than Partner 1.
- Partner 2 has 4 blocks. Partner 1 has the SAME number of blocks as Partner 2.
- Partner 2 has 4 blocks. Partner 1 has 2 fewer than Partner 2.
- Partner 1 has 2 blocks. Partner 2 has the SAME number of blocks as Partner 1.
- Partner 1 has 2 blocks. Partner 2 has 7 more blocks than Partner 1.

Learning Goal #1: Introducing New Problem Types

- **Mini-lesson:** Model Math Practice #1 and make sense of this new problem type and, in this case, connect to modeling / representations math practice standard #4.

Tim and Andy went fishing.

Tim caught 5 more fish than Andy.

Tim caught 8 fish.

How many fish did Andy catch?

Learning Goal #1: Introducing New Problem Types

- Problem Solving Phase (20-30 minutes) (Partner / Individual)

- Student's Role
  - Find the answer to the original problem and possibly a similar second problem
  - Find a way to draw / record what they did with their objects
  - Answer the questions in the problems

- Teacher's Role
  - Ask students to explain their thinking and actions
  - Prompt students to find a way to record their work in an organized way
  - Determine which 2-3 students' models and / or pictures to share

Learning Goal #1: Introducing New Problem Types

Problem Solving Phase: Students work independently or with partners to make sense of new problem type through modeling and drawing. Teacher supports / scaffolds instruction with questions focused on the learning goals.

1. Callie has 7 more toy trucks than Bob. Callie has 12 toy trucks. How many toy trucks does Bob have?

2. Jerry and Sandy each have a bag of Skittles. Jerry counted 16 Skittles in his bag. Sandy had 4 fewer Skittles in her bag than Jerry. How many Skittles were in Sandy's bag?

3. Maria and Jose went fishing. Jose caught a fish that was 8 inches longer than Maria's fish. If Jose's fish was 18 inches long, how long was Maria's?

Learning Goal #1: Introducing New Problem Types

Closure: Focus sharing on learning goals

- How can we represent story problems with efficient models / drawings?
- How can I use my words to explain the connections between my models / representations and the story problem?
- What reading skills help us to make sense of story problems?

Learning Goal #2:
Symbolic Representations for New Problem Types

Use "I can" statements to set learning goals

- I can represent the story problem with an efficient model and / or picture.
- I can represent what happens in the story and my model with a number sentence.
- I can explain the connections between my model, picture, and number sentence and the story problem.

Use "essential questions" to set learning goals

- How can we represent story problems with efficient models and pictures?
- How can we use our models and pictures to help us write number sentences?
- How can we use our words to explain the connections between the story problem, our models, and our number sentence?

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Choose / create / adapt problems:

- **Problem types that students are able to represent with models and pictures**

- **Quantities large enough that students don't just "know" what's missing**

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Warm Up: Reviewing when to use + and –

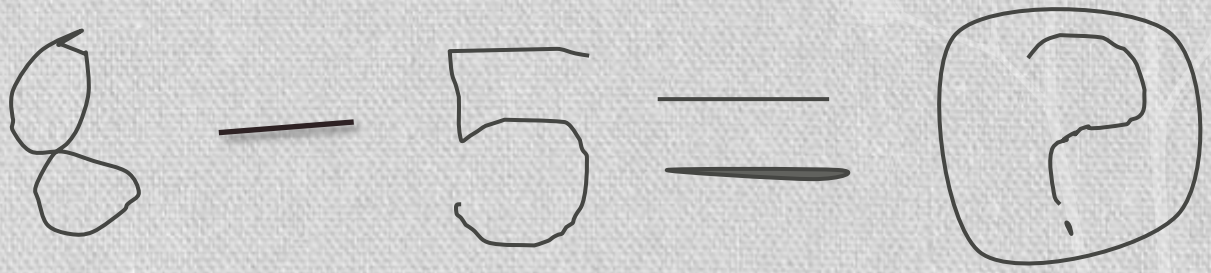
- I'll read a story, you show with your fingers whether you would add or subtract the numbers. Be ready to say the number sentence and how you knew to use a plus or minus in your number sentence.
  - Sam ate 3 cookies after lunch and 2 cookies after supper. How many cookies did Sam eat?
  - Beth had 5 dollars. Then she gave 2 dollars to her brother. How much money does Beth have now?
  - There were 6 birds in the tree. Then a new group of birds landed in the tree. Now there are 9 birds in the tree. How many birds were in the new group of birds?
  - Tommy's mom cooked some fish in a pan. Tommy's mom took 3 fish from the pan and put them on his plate. There were 6 fish left in the pan. How many fish did Tommy's mom cook?

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Mini-Lesson: Matching number sentences to the actions students recorded in their pictures.

- Using drawings from prior day's problems, model Math Practices #4: Model with Mathematics & #6: Attend to Precision.

Tim and Andy went fishing. Tim caught 5 more fish than Andy. Tim caught 8 fish. How many fish did Andy catch?



Learning Goal #2:
Symbolic Representations for "New" Problem Types

Problem Solving Phase: Students work independently or with a partner to model and write equations for a problem type with which they are quite comfortable and for a problem of the "new" type.

1. Callie has 7 toy trucks. Bob has 12 toy trucks. How many more toy trucks does Bob have than Callie?

2. Jerry and Sandy each have a bag of Starbursts. Jerry counted 16 candies in his bag. Sandy had 4 fewer candies in her bag than Jerry. How many Skittles were in Sandy's bag?

Learning Goal #2:
Symbolic Representations for "New" Problem Types

- Problem Solving Phase (20-30 minutes) (Partner / Individual)

- Student's Role
  - Solve the two problems and possibly a similar third problem
  - Find a way to draw / record what they did with their objects
  - Determine corresponding equations for each problem.
  - Answer the questions in the problems

- Teacher's Role
  - Ask students to explain their thinking and actions
  - Prompt students to find a way to record their work in an organized way
  - Determine which 2-3 students' models and / or pictures to share
  - Possibly choose a work that isn't quite finished to complete as a class

Learning Goal #2:
Symbolic Representations for "New" Problem Types

Closure: Focus the discussion on the learning goals

- How can we represent story problems with efficient models and pictures?
- How can we use our models and pictures to help us write number sentences?
- How can we use our words to explain the connections between the story problem, our models, and our number sentence?

Three Learning Goals

- Introducing "New" Problem Types
- Symbolic Representations for "New" Problem Types
- **Computation Strategies**

Computation Strategies in the Common Core

K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by **using objects or drawings**, and record each decomposition by a drawing or equation (e.g., 5=2+3)

K.OA.4 For any number 1 to 9, find the number that makes 10 when added to the given number, e.g., by **using objects or drawings**, and record the answer with a drawing or equation.

K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones; e.g., **by using objects or drawings** and record each composition or decomposition by a drawing or equation. Understand that these numbers are composed of ten ones and 1, 2, 3, 4, 5, 6, 7, 8, or 9 ones.

Computation Strategies in the Common Core

1.OA.3 **Apply properties of operations** as strategies to add and subtract.

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. **Use strategies** such as counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, creating equivalent but easier or known sums.

1.NBT.4 Add within 100, including adding a 2-digit number & a 1-digit number, and adding a 2-digit number and a multiple of 10, **using concrete models or drawings and strategies** based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

1.NBT.5 Given a 2-digit number, **mentally find** 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiplies of 10 in the range 10-90 from multiples of 10 in the range 10-90, **using concrete models or drawings and strategies** based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Computation Strategies in the Common Core

- 2.OA.2 Fluently add and subtract within 10 **using mental strategies**.
- 2.NBT.5 Fluently add and subtract within 100 **using strategies** based on place value, properties of operations, and/or the relationship between addition and subtraction.
- 2.NBT.6 Add up to four two-digit numbers **using strategies** based on place value and properties of operations.
- 2.NBT.7 Add and subtract within 1000, **using concrete models or drawings and strategies** based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.
- 2.NBT.8 **Mentally add** 10 or 100 to a given number 100-900, and **mentally subtract** 10 or 100 from a given number 100-900.
- 2.NBT.9 **Explain why addition and subtraction strategies work**, using place value and the properties of operations.

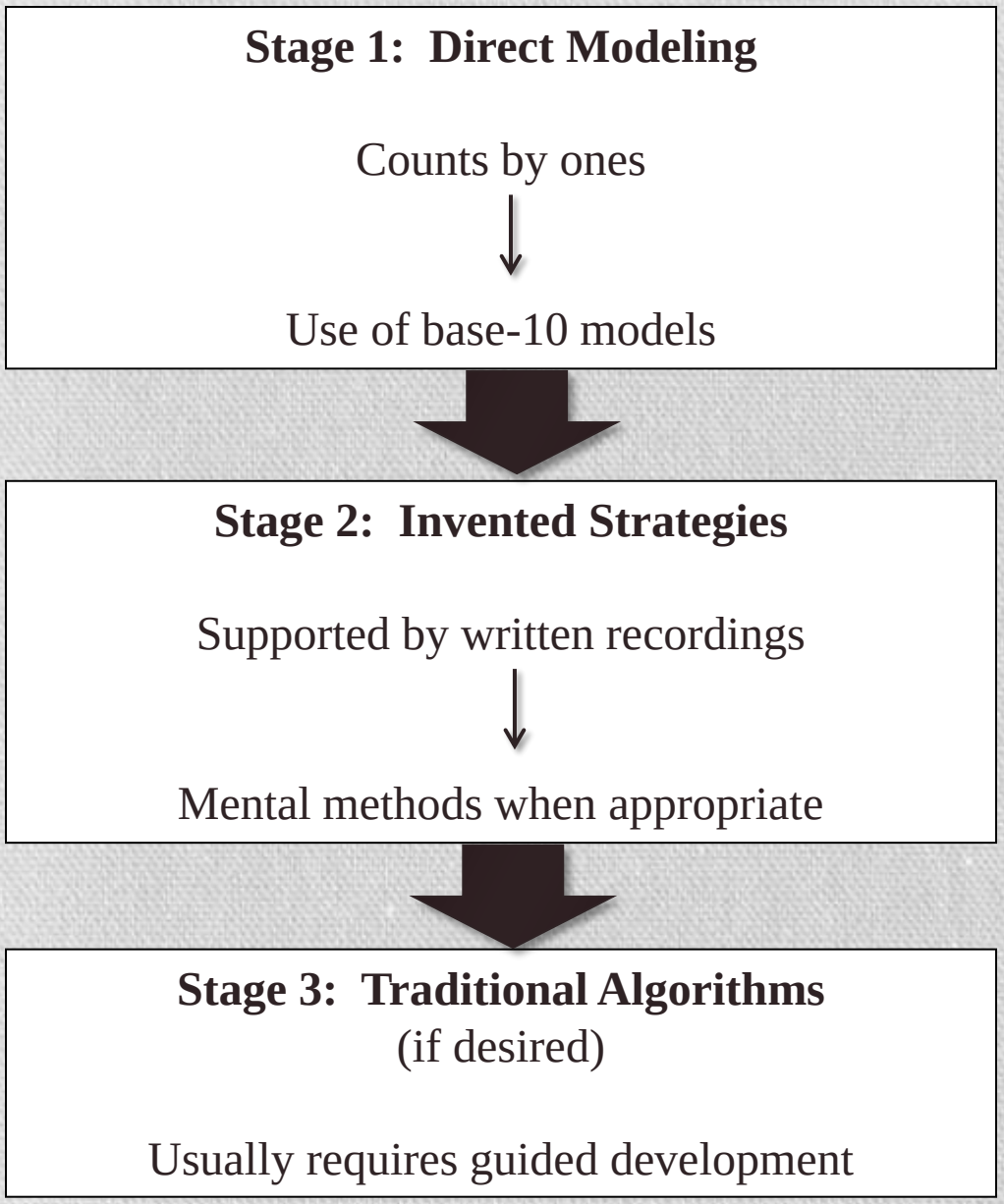
Computation Strategies in the Common Core

Computation Strategy

Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

Computation Algorithm

A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.



Van de Walle & Lovin, 2006, p. 158

Learning Goal #3: Computation Strategies

Use "I can" statements to set learning goals

- I can represent story problems with physical objects, pictures and equations.
- I can draw a picture that shows how I used the math tools to add or subtract without counting by ones.
- I can explain using words how I used the math tools to add or subtract without counting by ones.

Use "essential questions" to set learning goals

- How can we use tiny ten frames and pictures of tiny ten frames to solve problems without counting by ones?
- How can I show my strategy/thinking for adding or subtracting in my picture?
- How can I explain what is happening in my picture using my words?

Learning Goal #3: Computation Strategies

- **Content Standard: 1.OA.6**

  Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

  Use strategies such as:

  - counting on;
  - making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
  - decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9);
  - using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and
  - creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Learning Goal #3: Computation Strategies

Choose / create / adapt problems

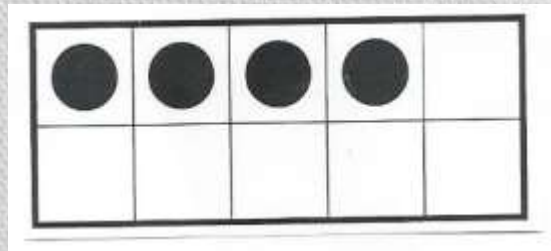
- Contexts that contain easily recognized operations
- Quantities that move students forward to more complex / efficient strategies
- Tools that support students thinking as they move forward to more complex / efficient strategies

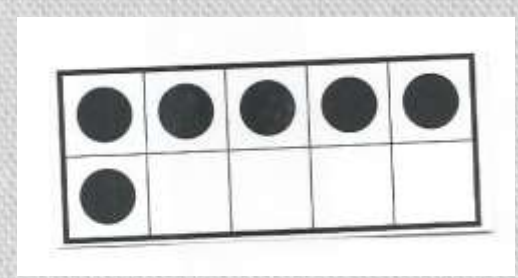
Learning Goal #3: Computation Strategies

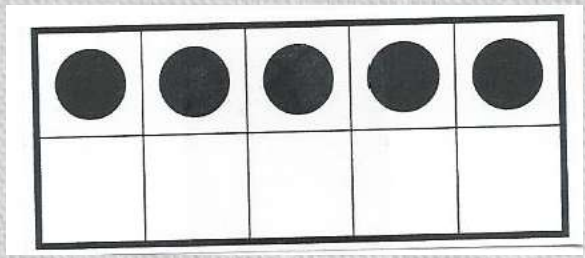
Opening / Warm-Up

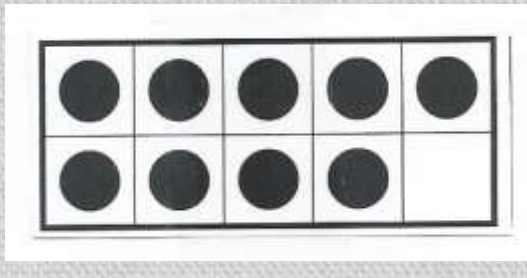
Tens frames flash:

- How many dots do you see?
- How did you see them?









Learning Goal #3: Computation Strategies

Opening / Warm-up

Jack and Jill went up the hill. Jack carried 8 apples in his pail and Jill had 7 apples in her pail.

How can we efficiently show this part of the story with our ten frames?

How will we draw what we did?

Learning Goal #3: Computation Strategies

Opening / Warm Up

Mama Bear found 4 blueberry bushes and Papa Bear found 9 blueberry bushes.

How can we efficiently show this part of the story with our ten frames?

How will we draw what we did?

Learning Goal #3: Computation Strategies

Problem Solving Phase: Students work independently or with partners to develop computation strategies and ways to record their thinking.

Charlotte put 5 cookies on the small tray. She put 7 cookies on the big tray. How many cookies did she have for the party?

Tanya put 8 gummy candies in one cup. She put 4 gummy candies in the other cup. How many gummy candies did she use?

Tony made two snakes from the linking cubes. One snack was 9 cubes long and the other was 5 cubes long. How many cubes did Tony use to make his snakes?

Tom has 2 boxes of crayons. Each box has 8 crayons in it. How many crayons does Tom have?

Learning Goal #3: Computation Strategies

Closure: Focus sharing on learning goals

- How can we use tiny ten frames and pictures of tiny ten frames to solve problems without counting by ones?
- How can I show my strategy/thinking for adding or subtracting in my picture?
- How can I explain what is happening in my picture using my words?

Exit Card

- How would you explain the similarities and differences between the three uses of story problems you learned about today?

- What do you still want to know about using story problems to meet specific learning goals?

References and Resources

- Progressions documents for CCSS:

  ime.math.arizona.edu/progressions

- Van de Walle & Lovin, 2006. *Teaching Student-Centered Mathematics, grades K-3. Pearson: New York.*
- *CCSS, Tables 1 & 2, pp. 88-89*

- *Contact me with questions / comments:*

Lori Williams

dr.loriwilliams@sbcglobal.net

williamsl@mpsd.k12.wi.us